MexiCOPAS

Mexican Cosmology Particles and Strings Schools

Introduction to the Standard Model

MEXICOPAS 2019

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León, Gto., June 2019

Outline

1. Fundamentals
   - Symmetries in Classical and Quantum Mechanics.
   - Irreducible representations (irreps) of $SU(2)$.
   - Irreps of the HLG: Chirality, Parity and Dirac Equation.
   - Quantum Field theory: complex scalar field.
2. Electroweak interactions: Glashow-Weinberg-Salam theory.
   - Minimal coupling principle in classical mechanics.
   - Gauge theories: Abelian and non-Abelian.
   - Quantum Electrodynamics
   - Fermi theory, IVB theory, parity violation and V-A structure of weak interactions.
   - GWS Theory. Spontaneous Breaking of Symmetries.
3. Strong interactions:QCD.
   - Irreducible representations of $SU(3)$
   - Classification of hadrons: Eightfold Way, Quark Model
   - Gauge theory of strong interactions: QCD.
   - Running of couplings: Confinement and asymptotic freedom.
   - Experimental evidence for color degrees of freedom.

Minimal coupling in classical mechanics.

Maxwell equations (c.g.s. units $\alpha = e^2/4\pi\hbar c \simeq 1/137$)

$$\nabla \cdot \boldsymbol{E} = \rho, \qquad \nabla \cdot \boldsymbol{B} = 0,$$

$$\nabla \times \boldsymbol{E} + \frac{1}{c}\frac{\partial \boldsymbol{B}}{\partial t} = 0, \qquad \nabla \times \boldsymbol{B} - \frac{1}{c}\frac{\partial \boldsymbol{E}}{\partial t} = \frac{1}{c}\boldsymbol{j}$$

$$\nabla \cdot \boldsymbol{B} = 0 \qquad \Rightarrow \qquad \boldsymbol{B} = \nabla \times \boldsymbol{A}$$

$$\nabla \times \boldsymbol{E} + \frac{1}{c}\frac{\partial \boldsymbol{B}}{\partial t} = 0 \qquad \Rightarrow \qquad \boldsymbol{E} = -\nabla\phi - \frac{1}{c}\frac{\partial \boldsymbol{A}}{\partial t}$$

$$\nabla \cdot \boldsymbol{E} = \rho \qquad \Rightarrow \qquad \Box\phi - \frac{1}{c}\frac{\partial}{\partial t}\left(\frac{1}{c}\frac{\partial \phi}{\partial t} + \nabla \cdot \boldsymbol{A}\right) = \rho,$$

$$\nabla \times \boldsymbol{B} - \frac{1}{c}\frac{\partial \boldsymbol{E}}{\partial t} = \frac{1}{c}\boldsymbol{j} \qquad \Rightarrow \qquad \Box\boldsymbol{A} - \nabla\left(\frac{1}{c}\frac{\partial \phi}{\partial t} + \nabla \cdot \boldsymbol{A}\right) = \frac{1}{c}\boldsymbol{j}.$$

Remark: ϕ and $\boldsymbol{A}$ are not unique: $\phi' = \phi + \frac{1}{c}\frac{\partial \Lambda}{\partial t}$, $\boldsymbol{A}' = \boldsymbol{A} - \nabla\Lambda$ with arbitrary $\Lambda(\boldsymbol{r}, t)$ yield the same fields $\boldsymbol{E}, \boldsymbol{B}$: Gauge invariance.

Covariant formulation

Define $F^{i0} = E_i = -F^{0i}$, $F^{ij} = -\epsilon_{ijk} B_k = -F^{ji}$, $j^\nu = (c\rho, \boldsymbol{j})$

$$\boldsymbol{\nabla} \cdot \boldsymbol{E} = \rho, \quad \boldsymbol{\nabla} \times \boldsymbol{B} - \frac{1}{c}\frac{\partial \boldsymbol{E}}{\partial t} = \frac{1}{c}\boldsymbol{j} \quad \Leftrightarrow \quad \partial_\mu F^{\mu\nu} = \frac{1}{c} j^\nu,$$

$$\boldsymbol{\nabla} \cdot \boldsymbol{B} = 0, \quad \boldsymbol{\nabla} \times \boldsymbol{E} + \frac{1}{c}\frac{\partial \boldsymbol{B}}{\partial t} = 0 \quad \Leftrightarrow \quad \partial^\rho F^{\mu\nu} + \partial^\mu F^{\nu\rho} + \partial^\nu F^{\rho\mu} = 0.$$

- In terms of the potential four-vector $A^\mu = (\phi, \boldsymbol{A})$

$$\left.\begin{array}{c} \boldsymbol{E} = -\boldsymbol{\nabla}\phi - \frac{1}{c}\frac{\partial \boldsymbol{A}}{\partial t} \\ \boldsymbol{B} = \boldsymbol{\nabla} \times \boldsymbol{A} \end{array}\right\} \quad \Leftrightarrow \quad F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu.$$

- Equations of motion in terms of A^μ:

$$\partial_\mu F^{\mu\nu} = \frac{1}{c} j^\nu \quad \Leftrightarrow \quad \Box A^\nu - \partial^\nu(\partial_\mu A^\mu) = \frac{1}{c} j^\nu$$

- Electromagnetic currents are conserved

$$\partial_\nu \partial_\mu F^{\mu\nu} = 0 \quad \Rightarrow \quad \partial_\nu j^\nu = 0.$$

- Gauge invariance:

$$\phi' = \phi + \frac{1}{c}\frac{\partial \Lambda}{\partial t}, \qquad \boldsymbol{A}' = \boldsymbol{A} - \boldsymbol{\nabla}\Lambda \qquad \Leftrightarrow \qquad A'_\mu = A_\mu + \partial_\mu \Lambda.$$

- F is invariant under this transformation : $(F^{\mu\nu})' = F^{\mu\nu}$.
- The e.o.m. takes its simplest form if $\partial \cdot A = 0$ (Lorentz gauge):

$$\Box A^\nu = j^\nu / c$$

- If A^μ is not in this gauge we work with $A'_\mu = A_\mu + \partial_\mu \Lambda$ with Λ chosen such that $\Box \Lambda = -\partial_\mu A^\mu$.
- Lorentz gauge does not completely removes the gauge freedom. If A_μ satisfies $\partial \cdot A = 0$ so do it $A'_\mu = A_\mu + \partial_\mu f$ whenever f satisfies $\Box f = 0$.
- For A^μ in the class of Lorentz gauges $(\partial \cdot A = 0)$ we can always choose $A_0 = 0$.
- This leave us with the conditions (Coulomb gauge)

$$A_0 = 0, \qquad \boldsymbol{\nabla} \cdot \boldsymbol{A} = 0,$$

- **The electromagnetic field has only two d.o.f.**

Charged particle in an electromagnetic field

- Newton equation + Lorentz force yields

$$m\frac{d^2\boldsymbol{r}}{dt^2} = q\left(\boldsymbol{E} + \frac{\boldsymbol{v}}{c} \times \boldsymbol{B}\right) = q\left(-\boldsymbol{\nabla}\phi - \frac{1}{c}\frac{\partial \boldsymbol{A}}{\partial t} + \frac{\boldsymbol{v}}{c} \times \boldsymbol{\nabla} \times \boldsymbol{A}\right).$$

- But $\boldsymbol{A} = \boldsymbol{A}(\boldsymbol{r}, t)$ thus

$$\frac{d\boldsymbol{A}}{dt} = \frac{\partial \boldsymbol{A}}{\partial t} + (\boldsymbol{v} \cdot \boldsymbol{\nabla})\boldsymbol{A}$$

$$(\boldsymbol{v} \times \boldsymbol{\nabla} \times \boldsymbol{A})_i = \boldsymbol{v} \cdot \partial_i \boldsymbol{A} - (\boldsymbol{v} \cdot \boldsymbol{\nabla})\boldsymbol{A}_i.$$

- The e.o.m. can be rewritten as

$$\frac{d}{dt}(m\dot{x}_i + \frac{q}{c}A_i) = -q\partial_i\phi + \frac{q}{c}\dot{x}_j\partial_i A_j$$

- A suitable Lagrangian for this equation is

$$L(\boldsymbol{r}, \dot{\boldsymbol{r}}, t) = \frac{1}{2}m\dot{\boldsymbol{r}}^2 - q\phi + \frac{q}{c}\dot{\boldsymbol{r}} \cdot \boldsymbol{A}.$$

- The corresponding Hamiltonian is

$$H(\boldsymbol{r},\boldsymbol{p},t) = \boldsymbol{p}\cdot\dot{\boldsymbol{r}} - L(\boldsymbol{r},\dot{\boldsymbol{r}},t)$$

with the canonical conjugate momentum

$$p_i \equiv \frac{\partial L}{\partial \dot{x}_i} = m\dot{x}_i + \frac{q}{c}A_i.$$

Finally

$$H(\boldsymbol{r},\boldsymbol{p},t) = \boldsymbol{p}\cdot\frac{\boldsymbol{p}-\frac{q}{c}\boldsymbol{A}}{m} - \frac{1}{2}m\frac{(\boldsymbol{p}-\frac{q}{c}\boldsymbol{A})^2}{m^2} + q\phi - \frac{q}{c}\frac{\boldsymbol{p}-\frac{q}{c}\boldsymbol{A}}{m}\cdot\boldsymbol{A}$$

$$H - q\phi = \frac{(\boldsymbol{p}-\frac{q}{c}\boldsymbol{A})^2}{2m}$$

Minimal Coupling

The Hamiltonian describing the **classical** dynamics of a particle of charge q in an **external** electromagnetic field is obtained from the free particle description replacing $H \to H - q\phi$, $\boldsymbol{p} \to \boldsymbol{p} - \frac{q}{c}\boldsymbol{A}$, or in covariant notation, $p^\mu \to p^\mu - \frac{q}{c}A^\mu$.

Gauge invariance

- Consider a theory with a global symmetry, e.g.

$$\mathcal{L} = \bar{\psi}(x)[i\gamma^\mu \partial_\mu - m]\psi(x)$$

- Invariant under $\psi \to \psi' = U(\Lambda)\psi \equiv e^{-iq\Lambda}\psi$ with $\Lambda = cte$.
- Notice: homogeneous transformation $\partial_\mu(U(\Lambda)\psi) = U(\Lambda)\partial_\mu\psi$.
- Noether's theorem: there is a conserved charge associated to this symmetry.
- Yang-Mills (1954): global symmetries are not consistent with the concept of localized fields $\Rightarrow$ **Local Symmetries**.

$$\psi \to \psi'(x) = U(\Lambda(x))\psi(x) = e^{-iq\Lambda(x)}\psi(x)$$

- Problem: non-homogeneous transformation

$$\partial_\mu\psi' = U(\Lambda(x))(\partial_\mu - iq\partial_\mu\Lambda)\psi \neq U(\Lambda(x))\partial_\mu\psi.$$

- The theory is not invariant.

- Is there a way to get an homogeneous transformation

$$(D_\mu\psi(x))' = U(\Lambda(x))D_\mu\psi(x)?$$

- Covariance requires $D_\mu = \partial_\mu + B_\mu(x)$

$$\begin{aligned}(D_\mu\psi(x))' = D'_\mu\psi'(x) &= (\partial_\mu + B'_\mu)[U(x)\psi(x)] \\ &= [U(x)\partial_\mu + \partial_\mu U(x) + B'_\mu U(x)]\psi(x) \\ &= U(x)[\partial_\mu + U^{-1}(x)\partial_\mu U(x) + U^{-1}(x)B'_\mu U(x)]\psi(x) \\ &= U(x)(\partial_\mu + B_\mu)\psi(x).\end{aligned}$$

- Homogeneous transformation requires

$$U^{-1}\partial_\mu U + U^{-1}B'_\mu U = B_\mu \Rightarrow B'_\mu = UB_\mu U^{-1} - (\partial_\mu U)U^{-1}$$

- In our case $U = e^{-iq\Lambda(x)}$ is a complex number thus

$$B'_\mu = B_\mu + iq\partial_\mu\Lambda(x)$$

- Defining $B_\mu = iqA_\mu$ we get

$$A'_\mu = A_\mu + \partial_\mu\Lambda.$$

- This is identical to the gauge transformation property of the Maxwell field.
- The covariant derivative $D_\mu = \partial_\mu + iqA_\mu$ is just the minimal coupling recipe (Lorentz force)
- Local gauge invariance principle connects the local phase to two separate properties of the photon field: gauge invariance and minimal coupling.
- Kinetic term for this vector field?. Two possible terms $F^{\mu\nu}F_{\mu\nu}$ and $\epsilon_{\mu\nu\alpha\beta}F^{\mu\nu}F^{\alpha\beta} \equiv F^{\mu\nu}\tilde{F}_{\mu\nu}$.
- But $F^{\mu\nu}\tilde{F}_{\mu\nu}$ violates parity and time reversal!

$$\mathcal{L}_A^{free} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + m_A^2 A^\mu A_\mu.$$

- But the m_A term breaks gauge invariance!

Local gauge invariance uniquely dictates the QED Lagrangian

$$\mathcal{L}_{QED} = \bar{\psi}(x)[i\gamma^\mu(\partial_\mu + iqA_\mu) - m]\psi(x) - \frac{1}{4}F^{\mu\nu}F_{\mu\nu}$$

Similar considerations for the complex scalar field yield the following Lagrangian for scalar electrodynamics

$$\mathcal{L} = (\partial^\mu - iqA^\mu)\phi^*(\partial_\mu + iqA^\mu)\phi - m^2\phi^*\phi - \frac{1}{4}F^{\mu\nu}F_{\mu\nu}$$

Some remarks

1. The charge q plays a double role:
   - It is a measure of the strength of the interaction
   - It is a measure of the size of the local transformation
2. The magnitude of the charge is not fixed by the local transformation. It can be different for different fields.
3. The Lagrangian is still invariant under **global** transformations $U = e^{-i\kappa\theta}$ with constant θ.
4. There is a corresponding conserved charge

$$Q = \kappa(N_a - N_b).$$

κ can be identified with the electric charge or with other conserved quantity: baryonic number, leptonic number etc.

More remarks

1. In QFT fields are operators, they transform as $\psi' = U^\dagger \psi U$.

$$
\begin{aligned}
[Q, \psi(x)] &= \kappa[N^a - N^b, \psi] \\
&= \kappa \sum_\lambda \int d^3p \left(u_{\boldsymbol{p}\lambda} f_p(x)[N^a, a_{\boldsymbol{p}\lambda}] - v_{\boldsymbol{p}\lambda} f_p^*(x)[N^b, b^\dagger_{\boldsymbol{p}\lambda}] \right) \\
&= -\kappa\psi(x).
\end{aligned}
$$

2. Similarly $[Q, \bar{\psi}(x)] = \kappa\bar{\psi}(x)$.
3. Considering $U(\theta) = e^{-iQ\theta}$ and infinitesimal transformations

$$
\begin{aligned}
\psi' &= (1 + iQ\theta)\psi(1 - iQ\theta) = \psi + i\theta[Q, \psi] + \mathcal{O}(\theta^2) \\
&= (1 - i\kappa\theta)\psi \Rightarrow U^\dagger(\theta)\psi U(\theta) = e^{-i\kappa\theta}\psi.
\end{aligned}
$$

4. In QFT the local transformation is done by $U(\theta) = e^{-iQ\theta}$.
5. For $\kappa = q$, it is convenient to normalize $q = eq_f$ $(e > 0)$ and the covariant derivative is $D_\mu = \partial_\mu + ieQA_\mu$, where now $Q = q_f(N^a - N^b)$ and $U(\theta) = e^{-ieQ\theta}$.

- There is a term for every charged fermion. Denoting the fermionic field with the name of the corresponding particle

$$\mathcal{L}_{QED} = \bar{e}(x)[i\gamma^\mu(\partial_\mu + ieQ_e A_\mu) - m_e]e(x)$$
$$+\bar{u}(x)[i\gamma^\mu(\partial_\mu + ieQ_u A_\mu) - m_u]u(x) + ... - \frac{1}{4}F^{\mu\nu}F_{\mu\nu}$$

- More compact notation: Denoting the fermionic field of the kind i as f_i we get

$$\mathcal{L}_{QED} = \sum_i \bar{f}_i(x)[i\gamma^\mu(\partial_\mu + ieQ_i A_\mu) - m_i]f_i(x) - \frac{1}{4}F^{\mu\nu}F_{\mu\nu}.$$

- Split into free particle and interaction terms

$$\mathcal{L}(x) = \sum_f \bar{f}(x)[i\gamma^\mu\partial_\mu - m_f]f(x) - \frac{1}{4}F^{\mu\nu}F_{\mu\nu} - \frac{1}{2\alpha}(\partial\cdot A)^2$$
$$-\sum_f eq_f \bar{f}(x)\gamma^\mu A_\mu(x)f(x)$$

Observables: Cross sections, decay widths

The cross section for the process $1 + 2 \to 3 + 4 + ... + n$ is

$$d\sigma = \frac{(2\pi)^4 \delta^4(p_1 + p_2 - \sum_{i=3}^{n} p_i)|\mathcal{M}_{fi}|^2}{4\sqrt{(p_1 \cdot p_2)^2 - m_1^2 m_2^2}} \frac{d^3 p_3}{(2\pi)^3 2E_3} \frac{d^3 p_4}{(2\pi)^3 2E_4} \cdots \frac{d^3 p_n}{(2\pi)^3 2E_n}$$

The decay width of a particle $1 \to 2 + 3 + ... + n$ is obtained as

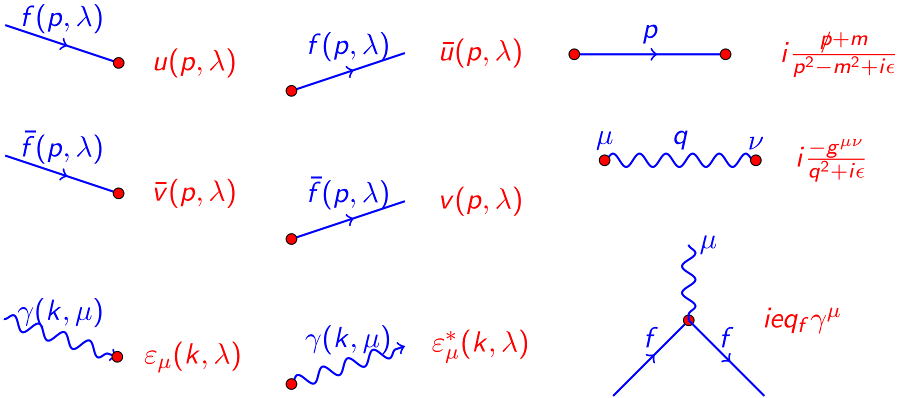
$$d\Gamma = \frac{(2\pi)^4 \delta^4(p_1 - \sum_{i=2}^{n} p_i)|\mathcal{M}_{fi}|^2}{2E_1} \frac{d^3 p_2}{(2\pi)^3 2E_2} \frac{d^3 p_3}{(2\pi)^3 2E_3} \cdots \frac{d^3 p_n}{(2\pi)^3 2E_n}$$

The invariant amplitude $\mathcal{M}_{fi}$ is calculated perturbatively with the Feynman Rules

Feynman Rules

- The invariant amplitude is expanded in powers of the coupling constant
$$\mathcal{M} = \alpha \mathcal{M}_1 + \alpha^2 \mathcal{M}_2 + \ldots$$
where $\alpha = g^2/4\pi << 1$.
- Terms in this series can be mapped to Feynman diagrams.
- Feynman diagrams are constructed from Feynman Rules (FR).
- There is a term in the FR for every factor in the Lagrangian.
- Consider QED of leptons ($f = e^-, \nu_e, \mu^-, \nu_\mu$)
$$\mathcal{L}(x) = \sum_f \bar{f}(x)[i\gamma^\mu \partial_\mu - m_f]f(x) - \frac{1}{4}F^{\mu\nu}F_{\mu\nu} - \frac{1}{2\alpha}(\partial \cdot A)^2$$
$$- \sum_f e q_f \bar{f}(x) \gamma^\mu A_\mu(x) f(x)$$
- Propagators$=i(\textit{kinetic term operator})^{-1}$ in momentum space.
- Vertex$= -i\mathcal{L}_{int}$ in momentum space.

Feynman Rules for Quantum Electrodynamics ($\alpha = 1$)



The contributions to the amplitude for a specific process are given by all the diagrams that can be drawn with the corresponding initial and final states.

Non-abelian gauge invariance

- Take N species of fermion fields $f_j,\ j = 1 \ldots N$

$$\mathcal{L} = \sum_{i=1}^{N} \bar{f}_j [i\gamma^\mu \partial_\mu - m_j] f_j = \bar{f} [i\gamma^\mu \partial_\mu - M_f] f$$

with $M_f = Diag(m_1, m_2, \ldots, m_N)$ and $\bar{f} = (\bar{f}_1, \bar{f}_2, \ldots, \bar{f}_N)$.

- In the case $m_i = m_j = m$, $M_f = m \mathbb{1}_{N \times N}$ and this (free) Lagrangian is invariante under the **global** SU(N)

$$f \to f' = Uf = e^{-iT^a \theta^a} f, \quad U \in SU(N)$$

- What if we assume local SU(N) symmetry: $\theta^a(x)$?
- Fermion fields do not transform homogeneously

$$\partial_\mu f' = \partial_\mu [U(x) f] = (U \partial_\mu + \partial_\mu U) f = U(\partial_\mu + U^{-1} \partial_\mu U) f \neq U(x) \partial_\mu f$$

- Homogeneous covariant derivative: $D_\mu f = (\partial_\mu + igA_\mu)f$:

$$\begin{aligned} D'_\mu f' &= (\partial_\mu + igA'_\mu)U(x)f = U[(\partial_\mu + U^{-1}\partial_\mu U) + igU^{-1}A'_\mu U]f \\ &= UD_\mu f = U(\partial_\mu + igA_\mu)f, \end{aligned}$$

- The vector gauge fields must satisfy

$$U^{-1}\partial_\mu U + igU^{-1}A'_\mu U = igA_\mu$$

- Change under the gauge transformation according to

$$A'_\mu = UA_\mu U^{-1} + \frac{i}{g}(\partial_\mu U)U^{-1}.$$

- Thus A is a $SU(N)$-valued vector field. The simplest realization is

$$A_\mu = T^a A^a_\mu.$$

- Kinetic term for the N vector Fields A^a_μ? Notice

$$D'_\mu f' = UD_\mu f = UD_\mu U^{-1}Uf = UD_\mu U^{-1}f'.$$

- The operator D_μ transforms covariantly $D'_\mu = U D_\mu U^{-1}$.
- Define the field

$$F_{\mu\nu} = \frac{-i}{g}[D_\mu, D_\nu)] = \partial_\mu A_\nu - \partial_\nu A_\mu + ig[A_\mu, A_\nu]$$

it also transforms covariantly

$$F'_{\mu\nu} = U F_{\mu\nu} U^{-1}$$

- $F^{\mu\nu}$ is strictly invariant only for $U(1)$ transformations.
- A gauge invariant kinetic term can be constructed as

$$\mathcal{L}_A = -\frac{1}{2} Tr(F^{\mu\nu} F_{\mu\nu}).$$

- Notice

$$\begin{aligned} F_{\mu\nu} &= \partial_\mu T^a A^a_\nu - \partial_\nu T^a A^a_\mu + ig[T^b A^b_\mu, T^c A^c_\nu] \\ &= T^a(\partial_\mu A^a_\nu - \partial_\nu A^a_\mu - g\ f_{abc} A^b_\mu A^c_\nu) \equiv T^a F^a_{\mu\nu}, \end{aligned}$$

where we used the group algebra $[T^a, T^b] = i\ f_{abc} T^c$ with f_{abc} antisymmetric.

- We use group generators normalized as $Tr(T^a Tb) = \frac{1}{2}\delta^{ab}$.
- With these conventions

$$\mathcal{L}_A = -\frac{1}{2} Tr(F^{\mu\nu} F_{\mu\nu}) = -\frac{1}{4} F^{a\mu\nu} F^a_{\mu\nu}.$$

- Again, a possible mass term for the gauge fields is forbidden by gauge invariance.
- The gauge invariant Lagrangian

$$\mathcal{L} = \bar{f}[(i\gamma^\mu(\partial_\mu \mathbb{1} + igA^a_\mu T^a) - m\mathbb{1}]f - \frac{1}{2} Tr(F^{\mu\nu} F_{\mu\nu})$$

Sumarizing

- Local $SU(N)$ gauge invariance requires the existence of N vector fields A^a_μ
- It does fix the masses of fermion fields up to a global constant $M_f = m\mathbb{1}_{N\times N}$
- Dynamics is dictated by the local gauge symmetry. We have N gauge fields but only one coupling constant g.

Gauge transformations

$$f \to f' = Uf = e^{-iT^a\theta^a(x)}f, \qquad A'_\mu = UA_\mu U^{-1} + \frac{i}{g}(\partial_\mu U)U^{-1}$$

Gauge invariant Lagrangian

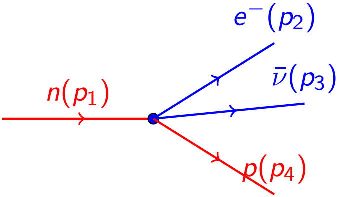
$$\mathcal{L} = \bar{f}[(i\gamma^\mu\partial_\mu - m)\mathbb{1}]f - \frac{1}{2}Tr(F^{\mu\nu}F_{\mu\nu}) - g\bar{f}[\gamma^\mu A^a_\mu T^a]f$$

Weak interactions: Fermi theory

- Nuclei are composed of protons and neutrons.
- Nuclear beta decay related to nucleon decay $n \to pe^- \bar{\nu}$.
- Fermi (1933): Following the current-current electromagnetic structure

$$\mathcal{L}_{int} = \frac{G_V}{\sqrt{2}} \bar{p}\gamma^\mu n \; \bar{e}\gamma_\mu \nu + \text{h.c.}$$

- Neutron beta decay $n(p_1) \to e(p_2)\bar{\nu}(p_3)p(p_4)$ is induced by the following Feynman diagram



A calculation of the decay width yields

$$\Gamma = \frac{G_V^2(m_n - m_p)^5}{30(2\pi)^3}$$

The measured mean time-life of the neutron is $\tau_n = 881{,}5\ seg$, thus the corresponding decay width is

$$\Gamma^n_{exp} = \frac{\hbar}{\tau} = \frac{6{,}582 \times 10^{-25} GeV}{881{,}5} = 7{,}46 \times 10^{-28} GeV$$

Using $m_p = 938{,}272\ MeV$, $m_n = 939{,}565\ MeV$ we get

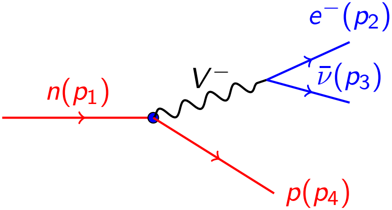
$$G_V = 3{,}9 \times 10^{-5} GeV^{-2}.$$

This crude estimate yields a very small coupling. Interaction is really weak.

1. Other structures posible: $\bar{p}\Gamma^{i}n\ \bar{e}\Gamma_{i}\nu$ with $\Gamma^{i} = 1, \gamma^{5}, \gamma^{\mu}\gamma^{5}, \sigma^{\mu\nu}$.
2. The scalar, vector and axial-vector currents yield similar couplings $G \approx 10^{-5}$.
3. Contact interaction yields a $G_{V}\delta(x)$ potential. Short range interaction.
4. EM interaction involve the exchange of virtual photons. Long range interactions $V(r) = e^{2}/r$ corresponding to a $1/q^{2}$ propagator.
5. The propagator for a massive particle goes like $1/(q^{2} - M^{2})$. The range of the potential is of the order of $1/M$.
6. At low energies ($q^{2} << M^{2}$), it yields a point interaction with coupling $\sim 1/M^{2} << 1$.
7. Is this happening for weak interactions? Can weak interaction be produced by the exchange of a new massive particle? [1]

[1] T.D.Lee, M. Rosenbluth, C.N.Yang; Phys.Rev.75 (1949) 9905.

Weak interactions: Intermediate Vector Boson



- The QED-like interacting lagrangian is

$$\mathcal{L}_{int} = g_V(\bar{p}\gamma^\mu n + \bar{e}\gamma^\mu \nu)V^-_\mu$$

- It yields the following amplitude ($k = (p_1 - p_4)^2$)

$$-i\mathcal{M} = \bar{u}(4)[ig_V\gamma^\mu]u(1)[i\frac{-g_{\mu\nu} + k_\mu k_\nu/m_W^2}{k^2 - M_V^2 + i\epsilon}\bar{u}(2)[ig_V\gamma^\nu]v(3)$$

$$\xrightarrow{k^2<<M_V^2} i\frac{g_V^2}{M_V^2}\bar{u}(4)\gamma^\mu u(1)\bar{u}(2)\gamma_\mu v(3)$$

- This is a Fermi interaction with

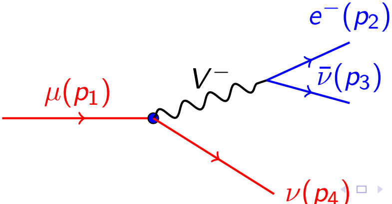
$$\frac{G_V}{\sqrt{2}} = \frac{g_V^2}{M_V^2} \Rightarrow M_V^2 = \frac{\sqrt{2} g_V^2}{G_V}$$

- Assuming QED-like coupling $\frac{g_V^2}{4\pi} \approx 10^{-2}$ we obtain

$$M_V = \sqrt{\frac{\sqrt{2} g_V^2}{G_V}} \approx 67\,\mathrm{GeV}$$

- Promising scheme. Including the muon-neutrino interaction it predicts muon decay

$$\mathcal{L}_{int} = g_V (\bar{p}\gamma^\mu n + \bar{e}\gamma^\mu \nu + \bar{\mu}\gamma^\alpha \nu) V_\alpha^-$$



- The decay width in this case is given by

$$\Gamma = \frac{1}{8(2\pi)^3}\frac{2g_V^4 m_\mu^5}{12M_V^4} = \frac{G_V^2 m_\mu^5}{96(2\pi)^3}.$$

- The measured muon lifetime is $\tau_\mu = 2{,}197 \times 10^{-6} seg$ corresponding to a decay width $\Gamma^\mu_{exp} = 3 \times 10^{-19} GeV$.
- Using $m_\mu = 0{,}105658 GeV$ we get

$$G_V = \sqrt{\frac{96(2\pi)^3\Gamma^\mu_{exp}}{m_\mu^5}} = 2{,}36 \times 10^{-5}\ GeV^{-2}.$$

- **Similar coupling supports the idea but fine tuning is needed.**

The $\tau - \theta$ puzzle

1. The τ and θ mesons were discovered in cosmic ray experiments in 1947.
2. They have identical masses ($m = 493{,}6\ MeV$) and lifetime ($\tau = 1{,}23 \times 10^{-8} seg$) but different decay modes.

$$\theta \to \pi^+\pi^0, \qquad \tau \to \pi^+\pi^-\pi^+$$

3. Dalitz (1953): Parity(2π)$= +1$ and Parity(3π)$= -1$. Identified as different particles.
4. Lee-Yang (1956): Consequences of parity violation in beta decays and other weak processes.
5. Suggest possible experimental tests of parity violation in Co^{60} beta decay, μ beta decay and $\pi \to e\bar{\nu}$.
6. C.S. Wu et. al. (1957): High parity violation in Co^{60} beta decay. Lederman et. al.(1957), Telegdi et.al. (1957): Strong violation of parity in the chain $\pi \to \mu\bar{\nu} \to e\bar{\nu}\nu\bar{\nu}$.
7. $\tau = \theta$ (now named K^+) and parity is strongly violated.

V-A structure of weak interaction

1. Marshak and Sudarshan (1957): available data suggest maximal parity violation for leptons. $V-A$ Lorentz structure.

$$\mathcal{L}_{int} = \frac{G_N}{\sqrt{2}}[\bar{p}\gamma^\mu(1-\frac{g_A}{g_V}\gamma^5)n][\bar{e}\gamma^\mu(1-\gamma^5)\nu] + \frac{G_F}{\sqrt{2}}[\bar{\mu}\gamma^\mu(1-\gamma^5)\nu][\bar{e}\gamma_\mu(1-\gamma^5)\nu] + h.c.$$

2. The decay width in this framework is

$$\Gamma = \frac{1}{8(2\pi)^3}\frac{G_F^2 m_\mu^5}{3}.$$

3. The corresponding Fermi constant from the measured muon lifetime is

$$G_F = \sqrt{\frac{192\pi^3\Gamma^\mu_{exp}}{m_\mu^5}} = 1{,}16\times 10^{-5}\ GeV^{-2}.$$

- Similarly the decay width for the neutron beta decay is given by $(\rho = \frac{g_A}{g_V})$

$$\Gamma = \frac{G_N^2 (m_n - m_p)^5}{15(2\pi)^3}(1 + 3\rho^2).$$

- The Fermi constant extracted from the neutron lifetime satisfy

$$G_N \sqrt{(1 + 3\rho^2)} = \sqrt{\frac{15(2\pi)^3 \Gamma^{\mu}_{exp}}{(M - m)^5}} = 2{,}77 \times 10^{-5} \ GeV^{-2}.$$

- A universal coupling $(G_N = G_F = 1{,}16 \times 10^{-5} \ GeV^{-2})$, requires the axial to vector coupling ratio

$$\rho = \frac{g_A}{g_V} = 1{,}25$$

for the nucleon weak current.

Effective theory for leptons at low energies ($E \leq GeV$)

- Gauge theory of e.m. interactions + effective theory for weak interaction ($\psi_{L/R} \equiv \frac{1}{2}(1 \mp \gamma^5)\psi \equiv \gamma_{L/R}\psi$)

$$\mathcal{L} = \bar{e}[i\gamma^\alpha(\partial_\alpha - ieA_\alpha) - m_e]e + \bar{\nu}[i\gamma^\alpha\partial_\alpha - m_\nu]\nu - \frac{1}{4}F^{\alpha\beta}F_{\alpha\beta}$$

$$+ \frac{g}{\sqrt{2}}(\bar{e}\gamma^\alpha\nu_L)V^-_\alpha + h.c - \frac{1}{4}V^{\alpha\beta}V_{\alpha\beta} + M_V^2 V^\alpha V_\alpha + e \to \mu,$$

- Weak interaction is chiral, involves only the left components

$$\bar{e}\gamma^\alpha\gamma_L\nu = \bar{e}(\gamma_R + \gamma_L)\gamma^\alpha\gamma_L\nu = \bar{e}\gamma_R\gamma^\alpha\gamma_L\nu = \bar{e}_L\gamma^\alpha\nu_L$$

- In terms of the chiral fields, taking $m_\nu = 0$,

$$\mathcal{L} = \bar{e}_L i\gamma^\alpha\partial_\alpha e_L + \bar{e}_R i\gamma^\alpha\partial_\alpha e_R + \bar{\nu}_L i\gamma^\alpha\partial_\alpha\nu_L + \bar{\nu}_R i\gamma^\alpha\partial_\alpha\nu_R$$

$$- ie(\bar{e}_L\gamma^\alpha e_L + \bar{e}_R\gamma^\alpha e_R)A_\alpha - m_e(\bar{e}_R e_L + \bar{e}_L e_R) - \frac{1}{4}F^{\alpha\beta}F_{\alpha\beta}$$

$$+ \frac{g}{\sqrt{2}}(\bar{e}_L\gamma^\alpha\nu_L)V^-_\alpha + h.c. - \frac{1}{4}V^{\alpha\beta}V_{\alpha\beta} + M_V^2 V^\alpha V_\alpha + e \to \mu,$$

- Right components of the fields do not feel weak interaction.
- Both (right and left) components of leptons feel e.m. interaction with the same strength.
- A gauge theory of weak interactions must involve only transformations of the left fields.
- Fermion mass terms would not be invariant.
- Vector bosons - $(\frac{1}{2}, \frac{1}{2})$ irrep of the HLG- are not chiral.
- If we pretend the IVB to be a gauge field, the mass term will be forbidden by gauge invariance.

Weak interactions of leptons: G-W-S Model

- The free particle Lagrangian reads

$$\begin{aligned}\mathcal{L} &= \bar{e}_L i\gamma^\alpha \partial_\alpha e_L + \bar{e}_R i\gamma^\alpha \partial_\alpha e_R + \bar{\nu}_L i\gamma^\alpha \partial_\alpha \nu_L + \bar{\nu}_R i\gamma^\alpha \partial_\alpha \nu_R \\ &= \left(\bar{\nu}_L, \bar{e}_L\right) i\gamma^\alpha \partial_\alpha \begin{pmatrix} \nu_L \\ e_L \end{pmatrix} + \bar{e}_R i\gamma^\alpha \partial_\alpha e_R + \bar{\nu}_R i\gamma^\alpha \partial_\alpha \nu_R \\ &\equiv \bar{L} i\gamma^\mu \partial_\mu L + \bar{e}_R i\gamma^\alpha \partial_\alpha e_R + \bar{\nu}_R i\gamma^\alpha \partial_\alpha \nu_R\end{aligned}$$

- Natural choice $SU(2)$: Left fields as $SU(2)$ doublets, right fields as singlets.

$$L \equiv \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}; \qquad R \equiv e_R, \nu_R$$

  It doesn't work (Glashow (1961)).

- Next minimal choice $SU(2) \otimes U(1)$. Group generators: $\{\frac{\tau}{2}, \frac{Y}{2}\}$ satisfy

$$[\frac{\tau_i}{2}, \frac{\tau_j}{2}] = i\epsilon_{ijk}\frac{\tau_k}{2}, \qquad [\frac{\tau_i}{2}, \frac{Y}{2}] = 0.$$

- Gauging the $SU(2)_L \otimes U(1)_Y$: covariant derivatives

$$\begin{aligned} D_\mu L &= \left(\partial_\mu - ig\frac{\sigma_i}{2}W^i_{\ \mu} - ig'\frac{Y}{2}B_\mu\right)L, \\ D_\mu R_i &= \left(\partial_\mu - ig'\frac{Y}{2}B_\mu\right)R_i. \end{aligned}$$

- Notice: e_L is not a singlet under $U(1)_Y$, i.e. $U(1)_Y \neq U(1)_R$.
- Lagrangian

$$\mathcal{L} = \bar{L}i\gamma^\mu D_\mu L + \sum_{i=1}^{2}\bar{R}_i i\gamma^\mu D_\mu R_i - \frac{1}{4}W^a_{\ \mu\nu}W^{a\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu}$$

Left field terms

$$\bar{L}i\gamma^\mu D_\mu L = (\bar{\nu}_L, \bar{e}_L)\, i\gamma^\mu \begin{pmatrix} \partial_\mu - i\frac{g}{2}W^3_{\ \mu} - ig'\frac{Y_{\nu_L}}{2}B_\mu, & -i\frac{g}{\sqrt{2}}W^+_{\ \mu} \\ -i\frac{g}{\sqrt{2}}W^-_{\ \mu}, & \partial_\mu + i\frac{g}{2}W^3_{\ \mu} - ig'\frac{Y_{e_L}}{2}B_\mu \end{pmatrix} \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}$$

where $W^\pm_{\ \mu} = \frac{1}{\sqrt{2}}(W^1_{\ \mu} \mp iW^2_{\ \mu})$.

$$\bar{L}i\gamma^\mu D_\mu L = (\bar{\nu}_L, \bar{e}_L)\, i\gamma^\mu \begin{pmatrix} (\partial_\mu - i\frac{g}{2}W^3_\mu - ig'\frac{Y_{\nu_L}}{2}B_\mu)\nu_L - i\frac{g}{\sqrt{2}}W^+_\mu e_L \\ -i\frac{g}{\sqrt{2}}W^-_\mu \nu_L + (\partial_\mu + i\frac{g}{2}W^3_\mu - ig'\frac{Y_{e_L}}{2}B_\mu)e_L \end{pmatrix}$$

$$= \bar{\nu}_L i\gamma^\mu\partial_\mu\nu_L + \bar{e}_L i\gamma^\mu\partial_\mu e_L + \bar{\nu}_L\gamma^\mu(\frac{g}{2}W^3_\mu + g'\frac{Y_{\nu_L}}{2}B_\mu)\nu_L$$

$$+\frac{g}{\sqrt{2}}\bar{\nu}_L\gamma^\mu W^+_\mu e_L + \frac{g}{\sqrt{2}}\bar{e}_L\gamma^\mu W^-_\mu \nu_L - \bar{e}_L\gamma^\mu(\frac{g}{2}W^3_\mu - g'\frac{Y_{e_L}}{2}B_\mu)e_L$$

$$\sum_{i=1}^{2}\bar{R}_i i\gamma^\mu D_\mu R_i = \bar{\nu}_R i\gamma^\mu\left(\partial_\mu - ig'\frac{Y_{\nu_R}}{2}B_\mu\right)\nu_R + \bar{e}_R i\gamma^\mu\left(\partial_\mu - ig'\frac{Y_{e_R}}{2}B_\mu\right)e_R$$

$$= \bar{\nu}_R i\gamma^\mu\partial_\mu\nu_R + \bar{e}_R i\gamma^\mu\partial_\mu e_R + \frac{g'}{2}\bar{\nu}_R\gamma^\mu Y_{\nu_R}B_\mu\nu_R + \frac{g'}{2}\bar{e}_R\gamma^\mu Y_{e_R}B_\mu e_R$$

Recovering QED: Define

$$W^3_\mu = \cos\theta Z_\mu + \sin\theta A_\mu$$
$$B_\mu = -\sin\theta Z_\mu + \cos\theta A_\mu$$

$$-\bar{e}_L\gamma^\mu(\frac{g}{2}W_\mu^3 - g'\frac{Y_{e_L}}{2}B_\mu)e_L + \frac{g'}{2}\bar{e}_R\gamma^\mu Y_{e_R}B_\mu e_R$$

$$\begin{aligned}
= &- \bar{e}_L\gamma^\mu(\frac{g}{2}\sin\theta - g'\frac{Y_{e_L}}{2}\cos\theta)A_\mu e_L + \frac{g'}{2}\bar{e}_R\gamma^\mu Y_{e_R}\cos\theta A_\mu e_R \\
&- \bar{e}_L\gamma^\mu(\frac{g}{2}\cos\theta + g'\frac{Y_{e_L}}{2}\sin\theta)Z_\mu e_L - \frac{g'}{2}\bar{e}_R\gamma^\mu Y_{e_R}\sin\theta Z_\mu e_R
\end{aligned}$$

Similarly for the neutrino terms

$$\bar{\nu}_L\gamma^\mu(\frac{g}{2}W_\mu^3 + g'\frac{Y_{\nu_L}}{2}B_\mu)\nu_L + \frac{g'}{2}\bar{\nu}_R\gamma^\mu Y_{\nu_R}B_\mu\nu_R$$

$$\begin{aligned}
= &\bar{\nu}_L\gamma^\mu(\frac{g}{2}\sin\theta + g'\frac{Y_{\nu_L}}{2}\cos\theta)A_\mu\nu_L + \frac{g'}{2}\bar{\nu}_R\gamma^\mu Y_{\nu_R}\cos\theta A_\mu\nu_R \\
&+ \bar{\nu}_L\gamma^\mu(\frac{g}{2}\cos\theta - g'\frac{Y_{\nu_L}}{2}\sin\theta)Z_\mu\nu_L - \frac{g'}{2}\bar{\nu}_R\gamma^\mu Y_{\nu_R}\sin\theta Z_\mu\nu_R
\end{aligned}$$

The measured electric charges of the electron and neutrino require

$$\frac{g}{2}\sin\theta - g'\frac{Y_{e_L}}{2}\cos\theta = e, \qquad -\frac{g'}{2}Y_{e_R}\cos\theta = e$$
$$\frac{g}{2}\sin\theta + g'\frac{Y_{\nu_L}}{2}\cos\theta = 0, \qquad \frac{g'}{2}Y_{\nu_R}\cos\theta = 0$$

- The last equation impose $Y_{\nu_R} = 0$. The remaining Eqs. yield

$$e = \frac{g}{2}\frac{Y_{e_R}}{Y_{\nu_L}}\sin\theta = -\frac{g'}{2}Y_{e_R}\cos\theta, \quad Y_{e_R} = Y_{e_L} + Y_{\nu_L}.$$

- It is conventional to assign $Y_L = -1 \Rightarrow Y_{e_R} = -2$, such that the Nishijima-Gell-Mann relation holds for all particles

$$Q = T^3 + \frac{Y}{2}.$$

- In this case

$$e = g\sin\theta = g'\cos\theta,$$

- **Notice that ν_R is a singlet of $SU(2)_L \otimes U(1)_Y$.**

Weinberg-Salam Model content so far

1. Massless QED

$$\mathcal{L}_{QED} = \bar{e} i \gamma^{\alpha} (\partial_{\alpha} + ieQA_{\alpha}) e + \bar{\nu} i \gamma^{\alpha} (\partial_{\alpha} + ieQA_{\alpha}) \nu - \frac{1}{4} F^{\alpha\beta} F_{\alpha\beta}$$

2. IVB correct interactions

$$\mathcal{L}_{IVB} = \frac{g}{\sqrt{2}} (\bar{e}_L \gamma^{\mu} W_{\mu}^{-} \nu_L + \bar{\nu}_L \gamma^{\mu} W_{\mu}^{+} e_L)$$

3. New interactions with neutral bosons: neutral currents

$$\mathcal{L}_{NC} = -\bar{e}_L \gamma^{\mu} (\frac{g}{2} \cos\theta - \frac{g'}{2} \sin\theta) Z_{\mu} e_L + g' \sin\theta \bar{e}_R \gamma^{\mu} Z_{\mu} e_R$$
$$+ \bar{\nu}_L \gamma^{\mu} (\frac{g}{2} \cos\theta + \frac{g'}{2} \sin\theta) Z_{\mu} \nu_L$$

4. Gauge bosons kinetic terms and (self-) interactions

$$\mathcal{L}_{GB} = -\frac{1}{4} W^{a}_{\ \mu\nu} W^{a\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu}$$

Problems: massless fermions and massless gauge bosons ($W^{\pm}, Z^0$)

1. Fermion mass: forbidden by chiral symmetry (chiral structure of weak interactions).
2. Gauge boson mass: forbidden by gauge symmetry.

- Glashow (1961): $SU(2) \otimes U(1)$ with explicit symmetry breaking by mass terms. Partial symmetry.
- Weinberg (1967), Salam (1967): $SU(2)_L \otimes (1)_Y$ with spontaneous symmetry breaking a la Higgs, following Nambu-Goldstone.

Spontaneous Symmetry Breaking: Nambu

- Let us consider a real scalar free theory

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m^2\phi^2$$

- We have been considering $m^2 > 0$ so far. Mandatory since $E = \sqrt{\boldsymbol{p}^2 + m^2} > 0$ for the particles.
- A QFT with $m^2 < 0$ is not tenable for free particles.
- The vacuum expectation value of this field vanishes

$$\langle 0|\phi|0\rangle = \int \frac{d^3p}{(2\pi)^3 2E_p}[\langle 0|a_{\boldsymbol{p}}|0\rangle e^{-ip.x} + \langle 0|a^\dagger_{\boldsymbol{p}}|0\rangle e^{ip.x}] = 0.$$

- Let us consider now an interacting real scalar field theory whose interactions are described by a potential $V(\phi)$

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi).$$

- Expanding the potential around a minimum ϕ_0

$$V(\phi) = V(\phi_0) + V'(\phi_0)(\phi - \phi_0) + \frac{1}{2}V''(\phi_0)(\phi - \phi_0)^2 + ...$$

- For the interacting theory the Lagrangian can be written as

$$\mathcal{L} = \frac{1}{2}\partial_\mu(\phi - \phi_0)\partial^\mu(\phi - \phi_0) - \frac{1}{2}V''(\phi_0)(\phi - \phi_0)^2 + ...$$

- Conventional interactions have a minimum at $\phi_0 = 0$. In this case the mass of the particle is $m^2 = V''(0)$.
- But the expansion (and the particle content) actually depend on the nature of the potential. Consider e.g.

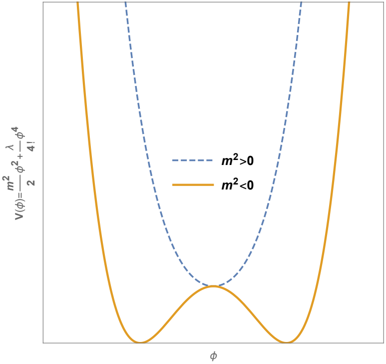
$$V(\phi) = \frac{1}{2}m^2\phi^2 + \frac{\lambda}{4!}\phi^4, \qquad \lambda > 0$$

- The extremals are located at

$$V'(\phi_0) = \phi_0(m^2 + \frac{\lambda}{6}\phi_0^2) = 0$$

- If $m^2 > 0$ the only solution is $\phi_0 = 0$. In this case $V''(\phi_0) = m^2 > 0$ and $\phi_0 = 0$ is indeed a minimum.

- But if $m^2 < 0$ we have another solutions at $\phi_0 = \pm\sqrt{-6m^2/\lambda}$ with $V''(\phi_0) = (m^2 - 3m^2) = -2m^2 > 0$, i.e. it is indeed a minimum. For $\phi_0 = 0$, $V''(\phi_0) = m^2 < 0$,i.e. we have a maximum. QFT expansion must be done around ϕ_0.



- Rewriting the Lagrangian in terms of $\xi = \phi - \phi_0$ we get

$$\begin{aligned}\mathcal{L} &= \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m^2\phi^2 - \frac{\lambda}{4!}\phi^4 \\ &= \frac{1}{2}\partial_\mu\xi\partial^\mu\xi - \frac{1}{2}(-2m^2)\xi^2 - \frac{1}{6}\lambda\phi_0\xi^3 - \frac{\lambda}{4!}\xi^4 - V(\phi_0)\end{aligned}$$

- Notice that $\mathcal{L}(\phi)$ is symmetric under $\phi \to -\phi$.
- Expansion around $\langle 0|\phi|0\rangle = 0$ makes no sense, particles would have $E = \sqrt{\boldsymbol{p}^2 + m^2}$, imaginary for $\boldsymbol{p}^2 < -m^2$,. The Hamiltonian is not Hermitian.
- We must choose a minimum of the potential to quantize the theory. We have two possibilities: $\phi_0 = \pm\sqrt{\frac{-6m^2}{\lambda}}$. The $\phi \to -\phi$ symmetry is broken with this choice (SSB).
- The particles have a mass $-2m^2$ and cubic interactions. The symmetry of the Lagrangian does not show up in the spectrum, it is hidden.

Spontaneous breaking of a continuous global symmetry

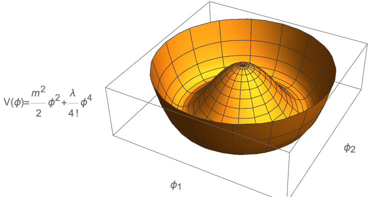
- Let us consider now a continuous symmetry. Take two real scalar fields $\phi = (\phi_1, \phi_2)$ with a similar potential

$$\mathcal{L} = \frac{1}{2}\partial_\mu \phi \cdot \partial^\mu \phi - \frac{m^2}{2}\phi^2 + \frac{\lambda}{4!}\phi^4.$$

- For $m^2 < 0$ now the minimum of the potential satisfy

$$\phi_{10}^2 + \phi_{20}^2 = \frac{-6m^2}{\lambda} \qquad \Rightarrow \qquad |\phi_0| = \sqrt{\frac{-6m^2}{\lambda}} \equiv v.$$

We have a continuously degenerated vacuum. Must choose one of them to quantize the theory.



- The Lagrangian and the vacuum have a $SO(2)$ symmetry.
- Choosing a particular vacuum breaks this symmetry. Take $\phi_0 = (0, v)$ and define $\xi = \phi_2 - v$

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi_1\partial^\mu\phi_1 + \frac{1}{2}\partial_\mu\xi\partial^\mu\xi - \frac{1}{2}(m^2 + \frac{\lambda v^2}{6})\phi_1^2 - \frac{1}{2}(m^2 + \frac{\lambda v^2}{2})\xi^2 + ..$$
$$= \frac{1}{2}\partial_\mu\phi_1\partial^\mu\phi_1 + \frac{1}{2}\partial_\mu\xi\partial^\mu\xi - \frac{1}{2}(-2m^2)\xi^2 + ...$$

- One of the fields turns out to be massive with a mass $-2m^2$.The other field is massless.
- Considering N real fields with the analogous potential yields a $SO(N)$ symmetric Lagrangian.
- The minimal is given also by $|\phi_0| = \sqrt{\frac{-6m^2}{\lambda}}$. The vacuum is $SO(N)$ symmetric.
- Choosing $\phi_0 = (0, 0, .., 0, v)$ breaks this symmetry down to $SO(N-1)$.
- We get $N-1$ massless fields (Goldstone bosons) and a massive field with mass $-2m^2$.

Goldstone Theorem

- Consider a general QFT with interacting scalar fields ϕ. Assume a potential $V(\phi)$ with a symmetry group G whose generators are L^a, $a = 1, 2, .., N$.
- The vacuum configurations are obtained from $\left.\frac{\delta V}{\delta \phi}\right|_{\phi_0 = \boldsymbol{v}} = 0$. There will be $M < N$ generators leaving the vacuum invariant

$$L^a{}_{ij} v_j = 0, \qquad a = 1, 2, ..., M$$

These generators span a subgroup $H \subset G$. The remaining $N - M$ generators satisfy $L^a{}_{ij} v_j \neq 0$.
- Under an infinitesimal group transformation:

$$\delta \phi_i = -i \theta^a L^a{}_{ij} \phi_j$$

the potential is invariant thus

$$-i \frac{\delta V}{\delta \phi_i} \theta^a L^a{}_{ij} \phi_j = 0.$$

- Taking a second variation we get

$$\frac{\delta^2 V}{\delta\phi_i\delta\phi_k} L^a{}_{ij}\phi_j + \frac{\delta V}{\delta\phi_i} L^a{}_{ik} = 0.$$

- For the vacuum configuration the last term vanishes

$$\left.\frac{\delta^2 V}{\delta\phi_i\delta\phi_k} L^a{}_{ij}\phi_j\right|_{\phi_0=\mathbf{v}} = 0.$$

- Expanding the potential around the vacuum $\boldsymbol{\phi_0}$

$$V(\phi) = V(\boldsymbol{\phi_0}) + \frac{1}{2}(M^2)_{ij}(\phi_i - v_i)(\phi_j - v_j) + \dots$$

  thus

$$(M^2)_{ik} L^a{}_{ij} v_j = 0, \qquad a = 1, 2, \dots.$$

- For $a = 1, \dots M$ we have $L^a{}_{ij} v_j = 0$ and this condition is satisfied.
- For $a = M + 1, \dots, N$ we know that $L^a{}_{ij} v_j \neq 0$. The matrix M^2_{ij} must have $N - M$ vanishing eigenvalues.
- **There are $N - M$ massless modes: Nambu-Goldstone.**

Spontaneous Breaking of Gauge Symmetries: Higgs

- Let us consider escalar QED

$$\mathcal{L} = D_\mu \phi^* D^\mu \phi - m^2 \phi^* \phi - \lambda (\phi^* \phi)^2 - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

- The Lagrangian is invariant under $U(1)$ local transformations

$$\phi \to e^{-i\theta(x)} \phi, \qquad A_\mu \to A_\mu - \frac{1}{e} \partial_\mu \theta(x).$$

- If $m^2 < 0$, the minimum is given by $|\phi_0| = \sqrt{-\frac{m^2}{2\lambda}} \equiv \frac{v}{\sqrt{2}}$.
- Convenient to write the field as $\phi(x) = \rho(x) e^{-i\xi(x)}$. A gauge transformation allows us to do this. The minimum are given by $\phi_0(x) = \frac{v}{\sqrt{2}} e^{-i\xi(x)}$.
- Redefine the field: use

$$\phi = \frac{e^{i\xi/v}}{\sqrt{2}} (\sigma + v)$$

- Then perform a gauge transformation

$$\phi \to \phi' = e^{-i\xi/v}\phi = \frac{1}{\sqrt{2}}(\sigma + v), \qquad A_\mu \to A'_\mu = A_\mu - \frac{1}{ev}\partial_\mu \xi,$$

to obtain the following Lagrangian

$$\begin{aligned} \mathcal{L} = & - \frac{1}{4}F'_{\mu\nu}F'^{\mu\nu} + \frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma + \frac{1}{2}e^2v^2A'_\mu A'^\mu \\ & + \frac{1}{2}e^2A'_\mu A'^\mu\sigma(2v+\sigma) - \frac{1}{2}\sigma^2(3\lambda v^2 + m^2) \\ & - \lambda v\sigma^3 - \frac{1}{4}\lambda\sigma^4 \end{aligned}$$

- The gauge field acquires a mass $m_A^2 = e^2v^2/2$.
- The ξ field "disappears" from the spectrum. This d.o.f reappears as the longitudinal mode of the gauge field.

SSB of non-Abelian gauge symmetries: $SU(2)_I$

- Consider now a complex scalar doublet $\Phi = \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix}$ of $SU(2)_I$.

$$\mathcal{L} = (D_\mu \Phi)^\dagger D^\mu \Phi - m^2 \Phi^\dagger \Phi - \lambda(\Phi^\dagger \Phi)^2 - \frac{1}{4} W^a_{\ \mu\nu} W^{a\mu\nu}$$

with $D^\mu \Phi = (\mathbb{1}\partial^\mu + igT^a W^a_\mu)\Phi$, is invariant under

$$\Phi \to \Phi' \equiv U\Phi = e^{-iT^a\theta^a}\Phi, \quad W_\mu \to W'_\mu = UW_\mu U^{-1} + \frac{i}{g}(\partial_\mu U)U^{-1}.$$

- If $m^2 < 0$, the minimum of the potential satisfies $|\Phi_0| = v$ with $v = \sqrt{\frac{-m^2}{2\lambda}}$. Parametrize the doublet as

$$\Phi = e^{\frac{i}{v}\boldsymbol{\xi}\cdot\boldsymbol{T}} \begin{pmatrix} 0 \\ S \end{pmatrix} \equiv e^{\frac{i}{v}\boldsymbol{\xi}\cdot\boldsymbol{T}} \tilde{\Phi}.$$

where $\{\xi_1, \xi_2, \xi_3, S\}$ are real fields.

- Choose the vacuum such that only S acquires v.e.v. and shift $S = (\sigma + v)$.

- Now perform a gauge transformation with $U = e^{-\frac{i}{v}\boldsymbol{\xi}\cdot\boldsymbol{T}}$. The resulting Lagrangian is

$$\mathcal{L} = (D'_\mu\tilde{\Phi})^\dagger D'\mu\tilde{\Phi} - m^2\tilde{\Phi}^\dagger\tilde{\Phi} - \lambda(\tilde{\Phi}^\dagger\tilde{\Phi})^2 - \frac{1}{4}W'^a{}_{\mu\nu}W'^{a\mu\nu}$$

with $D'\mu\tilde{\Phi} = (\partial^\mu + igW'^\mu)\tilde{\Phi}$.

- The mass terms for the gauge bosons are

$$\mathcal{L}_{GBM} = (igW'^\mu\tilde{\Phi}_0)^\dagger igW'^\mu\tilde{\Phi}_0$$

where $\tilde{\Phi}_0 = \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix}$. A straightforward calculation yields

$$\mathcal{L}_{GBM} = \frac{g^2v^2}{2}(W^1{}_\mu W^{1\mu} + W^2{}_\mu W^{2\mu} + W^3{}_\mu W^{3\mu}).$$

- Similarly, the scalar field σ has a mass $m_\sigma = -2m^2$.

Sumarizing SSB of $SU(2)$ gauge symmetry:

1. We started with 3 massless gauge fields and a complex doublet containing four real scalar fields.
2. The gauge symmetry is $SU(2)$, there are three generators T^a.
3. The choice of the vacuum completely breaks down this symmetry $T^a\tilde{\Phi}_0 \neq 0$. There are three Nambu-Goldstone bosons: ξ_1, ξ_2, ξ_3, and three massless gauge fields. The particle content is not obvious.
4. A gauge transformation clearly shows the particle content. The NGB convert into the longitudinal modes of the three gauge fields which this way become massive.

Weinberg-Salam: SSB of $SU(2)_L \otimes U(1)_Y$

- We add the complex scalar doublet to the previous Lagrangian

$$\mathcal{L} = \bar{L} i\gamma^\mu D_\mu L + \sum_{i=1}^{2} \bar{R}_i i\gamma^\mu D_\mu R_i - \frac{1}{4} W^a{}_{\mu\nu} W^{a\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu}$$
$$+ (D_\mu \Phi)^\dagger D^\mu \Phi - m^2 \Phi^\dagger \Phi - \lambda (\Phi^\dagger \Phi)^2,$$

with $m^2 < 0$ and the $SU(2)_L \otimes U(1)_Y$ covariant derivatives

$$D_\mu \Phi = \left(\partial_\mu - ig \frac{\sigma_i}{2} W^i_\mu - ig' \frac{Y}{2} B_\mu \right) \Phi,$$

- We parametrize the complex scalar doublet as

$$\Phi = \exp\left(-\frac{i}{2v} \boldsymbol{\xi} \cdot \boldsymbol{\sigma}\right) \tilde{\Phi}, \qquad \tilde{\Phi} = \begin{pmatrix} 0 \\ \frac{1}{\sqrt{2}}(v + H) \end{pmatrix}.$$

- Now perform a gauge transformation with $U = e^{-\frac{i}{2v} \boldsymbol{\xi} \cdot \boldsymbol{\sigma}}$, eliminating the ξ_a fields from the Lagrangian.

- The resulting Lagrangian is

$$\mathcal{L} = \bar{L} i\gamma^\mu D'_\mu L + \sum_{i=1}^{2} \bar{R}_i i\gamma^\mu D'_\mu R_i - \frac{1}{4} W'^a_{\ \mu\nu} W'^{a\mu\nu} - \frac{1}{4} B'_{\mu\nu} B'^{\mu\nu}$$
$$+ (D_\mu \tilde{\Phi})^\dagger D^\mu \tilde{\Phi} - m^2 \tilde{\Phi}^\dagger \tilde{\Phi} - \lambda(\tilde{\Phi}^\dagger \tilde{\Phi})^2.$$

where

$$\begin{aligned} D'_\mu L &= \left(\partial_\mu - ig\frac{\sigma_a}{2} W'^a_{\ \mu} - ig'\frac{Y}{2} B'_\mu\right) L, \\ D'_\mu R_i &= \left(\partial_\mu - ig'\frac{Y}{2} B'_\mu\right) R_i \\ D'_\mu \tilde{\Phi} &= \left(\partial_\mu - ig\frac{\sigma_a}{2} W'^a_{\ \mu} - ig'\frac{Y}{2} B'_\mu\right) \tilde{\Phi}. \end{aligned}$$

The $|D'_\mu \tilde{\Phi}|^2$ term reads (we skip the $'$ label for gauge bosons) :

$$\left| \begin{pmatrix} \partial^\mu - \frac{i}{2}(gW^3_\mu + g'YB_\mu) & -\frac{i}{\sqrt{2}} gW^+_\mu \\ -\frac{i}{\sqrt{2}} gW^-_\mu & \partial^\mu - \frac{i}{2}(-gW^3_\mu + g'YB_\mu) \end{pmatrix} \begin{pmatrix} 0 \\ \frac{v+H}{\sqrt{2}} \end{pmatrix} \right|^2$$

A straightforward calculation yields

$$|D_\mu\tilde{\Phi}|^2 = \frac{g^2}{4}W_\mu^+ W^{-\mu}(v+H)^2 + \frac{1}{2}|\partial^\mu H - \frac{i}{2}(-gW_\mu^3 + g' Y_H B_\mu)(v+H$$

$$= \frac{1}{2}\partial^\mu H \partial_\mu H + \frac{v^2}{8}(gW_\mu^3 - g' Y_H B_\mu)^2 + \frac{v^2 g^2}{4}W_\mu^+ W^{-\mu} + \dots$$

From the previously considered mixing we get

$$gW_\mu^3 - g' Y_H B_\mu = (g\cos\theta + g' Y_H \sin\theta)Z_\mu + (g\sin\theta - g' Y_H\cos\theta)A_\mu$$
$$= (g\cos\theta + g' Y_H \sin\theta)Z_\mu - g'\cos\theta(Y_H - 1)A_\mu,$$

where we used $g\sin\theta = g'\cos\theta$ in the last row.
If we choose $Y_H = 1$, the photon remain massless. It can be easily shown that

$$\cos\theta = \frac{g}{\sqrt{g^2+g'^2}}, \qquad \sin\theta = \frac{g'}{\sqrt{g^2+g'^2}},$$

$$g\cos\theta + g'\sin\theta = \sqrt{g^2+g'^2} = \frac{g}{\cos\theta}.$$

- Finally

$$|D_\mu \tilde{\Phi}|^2 = \frac{1}{2}\partial^\mu H \partial_\mu H + \frac{g^2 v^2}{8\cos^2\theta} Z^\mu Z_\mu + \frac{v^2 g^2}{4} W_\mu^+ W^{-\mu} + \ldots$$

and the masses of the gauge bosons are given by

$$m_W^2 = \frac{g^2 v^2}{4}, \qquad m_Z^2 = \frac{m_W^2}{\cos^2\theta}.$$

- The massless mode is

$$A_\mu = \frac{g' B_\mu + g W_\mu^3}{\sqrt{g^2 + g'^2}}$$

- The Fermi coupling, already fixed from muon decay is

$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8 m_W^2} = \frac{1}{2v^2} \Rightarrow v = \sqrt{\frac{1}{\sqrt{2}G_F}} = 246 \ GeV$$

Fermion masses

- The problem of the gauge boson masses solved by the spontaneous breaking of gauge symmetries.
- We can trace the origin of the mass term to the interactions of the Higgs doublet with gauges bosons introduced by the covariant derivatives.
- Still remain the problem of fermion masses. Forbidden by chiral symmetry.
- So far, we have no interactions of the Higgs doublet with fermions.
- The only constraint for these interactions is $SU(2)_L \otimes U(1)_Y$ gauge symmetry and renormalizability.
- The lowest dimension product of L and Φ invariant under $SU(2)_L$ is $\bar{L}\Phi$ (and its h.c. $\Phi^\dagger L$).

- Since e_R is an $SU(2)_L$ singlet, the interaction

$$V_{int} = g_e \bar{L} \Phi e_R + h.c. = g_e(\bar{\nu}_L \phi^+ e_R + \bar{e}_L \phi^0 e_R) + h.c.$$

  is $SU(2)_L$ invariant.
- The $U(1)_Y$ quantum numbers are additive and $Y_{\bar{f}} = -Y_f$. This term is also $U(1)_Y$ invariant.
- After SSB this term yields

$$V_{int} = g_e(\bar{L}\tilde{\Phi} e_R + \bar{e}_R \tilde{\Phi}^\dagger L) = \frac{g_e}{\sqrt{2}}(\bar{e}_L e_R + \bar{e}_R e_L)(v + H)$$

- After SSB the fermion acquire a mass $m_e = \frac{g_e v}{\sqrt{2}}$.

Dirac Neutrino masses

- Right neutrinos have $Y_{\nu_R} = 0$ and are also $SU(2)_L$ singlets. Not required at all by the gauge symmetry.
- We know that for $SU(2)$, the $\bar{\mathbf{2}}$ irrep is equivalent to the $\mathbf{2}$.
- The field $\hat{\Phi} = i\sigma_2 \Phi^*$ transforms like Φ.
- Involves the conjugated scalar fields, has the opposite $U(1)_Y$ quantum numbers
- The lowest dimension gauge invariant interaction is

$$V_{int} = g_\nu \bar{L} \hat{\Phi} \nu_R + h.c.$$

- After SSB this term yields

$$V_{int} = g_\nu (\bar{L} \tilde{\Phi} \nu_R + \bar{\nu_R} \tilde{\Phi}^\dagger L) = \frac{g_\nu}{\sqrt{2}} (\bar{\nu_L} \nu_R + \bar{\nu_R} \nu_L)(v + H)$$

- After SSB the neutrino acquires a Dirac mass $m_\nu = \frac{g_\nu v}{\sqrt{2}}$. Measured masses $m_\nu < 1eV$.
- Right neutrino is a gauge singlet. Majorana masses $\bar{\nu_R} \nu_R$ are possible.

Summary: $SU(2)_L \otimes U(1)_Y \to U(1)_{em}$.

The complete one-generation leptons GWS Lagrangian is

$$\mathcal{L} = \bar{L} i\gamma^\mu D_\mu L + \bar{e_R} i\gamma^\mu D_\mu e_R + \bar{\nu_R} i\gamma^\mu D_\mu \nu_R - \frac{1}{4} W^a_{\ \mu\nu} W^{a\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu}$$
$$+ (D_\mu \tilde{\Phi})^\dagger D^\mu \tilde{\Phi} - m^2 \tilde{\Phi}^\dagger \tilde{\Phi} - \lambda (\tilde{\Phi}^\dagger \tilde{\Phi})^2 - (g_e \bar{L} \tilde{\Phi} e_R + g_\nu \bar{L} \hat{\Phi} \nu_R + h.c.)$$

where

$$D_\mu L = \left(\partial_\mu - ig \frac{\sigma_a}{2} W^a_{\ \mu} - ig' \frac{Y}{2} B_\mu \right) L, \quad D_\mu R_i = \left(\partial_\mu - ig' \frac{Y}{2} B_\mu \right) R_i$$

$$D_\mu \tilde{\Phi} = \left(\partial_\mu - ig \frac{\sigma_a}{2} W^a_{\ \mu} - ig' \frac{Y}{2} B_\mu \right) \tilde{\Phi}, \quad \tilde{\Phi} = \begin{pmatrix} 0 \\ \frac{v+H}{\sqrt{2}} \end{pmatrix}, \quad \hat{\Phi} = i\sigma^2 \tilde{\Phi}^*.$$

- Neutrinos discovered in 1956 (Cowan-Reines). There are two types neutrinos : ν_e and ν_μ (Lederman-Schwartz-Steinberger Brookhaven 1962).
- τ lepton discovered in 1975 (Perl et.al. SLAC) and there is an associated neutrino, ν_τ (DONUT Coll. FERMILAB, 2000).

- These lepton have same electroweak interactions. There is a replication of families with exactly the same terms

$$\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L, \qquad \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L, \qquad \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L.$$

- GWS predictions: i) Neutral currents; ii) Gauge bosons self-interactions, iii) Higgs. Weak coupling related to the e.m. coupling through the weak mixing angle θ.
- Except for $v = 246\ GeV$ no new information at the time, but plethora of new physics for experimentalists.
- Unknown parameters: $\sin\theta, \lambda$. The Yukawa couplings are given by the fermion masses, e.g. $g_e = \sqrt{2} m_e / v$.

Neutral Currents

- New neutral currents due to the exchange of the Z^0 boson

$$\mathcal{L}_{NC} = [g_L^e \bar{e}_L \gamma^\mu e_L + g_R^e \bar{e}_R \gamma^\mu e_R + g_L^\nu \bar{\nu}_{eL} \gamma^\mu \nu_{eL}] Z_\mu \quad + \mu \quad + \tau$$
$$= [\bar{e}\gamma^\mu (g_V^e - g_A^e \gamma^5) e + \bar{\nu}\gamma^\mu (g_V^\nu - g_A^\nu \gamma^5)\nu] Z_\mu \quad + \mu \quad + \tau$$

with

$$g_L^f = \frac{g}{\cos\theta}\left[T_3^f - Q_f \sin^2\theta\right], \qquad g_R^f = \frac{g}{\cos\theta} Q_f sin^2\theta,$$
$$g_V^f = \frac{g}{2\cos\theta} T_3^f, \qquad g_A^f = \frac{g}{2\cos\theta}\left[T_3^f - 2Q_f \sin^2\theta\right]$$

- NC discovered at LEP in 1973 in the $\nu_\mu e^- \to \nu_\mu e^-$ process: $sin^2\theta \approx 0{,}2$.
- With this value:

$$M_W = \sqrt{\frac{\pi\alpha_{em}}{\sqrt{2}G_F \sin^2\theta}} \approx 80 GeV, \qquad \Rightarrow \qquad M_Z \approx 90 GeV.$$

- The W^+ boson was discovered at LEP in 1983 with a mass $M_W = 80 GeV$.
- The Z^0 boson was discovered at LEP in 1983 with a mas $M_Z = 91 GeV$.
- The Higgs boson mass is: $M_H^2 = -2m^2 = 2\lambda v^2$. Free parameter, there are no direct predictions for this observable.
- However, perturbative coupling requires $\lambda << 1$. A light Higgs boson is expected.
- It enters many observables and gradually the mass was bounded from electroweak precision measurements.
- Finally it was discovered at CERN in 2012 with a mass $M_H = 125 GeV$.
- With this value $\lambda = \frac{M_H^2}{2v^2} = 0{,}125$.

Weak interactions of hadrons: beyond beta decay.

- Problem: Eightfold Way currents yield inconsistent predictions for $K^+ \to \mu^+\nu_\mu$ and $\pi^+ \to \mu^+\nu_\mu$ decays

$$\tau_{K^+} = 1{,}2 \times 10^{-8} seg \quad \Rightarrow \quad \Gamma(K^+) = 5{,}3 \times 10^{-14} MeV,$$
$$\tau_{\pi^+} = 2{,}6 \times 10^{-8} seg \quad \Rightarrow \quad \Gamma(\pi^+) = 2{,}5 \times 10^{-14} MeV,$$

$$BR(K^+ \to \mu^+\nu_\mu) = 0{,}63 \quad \Rightarrow \quad \Gamma(K^+ \to \mu^+\nu_\mu) = 3{,}3 \times 10^{-14} MeV$$
$$BR(\pi^+ \to \mu^+\nu_\mu) = 0{,}99 \quad \Rightarrow \quad \Gamma(\pi^+ \to \mu^+\nu_\mu) = 2{,}5 \times 10^{-14} MeV$$

- The ratio of these decay widths is

$$\frac{\Gamma(K^+ \to \mu^+\nu_\mu)}{\Gamma(\pi^+ \to \mu^+\nu_\mu)}\Big|_{exp} = 1{,}3.$$

- Current-current interaction for $K^+(Q) \to \mu^+(p_1)\nu_\mu(p_2)$ yields

$$-i\mathcal{M}_K = \langle K^+|J_\mu|0\rangle \bar{u}(p_2)\gamma^\mu(1-\gamma^5)v(p_1) = g_K Q_\mu \bar{u}(p_2)\gamma^\mu(1-\gamma^5)v(p_1)$$

$$|\mathcal{M}_K|^2 = 4g_K^2 m_\mu^2 (M_K^2 - m_\mu^2), \qquad \Rightarrow \qquad \Gamma_K = \frac{g_K^2 m_\mu^2}{4\pi}\frac{(M_K^2 - m_\mu^2)^2}{M_K^3}$$

- Analogous results for the pion decay. The ratio

$$\frac{\Gamma(K^+ \to \mu^+\nu_\mu)}{\Gamma(\pi^+ \to \mu^+\nu_\mu)} = \frac{g_K^2 m_\pi^3 (M_K^2 - m_\mu^2)^2}{g_\pi^2 M_K^3 (m_\pi^2 - m_\mu^2)^2} \approx \frac{g_K^2}{g_\pi^2} \times 17{,}6.$$

- SU(3) symmetry yields $g_K \approx g_\pi$ and we obtain a huge ratio compared to the experimental result.
- Nicola Cabibbo (1963): Consistent results are obtained for weak decays of hadrons if we assume i) Eightfold way (conserved vector currents) + axial currents ; ii) V-A structure for leptons ; iii) rotated currents

$$J_\mu = cos\theta_c(V_\mu^{\Delta S=0} - A_\mu^{\Delta S=0}) + \sin\theta_c(V_\mu^{\Delta S=1} - A_\mu^{\Delta S=1})$$

- This amounts to $g_K \to g_s \sin\theta_c$, $g_\pi \to g_s \cos\theta_c$ in the previous calculation. The measured ratio yields

$$\frac{\Gamma(K^+ \to \mu^+\nu_\mu)}{\Gamma(\pi^+ \to \mu^+\nu_\mu)} = \tan^2\theta_c \frac{m_\pi^3(M_K^2 - m_\mu^2)}{M_K^3(m_\pi^2 - m_\mu^2)} = 1{,}32 \Rightarrow \sin\theta_c \approx 0{,}20$$

Weak interactions of quarks

- Many other observables involving weak decays of hadrons consistent with this hypothesis. Weak interactions do not respect $SU(3)$ symmetry.
- Today we understand weak interactions of hadrons in terms of weak interactions of quarks.
- After deep inelastic scattering experiments (early 70's) the existence of quarks was firmly stablished, hadrons composed of u, d, s were known.
- The quark doublet of the weak interaction is

$$q_L = \begin{pmatrix} u' \\ d' \end{pmatrix}_L$$

  Right fields u'_R, d'_R, s'_R are $SU(2)_L$ singlets.
- We use the dash to remark that these are quark states with well defined transformation properties under $SU(2)_L \otimes U(1)_Y$.
- Do not necessarily coincide with the pure states $\mathbf{3}, \bar{\mathbf{3}}$ of $SU(3)$.

- In the next slides we skip the dash to simplify notation. Will come back to this point when we discuss mass terms.
- The $U(1)_Y$ quantum numbers can be obtained from the Gell-Mann-Nishijima relation $Q = T_3 + \frac{Y}{2}$

$$Y(u_L) = \frac{1}{3}, \quad Y(u_R) = \frac{4}{3}, \quad Y(d_L) = \frac{1}{3}, \quad Y(d_R) = -\frac{2}{3}.$$

The additional terms in the GWS Lagrangian are

$$\begin{aligned}\mathcal{L} =& \bar{q_L} i\gamma^\mu D_\mu q_L + \bar{u_R} i\gamma^\mu D_\mu u_R + \bar{d_R} i\gamma^\mu D_\mu d_R \\ & - (g_u \bar{q_L} \hat{\Phi} u_R + g_d \bar{q_L} \tilde{\Phi} d_R + h.c.)\end{aligned}$$

where

$$D_\mu q_L = \left(\partial_\mu - ig\frac{\sigma_a}{2} W^a{}_\mu - ig'\frac{Y}{2} B_\mu\right) q_L, \quad D_\mu q_{R_i} = \left(\partial_\mu - ig'\frac{Y}{2} B_\mu\right) q_{R_i}$$

$$\tilde{\Phi} = \begin{pmatrix} 0 \\ \frac{v+H}{\sqrt{2}} \end{pmatrix}, \quad \hat{\Phi} = i\sigma^2 \tilde{\Phi}^* = \begin{pmatrix} \frac{v+H}{\sqrt{2}} \\ 0 \end{pmatrix}.$$

Left field terms

$$\bar{L}i\gamma^\mu D_\mu L = (\bar{u}_L, \bar{d}_L)\, i\gamma^\mu \begin{pmatrix} \partial_\mu - i\frac{g}{2}W^3{}_\mu - ig'\frac{Y_{u_L}}{2}B_\mu, -i\frac{g}{\sqrt{2}}W^+{}_\mu \\ -i\frac{g}{\sqrt{2}}W^-{}_\mu, \partial_\mu + i\frac{g}{2}W^3{}_\mu - ig'\frac{Y_{d_L}}{2}B_\mu \end{pmatrix} \begin{pmatrix} u_L \\ d_L \end{pmatrix}$$

$$= (\bar{u}_L, \bar{d}_L)\, i\gamma^\mu \begin{pmatrix} (\partial_\mu - i\frac{g}{2}W^3_\mu - ig'\frac{Y_{u_L}}{2}B_\mu)u_L - i\frac{g}{\sqrt{2}}W^+_\mu d_L \\ -i\frac{g}{\sqrt{2}}W^-_\mu u_L + (\partial_\mu + i\frac{g}{2}W^3_\mu - ig'\frac{Y_{d_L}}{2}B_\mu)d_L \end{pmatrix}$$

$$= \bar{u}_L i\gamma^\mu\partial_\mu u_L + \bar{d}_L i\gamma^\mu\partial_\mu d_L + \bar{u}_L\gamma^\mu(\frac{g}{2}W^3_\mu + g'\frac{Y_{u_L}}{2}B_\mu)u_L$$

$$+\frac{g}{\sqrt{2}}\bar{u}_L\gamma^\mu W^+_\mu d_L + \frac{g}{\sqrt{2}}\bar{d}_L\gamma^\mu W^-_\mu u_L - \bar{d}_L\gamma^\mu(\frac{g}{2}W^3_\mu - g'\frac{Y_{d_L}}{2}B_\mu)d_L$$

For the right fields we get

$$\bar{u}_R i\gamma^\mu\left(\partial_\mu - ig'\frac{Y_{u_R}}{2}B_\mu\right)u_R + \bar{d}_R i\gamma^\mu\left(\partial_\mu - ig'\frac{Y_{d_R}}{2}B_\mu\right)d_R =$$

$$= \bar{u}_R i\gamma^\mu\partial_\mu u_R + \bar{d}_R i\gamma^\mu\partial_\mu d_R + \frac{g'}{2}\bar{u}_R\gamma^\mu Y_{u_R}B_\mu u_R + \frac{g'}{2}\bar{d}_R\gamma^\mu Y_{d_R}B_\mu d_R$$

Recovering QED: Recall

$$W^3_\mu = \cos\theta Z_\mu + \sin\theta A_\mu$$
$$B_\mu = -\sin\theta Z_\mu + \cos\theta A_\mu$$

$$\bar{u}_L\gamma^\mu(\frac{g}{2}W^3_\mu + g'\frac{Y_{u_L}}{2}B_\mu)u_L + \frac{g'}{2}\bar{u}_R\gamma^\mu Y_{u_R}B_\mu u_R =$$
$$=\bar{u}_L\gamma^\mu(\frac{g}{2}\sin\theta + g'\frac{Y_{u_L}}{2}\cos\theta)A_\mu u_L + \frac{g'}{2}\bar{u}_R\gamma^\mu Y_{u_R}\cos\theta A_\mu u_R$$
$$+\bar{u}_L\gamma^\mu(\frac{g}{2}\cos\theta - g'\frac{Y_{u_L}}{2}\sin\theta)Z_\mu u_L - \frac{g'}{2}\bar{u}_R\gamma^\mu Y_{u_R}\sin\theta Z_\mu u_R$$

Similarly for the d terms

$$-\bar{d}_L\gamma^\mu(\frac{g}{2}W^3_\mu - g'\frac{Y_{d_L}}{2}B_\mu)d_L + \frac{g'}{2}\bar{d}_R\gamma^\mu Y_{d_R}B_\mu d_R =$$
$$=-\bar{d}_L\gamma^\mu(\frac{g}{2}\sin\theta - g'\frac{Y_{d_L}}{2}\cos\theta)A_\mu d_L + \frac{g'}{2}\bar{d}_R\gamma^\mu Y_{d_R}\cos\theta A_\mu d_R$$
$$-\bar{d}_L\gamma^\mu(\frac{g}{2}\cos\theta + g'\frac{Y_{d_L}}{2}\sin\theta)Z_\mu d_L - \frac{g'}{2}\bar{d}_R\gamma^\mu Y_{d_R}\sin\theta Z_\mu d_R$$

- The electric charges of the u, d quarks require

$$\frac{g}{2}\sin\theta + g'\frac{Y_{u_L}}{2}\cos\theta = e\ q_u, \qquad \frac{g'}{2}Y_{u_R}\cos\theta = e\ q_u$$
$$-\frac{g}{2}\sin\theta + g'\frac{Y_{d_L}}{2}\cos\theta = e\ q_d, \qquad \frac{g'}{2}Y_{d_R}\cos\theta = e\ q_d$$

- These equations can be summarized as

$$eQ_f = \ g\sin\theta\, T^3_{f_\chi} + g'\cos\theta\frac{Y_{f_\chi}}{2}$$

where $f = u, d$, and $\chi = L, R$.

- These equations yield

$$e = g\sin\theta = g'\cos\theta,$$

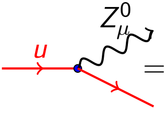
Neutral Currents

- New neutral currents due to the exchange of the Z^0 boson

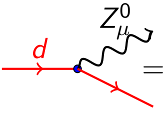
$$
\begin{aligned}
\mathcal{L}_{NC} &= [g_L^u(\bar{u}_L, \bar{d}_L)\gamma^\mu G^Z_{f_L}\begin{pmatrix} u_L \\ d_L \end{pmatrix} + (\bar{u}_R, \bar{d}_R)\gamma^\mu G^Z_{f_R}\begin{pmatrix} u_R \\ d_R \end{pmatrix}]Z_\mu \\
&= [\bar{u}\gamma^\mu(g_V^u - g_A^u\gamma^5)u + \bar{d}\gamma^\mu(g_V^d - g_A^d\gamma^5)d]Z_\mu
\end{aligned}
$$

with

$$
\begin{aligned}
G^Z_{f_\chi} &= \frac{g}{\cos\theta} T^3_{f_\chi} - e\tan\theta Q_f, \\
g_V^f &= \frac{g}{2\cos\theta} T_3^f, \qquad g_A^f = \frac{g}{2\cos\theta} T_3^f - e\tan\theta Q_f
\end{aligned}
$$



$= -i\gamma^\mu(g_V^u - g_A^u\gamma^5)$

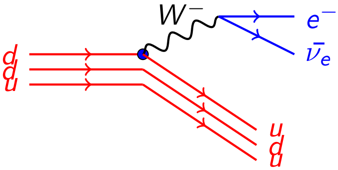


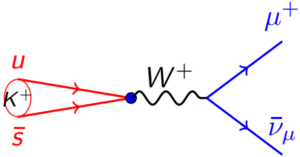
$= -i\gamma^\mu(g_V^d - g_A^d\gamma^5)$

Charged Currents and hadron weak decays

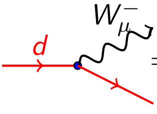
$$\frac{g}{\sqrt{2}}\bar{u}_L\gamma^\mu W_\mu^+ d_L + \frac{g}{\sqrt{2}}\bar{d}_L\gamma^\mu W_\mu^- u_L$$

Neutron beta decay: $n \to p\ e^-\ \nu_e$ Meson decay: $\frac{K^+ \to \mu^+\ \nu_\mu}{\pi^+ \to \mu^+\ \nu_\mu}$,

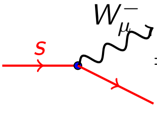




Not enough, Cabibbo factor missing: $d \to d' = \cos\theta d + \sin\theta s$.



$= -i\frac{g}{\sqrt{2}}\cos\theta_c\gamma_\mu\gamma_L$

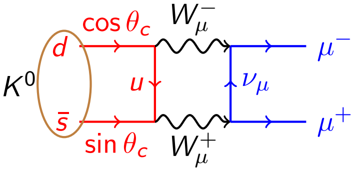


$= -i\frac{g}{\sqrt{2}}\sin\theta_c\gamma_\mu\gamma_L$

Phenomenology requires θ_c: what is its origin?

Cabibbo angle and Charm quark

- Some reactions induced by weak interactions were not observed e.g. $K^0 \to \mu^+\mu^-$



$$\mathcal{M} \approx G_F^2 \sin\theta_c \cos\theta_c$$

- One loop calculation beyond these lectures, but upper bounds (at the time) on the decay width much smaller than predicted.
- Glashow-Illiopoulos-Maiani (1970): this and other suppressed processes can be understood if a new "down-type" quark exists: Charm quark "c".
- Two generations of quarks for weak interactions.

$$q'_{1L} = \begin{pmatrix} u' \\ d' \end{pmatrix}_L, \qquad q'_{2L} \begin{pmatrix} c' \\ s' \end{pmatrix}_L$$

Weak eigenstates vs mass eigenstates: Cabibbo angle.

- The most general dimension 4, $SU(2)_L \otimes U(1)_Y$ invariant Yukawa Lagrangian is:

$$V_{Yuk} = y_{11}\bar{q'_{1L}}\Phi^c u'_R + y_{12}\bar{q'_{1L}}\Phi^c c'_R + y_{21}\bar{q'_{2L}}\Phi^c u'_R + y_{22}\bar{q'_{2L}}\Phi^c c'_R$$
$$+ g_{11}\bar{q'_{1L}}\Phi d'_R + g_{12}\bar{q'_{1L}}\Phi s'_R + g_{21}\bar{q'_{2L}}\Phi d'_R + g_{22}\bar{q'_{2L}}\Phi s'_R + h.c..$$

- After spontaneous symmetry breaking

$$V_{Yuk} = \frac{v}{\sqrt{2}}[y_{11}\bar{u'_L}u'_R + y_{12}\bar{u'_L}c'_R + y_{21}\bar{c'_L}u'_R + y_{22}\bar{c'_{2L}}c'_R$$
$$+ g_{11}\bar{d'_L}d'_R + g_{12}\bar{d'_L}s'_R + g_{21}\bar{s'_L}d'_R + g_{22}\bar{s'_L}s'_R] \ + \ H - interaction$$
$$= \frac{v}{\sqrt{2}}\left[(\bar{u'_L}, \bar{c'_L})\begin{pmatrix} y_{11}, y_{12} \\ y_{21}, y_{22}\end{pmatrix}\begin{pmatrix} u'_R \\ c'_R\end{pmatrix} + (\bar{d'_L}, \bar{s'_L})\begin{pmatrix} g_{11}, g_{12} \\ g_{21}, g_{22}\end{pmatrix}\begin{pmatrix} d'_R \\ s'_R\end{pmatrix}\right]$$
$$\equiv \bar{U}'_L M^U U'_R + \bar{D}'_L M^D D'_R.$$

- The mass matrices M^U, M^D are not diagonal. Weak eigenstates are not mass eigenstates.

- Calculation of physical processes requires to use mass eigenstates.
- Every square matrix can be diagonalized by two unitary matrices

$$A_L M^U A_R^\dagger = M^U_{diag}, \qquad B_L M^D B_R^\dagger = M^D_{diag}$$

with $M^U_{diag} = Diag[m_u, m_c]$; $M^D_{diag} = Diag[m_d, m_s]$ and $A_{R/L}, B_{R/L}$ are unitary matrices.
- Then

$$\begin{aligned} V_{Yuk} &= \bar{U}'_L A_L^\dagger A_L M^U A_R^\dagger A_R U'_R + \bar{D}'_L B_L^\dagger B_L M^D B_R^\dagger B_R D'_R \\ &= \bar{U}_L M^U_{diag} U_R + \bar{D}_L M^D_{diag} D_R \\ &= m_u \bar{u}_L u_R + m_c \bar{c}_L c_R + m_d \bar{d}_L d_R + m_s \bar{s}_L s_R, \end{aligned}$$

where

$$U_L = A_L U'_L, \qquad U_R = A_R U'_R, \qquad D_L = B_L U'_L, \qquad D_R = B_R U'_R$$

- Notice that the free field terms are invariant under these transformations

$$
\begin{aligned}
&\bar{u'_L} i\gamma^\mu \partial_\mu u'_L + \bar{c'_L} i\gamma^\mu \partial_\mu c'_L + \bar{d'_L} i\gamma^\mu \partial_\mu d'_L + \bar{s'_L} i\gamma^\mu \partial_\mu s'_L \\
&+\bar{u'_R} i\gamma^\mu \partial_\mu u'_R + \bar{c'_R} i\gamma^\mu \partial_\mu c'_R + \bar{d'_R} i\gamma^\mu \partial_\mu d'_R + \bar{s'_R} i\gamma^\mu \partial_\mu s'_R \\
=&\bar{U}'_L i\gamma^\mu \partial_\mu U'_L + \bar{D}'_L i\gamma^\mu \partial_\mu D'_L + \bar{U}'_R i\gamma^\mu \partial_\mu U'_R + \bar{D}'_R i\gamma^\mu \partial_\mu D'_R \\
=&\bar{U}_L A_L i\gamma^\mu \partial_\mu A_L^\dagger U_L + \bar{D}_L B_L i\gamma^\mu \partial_\mu B_L^\dagger D_L \\
&+ \bar{U}_R A_R i\gamma^\mu \partial_\mu A_R^\dagger U_R + \bar{D}_R B_R i\gamma^\mu \partial_\mu B_R^\dagger D_R \\
=&\bar{U}_L i\gamma^\mu \partial_\mu U_L + \bar{D}_L i\gamma^\mu \partial_\mu D_L + \bar{U}_R i\gamma^\mu \partial_\mu U_R + \bar{D}_R i\gamma^\mu \partial_\mu D_R .
\end{aligned}
$$

- Similarly, for the electromagnetic interactions

$$
\begin{aligned}
&[\frac{2}{3}(\bar{u'_L}\gamma^\mu u'_L + \bar{c'_L}\gamma^\mu c'_L) - \frac{1}{3}(\bar{d'_L}\gamma^\mu d'_L + \bar{s'_L}\gamma^\mu s'_L)]A_\mu + L \to R \\
=&(\frac{2}{3}\bar{U}'_L\gamma^\mu U'_L - \frac{1}{3}\bar{D}'_L\gamma^\mu D'_L)A_\mu + L \to R \\
=&(\frac{2}{3}\bar{U}_L A_L\gamma^\mu A_L^\dagger U_L - \frac{1}{3}\bar{D}_L B_L\gamma^\mu B_L^\dagger D_L)A_\mu + L \to R \\
=&(\frac{2}{3}\bar{U}_L\gamma^\mu U_L - \frac{1}{3}\bar{D}_L\gamma^\mu D_L)A_\mu + L \to R
\end{aligned}
$$

- Notice that invariance holds because A_μ has the same coupling to all type-u quarks and same coupling to all type-d quarks.
- The coupling of Z^0 to quarks also satisfy these requirements. Neutral currents are also invariant.
- For the charged currents

$$
\begin{aligned}
\mathcal{L}_{ch} =& \frac{g}{\sqrt{2}} \left[(\bar{u}'_L \gamma^\mu d'_L + \bar{c}'_L \gamma^\mu s'_L) W^+_\mu + (\bar{d}'_L \gamma^\mu u'_L + \bar{s}'_L \gamma^\mu c'_L) W^-_\mu \right] \\
&= \frac{g}{\sqrt{2}} \left[\bar{U}'_L \gamma^\mu D'_L W^+_\mu + \bar{D}'_L \gamma^\mu U'_L W^-_\mu \right] \\
&= \frac{g}{\sqrt{2}} \left[\bar{U}_L A_L \gamma^\mu B^\dagger_L D_L W^+_\mu + \bar{D}_L B_L \gamma^\mu A^\dagger_L U W^-_\mu \right] \\
&= \frac{g}{\sqrt{2}} \left[\bar{U}_L V \gamma^\mu D_L W^+_\mu + \bar{D}_L V^\dagger \gamma^\mu U W^-_\mu \right] \\
&= \frac{g}{\sqrt{2}} \left[(\bar{u}_L, \bar{c}_L) \begin{pmatrix} V_{ud}, V_{us} \\ V_{cd}, V_{cs} \end{pmatrix} \gamma^\mu \begin{pmatrix} d_L \\ s_L \end{pmatrix} W^+_\mu + h.c, \right] \\
=& \frac{g}{\sqrt{2}} \left[(V_{ud} \bar{u}_L \gamma^\mu d_L + V_{us} \bar{u}_L \gamma^\mu s_L + V_{cd} \bar{c}_L \gamma^\mu d_L + V_{cs} \bar{c}_L \gamma^\mu s_L) W^+_\mu + h.c \right]
\end{aligned}
$$

- The matrix $V = A_L B_L^\dagger$ is a unitary matrix:

$$VV^\dagger = A_L B_L^\dagger B_L A_L^\dagger = A_L A_L^\dagger = 1.$$

- Under quarks fields redefinition by a phase $q_i \to e^{i\phi_i} q_i$, all other terms in the Lagrangian are invariant but

$$\begin{pmatrix} V_{ud}, V_{us} \\ V_{cd}, V_{cs} \end{pmatrix} \to \begin{pmatrix} V_{ud}\, e^{-i(\phi_u - \phi_d)}, V_{us}\, e^{-i(\phi_u - \phi_s)} \\ V_{cd}\, e^{-i(\phi_c - \phi_d)}, V_{cs}\, e^{-i(\phi_c - \phi_s)} \end{pmatrix}$$

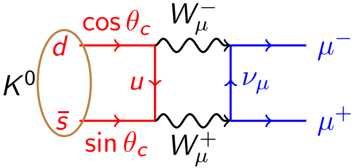
- Phases of the V_{ij} elements can be removed by an appropriate field redefinition leaving an orthogonal matrix

$$V = \begin{pmatrix} \cos\theta_c, \sin\theta_c \\ -\sin\theta_c, \cos\theta_c \end{pmatrix}$$

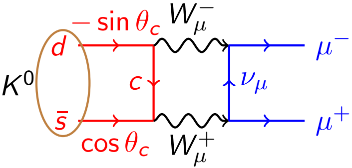
- Cabibbo mixing is due to the dissalingnement between weak eigenstates and mass eigenstates.

Cabibbo angle and Charm quark: GIM suppression

- Consequences for $K^0 \to \mu^+\mu^-$



$\mathcal{M}_u \approx G_F^2 \sin\theta_c \cos\theta_c$



$\mathcal{M}_c \approx -G_F^2 \sin\theta_c \cos\theta_c$

- We can understand the small decay width from strong cancellations of these diagrams due to the Cabibbo matrix.
- The $\bar{c}c(3096)$ meson discovered in 1974 by two independent Collaborations.: SLAC (ψ) and BNL (J); today known as J/ψ.
- Kobayashi-Maskawa (1973): CP violation observed in Kaon decays requires a phase in the Cabibbo matrix.
- The only possibility is that another generation of quarks exists.

Three generations: Cabibbo-Kobayashi-Maskawa matrix.

- A new quark (b) discovered at FERMILAB in 1977 in the $\bar{b}b$ system named $\Upsilon(9{,}41 GeV)$.
- Its weak partner (t) was discovered at FERMILAB in 1995 with a mass $m_t = 175 GeV$.
- There are three generations of quark weak eigenstates:

$$q'_{1L} = \begin{pmatrix} u' \\ d' \end{pmatrix}_L, \qquad q'_{2L} = \begin{pmatrix} c' \\ s' \end{pmatrix}_L, \qquad q'_{3L} = \begin{pmatrix} t' \\ b' \end{pmatrix}_L$$

- A similar analysis yields the unitary 3×3 CKM matrix

$$\begin{aligned}
\mathcal{L}_{ch} &= \frac{g}{\sqrt{2}} \left[\bar{U}_L V \gamma^\mu D_L W_\mu^+ + \bar{D}_L V^\dagger \gamma^\mu U W_\mu^- \right] \\
&= \frac{g}{\sqrt{2}} \left[(\bar{u}_L, \bar{c}_L, \bar{t}_L) \begin{pmatrix} V_{ud}, V_{us}, V_{ub} \\ V_{cd}, V_{cs}, V_{cb} \\ V_{td}, V_{ts}, V_{tb} \end{pmatrix} \gamma^\mu \begin{pmatrix} d_L \\ s_L \\ b_L \end{pmatrix} W_\mu^+ + h.c, \right]
\end{aligned}$$

- A 3×3 unitary matrix has only 9 real free parameters. Recall

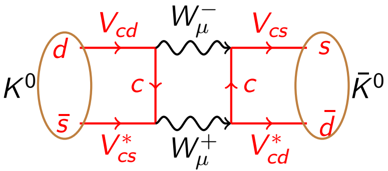
$$U = e^{iG} \qquad with \qquad G^\dagger = G.$$

- There are six phases in the quark field redefinitions: $V_{ij} \to e^{-i(\phi_i - \phi_j)} V_{ij}$. Only five differences $\phi_i - \phi_j$ are independent.
- We are left with $9 - 5 = 4$ free parameters: 3 rotation angles and a phase.
- Many alternative parametrizations. CKM original one is $V = R_{23} I_\delta R_{13} I_\delta^\dagger R_{12}$, with R_{ij} rotation matrices and $I_\delta = Diag(1, 1, e^{i\delta})$. Explicitly

$$V = \begin{pmatrix} 1, 0, 0 \\ 0, c_{23}, s_{23} \\ 0, -s_{23}, c_{23} \end{pmatrix} \begin{pmatrix} c_{13}, 0, s_{23} e^{-i\delta} \\ 0, 1, 0 \\ -s_{13} e^{i\delta}, 0, c_{23} \end{pmatrix} \begin{pmatrix} c_{23}, s_{23}, 0 \\ -s_{23}, c_{23}, 0 \\ 0, 0, 1 \end{pmatrix}$$

$$\begin{pmatrix} V_{ud}, V_{us}, V_{ub} \\ V_{cd}, V_{cs}, V_{cb} \\ V_{td}, V_{ts}, V_{tb} \end{pmatrix} = \begin{pmatrix} c_{12}c_{13}, & s_{12}c_{13}, & s_{13}e^{-i\delta} \\ -s_{13}c_{23} - c_{12}s_{23}c_{13}e^{i\delta}, & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta}, & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta}, & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta}, & c_{23}c_{13} \end{pmatrix}$$

- All these parameters have been measured.
- The phase δ induces CP violation: Neutral meson systems are actually admixtures of states with well defined CP (QCD eigenstates).



- Plenty of physics here we have no time to discuss.