MexiCOPAS

Mexican Cosmology Particles and Strings Schools

Introduction to the Standard Model

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Outline

1. Fundamentals
   - Symmetries in Classical and Quantum Mechanics.
   - Irreducible representations (irreps) of $SU(2)$.
   - Irreps of the HLG: Chirality, Parity and Dirac Equation.
   - Quantum Field theory: complex scalar field.
2. Electroweak interactions: Glashow-Weinberg-Salam theory.
   - Minimal coupling principle in classical mechanics.
   - Gauge theories: Abelian and non-Abelian.
   - Quantum Electrodynamics
   - Fermi theory, IVB theory, parity violation and V-A structure of weak interactions
   - GWS Theory. Spontaneous Breaking of Symmetries.
3. Strong interactions:QCD.
   - Irreducible representations of $SU(3)$
   - Classification of hadrons: Eightfold Way, Quark Model
   - Gauge theory of strong interactions: QCD.
   - Running of couplings: Confinement and asymptotic freedom.
   - Experimental evidence for color degrees of freedom.

Strong interactions: historical notes

- Atomic physics: nuclei with charge Ze. There are Z protons in the nucleus.
- Nucleus charge radius below 1 $fm = 10^{-15} m$. Instability due to Coulomb repulsion.
- There must be something that glue protons and overcome Coulomb repulsion: neutrons.
- Neutrons discovered by Chadwick in 1931. Similar mass to the proton: $M_p = 938\ MeV$, $M_n = 939\ MeV$.
- Heisenberg (1932) : similar proton and neutron mass suggests an $SU(2)$ symmetry: "Isotopic-spin".
- H. Yukawa (1935): there could be a mediator of this nucleon interaction: Pion. Mass estimated around 150 MeV.
- Charged pions discovered in 1947: $M_\pi = 139 MeV$. Neutral pions and Kaons discovered in 1949: similar masses to the charged partners.

- Effective Lagrangians with isospin symmetry for nuclear interactions. Coupling constant $g_{NN\pi}$ turns out to be large: non-perturbative interactions.
- Unexpected ("strange") particles discovered in 1949: Kaons, $M_{K^{\pm}} = 495\ MeV$.
- 1950-1960: a zoo of new particles discovered.
- Mass spectrum suggest they can be organized in isospin multiplets with defined J^{PC} quantum numbers.
- Gell-Mann/Neeman (1961): Eightfold Way, particles fit in the $\mathbf{8}$ and $\mathbf{10}$ multiplets of $SU(3)$.
- Gell-Mann/Zweig (1964): $SU(3)$ Quark Model. Fundamental representations $\mathbf{3}$ and $\bar{\mathbf{3}}$ of $SU(3)$ could be realized in nature.
- Known hadrons require three "flavors": u, d, s transforming in the $\mathbf{3}$ and their antiparticles $\bar{u}$, $\bar{d}$, $\bar{s}$ transforming in the $\bar{\mathbf{3}}$.
- Unconventional fractional electric charges.

- 1965: Struminsky, Bogolubov-Struminsky-Tavkhelidze, Greenberg, Han-Nambu: Pauli principle violation in the quark model.
- Gross-Wilczek and Politzer (1973): calculation of the Beta function of $SU(3)_c$. Confinement and asymptotic freedom.

Exponential map for unitary matrices

Theorem

Every unitary matrix can be written in the exponential form $U = e^{iG}$ with G a Hermitian matrix.

Proof.

U is invertible, thus there is a matrix S satisfying $SUS^\dagger = U_D \equiv \text{Diag}(\lambda_1, \ldots, \lambda_n)$. Furthermore, U_D is unitary, thus $U_D U_D^\dagger = \mathbb{1} = \text{Diag}(|\lambda_1|^2, |\lambda_2|^2, \ldots, |\lambda_n|^2)$. We can write $\lambda_k = e^{i\alpha_k}$ with $\alpha_k \in \mathbb{R}$, hence $U_D = \text{Diag}(e^{i\alpha_1}, \ldots, e^{i\alpha_n}) = e^{i\text{Diag}(\alpha_1,\ldots,\alpha_n)} \equiv e^{iG_D}$.

Inverting the matrices we get

$U = S^\dagger U_D S = S^\dagger e^{iG_D} S = e^{iS^{-\dagger} G_D S} \equiv e^{iG}$

Finally

$$G = S^\dagger G_D S \qquad \Rightarrow \qquad G^\dagger = S^\dagger G_D^\dagger S = G$$

□

$SU(3)$= unitary 3×3 matrices of unit determinant

- Use the exponential map and write

$$G = \begin{pmatrix} g_{11} & g_{12} & g_{13} \\ g_{12}^* & g_{22} & g_{23} \\ g_{13}^* & g_{23}^* & g_{33} \end{pmatrix}$$

- There are 9 free parameters (g_{ii} are real). Besides

$$\begin{aligned} \det(e^{iG}) &= \det(Se^{iG}S^{-1}) = \det\left(e^{iSGS^{-1}}\right) = \det\left(e^{iG_D}\right) = \prod_{k=1}^{n} e^{i\alpha_k} \\ &= \exp(i\sum_{k=1}^{n}\alpha_k) = e^{i\mathrm{tr}(G_D)} = e^{i\mathrm{tr}(SGS^{-1})} = e^{i\mathrm{tr}\, G} = 1 \end{aligned}$$

- The trace condition is real hence an $SU(3)$ matrix in general depend on eight independent parameters.

The most general form of G is

$$G = \begin{pmatrix} a_1 & a_2 - ia_3 & a_4 - ia_5 \\ a_2 + ia_3 & a_6 & a_7 - ia_8 \\ a_4 + ia_5 & a_7 + ia_8 & -a_1 - a_6 \end{pmatrix}$$

We can write G as a linear combination of eight independent matrices

$$\begin{aligned} G &= a_2 \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_3 \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + \frac{a_1 - a_6}{2} \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_4 \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} \\ &+ a_5 \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix} + a_7 \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} + a_8 \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix} + \frac{a_1 + a_6}{2} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix} \end{aligned}$$

Define the generators in terms of the matrices

$$T_a = \frac{\lambda_a}{2},$$

The λ_a matrices were introduced by Murray Gell-Mann:

$$\lambda_1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \lambda_2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \lambda_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \lambda_4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

$$\lambda_5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix} \quad \lambda_6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \quad \lambda_7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix} \quad \lambda_8 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$$

The generators are normalized to

$$Tr(T_a T_b) = \frac{1}{2}\delta_{ab}.$$

Gell-Mann matrices satisfy the following algebra

$$[T_a, T_b] = if_{abc} T_c \qquad \{T_a, T_b\} = \frac{1}{3}\delta_{ab}\mathbb{1} + d_{abc} T_c.$$

$f_{abc} \equiv$ structure constants, f totally antisymmetric .

The non-vanishing values are

$$f_{123} = 1, \quad f_{458} = f_{678} = \frac{\sqrt{3}}{2},$$
$$f_{147} = f_{165} = f_{246} = f_{257} = f_{345} = f_{376} = \frac{1}{2}.$$

The d_{abc} constants are totally symmetric and the non-null values are

$$d_{146} = d_{157} = d_{256} = d_{344} = d_{355} = \frac{1}{2}, \quad d_{118} = d_{228} = d_{338} = \frac{1}{\sqrt{3}},$$

$$d_{448} = d_{558} = d_{668} = d_{778} = -\frac{1}{2\sqrt{3}}, \quad d_{247} = d_{366} = d_{377} = -\frac{1}{2}$$

Summarizing, every $SU(3)$ matrix can be written as

$$U = e^{i\theta_a T_a}$$

where θ_a $a = 1, 2, ...8$ are real numbers and T_a are the classical group generators satisfying the Lie Algebra

$$[T_a, T_b] = i\ f_{abc} T_c$$

Quantum realm: Irreps of $SU(3)$

- There are two elements in the Cartan subalgebra

$$H^1 = T_3, \qquad H^2 = T_8.$$

- We will have two-dimensional space of the corresponding eigenvalues.
- Idea: use the $SU(2)$ subgroups to construct the irreps. Ladder operators connect all the states in an irrep.
- Recall the $SU(2)$ structure

$$J_\pm = J_x \pm iJy, \qquad [J_+, J_-] = 2J_z, \qquad [J_z, J_\pm] = \pm J_\pm.$$

- Define the ladder operators (notice normalization $1/\sqrt{2}$):

$$E^1_\pm = \frac{1}{\sqrt{2}}(T_1 \pm iT_2) \qquad E^2_\pm = \frac{1}{\sqrt{2}}(T_4 \pm iT_5) \qquad E^3_\pm = \frac{1}{\sqrt{2}}(T_6 \mp iT_7)$$

- A calculation yields

$$[E^1_+, E^1_-] = H^1 \equiv E^1_z$$

$$[E^2_+, E^2_-] = \frac{1}{2}H^1 + \frac{\sqrt{3}}{2}H^2 \equiv E^2_z$$

$$[E^3_+, E^3_-] = \frac{1}{2}H^1 - \frac{\sqrt{3}}{2}H^2 \equiv E^3_z$$

$$[E^1_z, E^1_\pm] = \pm E^1_\pm$$

$$[E^2_z, E^2_\pm] = \pm E^2_\pm$$

$$[E^3_z, E^3_\pm] = \pm E^3_\pm$$

- The set of operators

$$\{E^1_z, E^1_\pm\}, \qquad \{E^2_z, E^2_\pm\}, \qquad \{E^3_z, E^3_\pm\}$$

form three $SU(2)$ subgroups of $SU(3)$. On the other side

$$[H^1, E^1_\pm] = \pm E^1_\pm$$

$$[H^1, E^2_\pm] = \pm\frac{1}{2}E^2_\pm$$

$$[H^1, E^3_\pm] = \pm\frac{1}{2}E^3_\pm$$

$$[H^2, E^1_\pm] = 0$$

$$[H^2, E^2_\pm] = \pm\frac{\sqrt{3}}{2}E^2_\pm$$

$$[H^2, E^3_\pm] = \mp\frac{\sqrt{3}}{2}E^3_\pm$$

Definition

Define the **weights** as the the eigenvalues μ^i de H^i

$$H^i|\mu^1,\mu^2\rangle = \mu^i|\mu^1,\mu^2\rangle.$$

and the **weight vector** as $\boldsymbol{\mu} = (\mu^1,\mu^2)$.

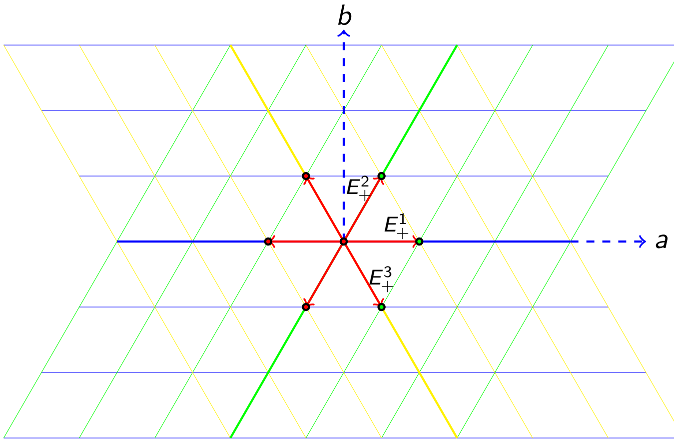
If $\boldsymbol{\mu} = (a,b)$ is a weight vector

$$H^1|a,b\rangle = a|a,b\rangle, \qquad H^2|a,b\rangle = b|a,b\rangle$$

then

- $E^1_\pm|a,b\rangle = 0$ ó $E^1_\pm|a,b\rangle$ is an eigenstate of (H^1,H^2) with eigenvalues $(a,b)\pm(1,0)$.
- $E^2_\pm|a,b\rangle = 0$ ó $E^2_\pm|a,b\rangle$ is an eigenstate of (H^1,H^2) with eigenvalues $(a,b)\pm(\frac{1}{2},\frac{\sqrt{3}}{2})$.
- $E^3_\pm|a,b\rangle = 0$ ó $E^3_\pm|a,b\rangle$ is an eigenstate of (H^1,H^2) with eigenvalues $(a,b)\pm(\frac{1}{2},-\frac{\sqrt{3}}{2})$.

Eigenvalues of (H^1, H^2) and the action of ladder operators



Definition

We define the *root* vector $\boldsymbol{\beta}^j_{\pm}$ as the vector whose components are the numbers $\beta^j_{i\pm}$ $i = 1, 2$ arising from the commutation relation of the $E^j_{\pm}$ operator with all the H^i, i.e. ,

$$\left[H^i, E^j_{\pm}\right] = \pm\beta^j_{i\pm} E^j_{\pm}$$

Roots of $SU(3)$: $\boldsymbol{\beta}^1_{\pm} = \pm(1, 0)$, $\boldsymbol{\beta}^2_{\pm} = \pm(\frac{1}{2}, \frac{\sqrt{3}}{2})$, $\boldsymbol{\beta}^3_{\pm} = \pm(\frac{1}{2}, -\frac{\sqrt{3}}{2})$.
Using

$$H^i E^j_{\pm}|\boldsymbol{\mu}\rangle = \left(\left[H^i, E^j_{\pm}\right] + E^j_{\pm} H^i\right)|\boldsymbol{\mu}\rangle = (\pm\beta^j_{i\pm} + \mu^i) E^j_{\pm}|\boldsymbol{\mu}\rangle$$

we get

$$E^j_{\pm}|\boldsymbol{\mu}\rangle = \begin{cases} 0 & or \\ N_{\boldsymbol{\beta}^j_{\pm}, \boldsymbol{\mu}} |\boldsymbol{\mu} \pm \boldsymbol{\beta}^j_{\pm}\rangle. \end{cases}$$

We can obtain all the states $\boldsymbol{\mu}$ in an irrep using the roots if we know a state in this irrep.

Definition

Llamamos **raíz positiva** β^j a la raíz cuyo primer elemento no nulo es positivo.

Para $SU(3)$: $\beta^1 = (1, 0)$ $\quad \beta^2 = (\frac{1}{2}, \frac{\sqrt{3}}{2})$ $\quad \beta^3 = (\frac{1}{2}, -\frac{\sqrt{3}}{2})$.
Notemos que

$$\begin{aligned}
[E_+^1, E_-^1] &= H^1 = E_z^1 = \beta^1 \cdot \boldsymbol{H} \qquad \boldsymbol{H} = (H^1, H^2) \\
[E_+^2, E_-^2] &= \frac{1}{2}H^1 + \frac{\sqrt{3}}{2}H^2 = E_z^2 = \beta^2 \cdot \boldsymbol{H} \\
[E_+^3, E_-^3] &= \frac{1}{2}H^1 - \frac{\sqrt{3}}{2}H^2 = E_z^3 = \beta^3 \cdot \boldsymbol{H}
\end{aligned}$$

$$\left[E_+^j, E_-^j\right] = E_z^j = \frac{\beta^j \cdot \boldsymbol{H}}{|\beta^j|^2}$$

Los estados $|\boldsymbol{\mu}\rangle$ son eigenestados de E_z^j:

$$E_z^j|\boldsymbol{\mu}\rangle = \frac{\boldsymbol{\beta}^j \cdot \boldsymbol{\mu}}{|\boldsymbol{\beta}^j|^2}|\boldsymbol{\mu}\rangle.$$

Por otro lado

$$E_z^j(E_\pm^j|\boldsymbol{\mu}\rangle) = \left(\left[E_z^j, E_\pm^j\right] + E_\pm^j E_z^j\right)|\boldsymbol{\mu}\rangle = \left(\pm E_\pm^j + E_\pm^j E_z^j\right)|\boldsymbol{\mu}\rangle = \left(\frac{\boldsymbol{\beta}^j \cdot \boldsymbol{\mu}}{|\boldsymbol{\beta}^j|^2} \pm 1\right) E_\pm^j|\boldsymbol{\mu}\rangle$$

Para cada $SU(2)$ podemos aplicar E_+^j solo un cierto número p de veces después del cual se anula. Para este estado

$$E_z^j\left((E_+^j)^p|\boldsymbol{\mu}\rangle\right) = \left(\frac{\boldsymbol{\beta}^j \cdot \boldsymbol{\mu}}{|\boldsymbol{\beta}^j|^2} + p^j\right)(E_+^j)^p|\boldsymbol{\mu}\rangle$$

En forma similar solo podemos aplicar q veces el operador E_-^j sobre un estado $|\boldsymbol{\mu}\rangle$ despues del cual se anula. Para este estado:

$$E_z^j\left[(E_-^j)^q|\boldsymbol{\mu}\rangle\right] = \left(\frac{\boldsymbol{\beta}^j \cdot \boldsymbol{\mu}}{|\boldsymbol{\beta}^j|^2} - q^j\right)(E_-^j)^q|\boldsymbol{\mu}\rangle$$

Si denotamos por $J^{(j)}$ al máximo eigenvalor de E_z^j, entonces el mínimo eigenvalor es $-J^{(j)}$, esto es:

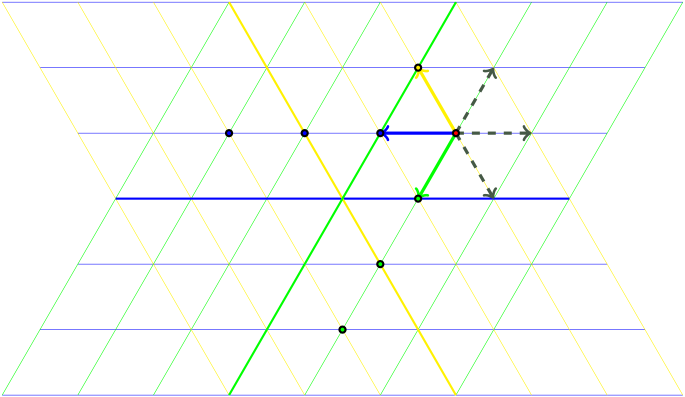
$$J^{(j)} = \frac{\boldsymbol{\beta}^j \cdot \boldsymbol{\mu}}{|\boldsymbol{\beta}^j|^2} + p^j, \qquad -J^{(j)} = \frac{\boldsymbol{\beta}^j \cdot \boldsymbol{\mu}}{|\boldsymbol{\beta}^j|^2} - q^j$$

Con lo cual

$$J^{(j)} = \frac{p^j + q^j}{2} \qquad \text{y} \qquad \frac{\boldsymbol{\beta}^j \cdot \boldsymbol{\mu}}{|\boldsymbol{\beta}^j|^2} = -\frac{p^j - q^j}{2}$$

Irreps de SU(3). Idea:

- Partir del peso con $p^j = 0, \quad j = 1, 2, 3$, en cuyo caso q^j es máximo y define la irrep de la correspondiente subálgebra $SU(2)$.
- Usar los operadores de escalera E^j_- .



Definition

Definimos el peso máximo $|\boldsymbol{\mu}^*\rangle$ de una representación irreducible de $SU(3)$ como aquel para el cual $E_+^j|\boldsymbol{\mu}^*\rangle = 0$ para todo j.

Para el peso máximo ($p^j = 0$) tenemos

$$2\frac{\boldsymbol{\beta}^j \cdot \boldsymbol{\mu}^*}{|\boldsymbol{\beta}^j|^2} = q^j$$

1. Partiendo de un estado de peso máximo podemos encontrar todos los estados en la irrep a la que el peso máximo pertenece actuando con los operadores de escalera E_-^j.
2. Para encontrar el peso máximo solo necesitamos un conjunto linealmente independiente en el espacio de las raices positivas $\boldsymbol{\beta}^i$.

Definition

Denotamos como *raíces simples* a cualquier subconjunto linealmente independiente $\{\boldsymbol{\alpha}^i\}$ del conjunto de raíces positivas $\{\boldsymbol{\beta}^j\}$.

Para $SU(3)$ escogeremos como las raíces simples al conjunto

$$\boldsymbol{\alpha}^1 = \boldsymbol{\beta}^2 = (\frac{1}{2}, \frac{\sqrt{3}}{2}) \qquad \boldsymbol{\alpha}^2 = \boldsymbol{\beta}^3 = (\frac{1}{2}, -\frac{\sqrt{3}}{2})$$

Estas raíces satisfacen $2\frac{\boldsymbol{\alpha}^i \cdot \boldsymbol{\mu}^*}{|\boldsymbol{\alpha}^i|^2} = q^i$ y esta es la mínima información necesaria para reconstruir una irrep, por lo tanto

Las irreps de SU(3) están caracterizados por dos números $q^1, q^2 \in \mathbb{Z}^+$ que satisfacen

$$2\frac{\boldsymbol{\alpha}^i \cdot \boldsymbol{\mu}^*}{|\boldsymbol{\alpha}^i|^2} = q^i$$

donde $\boldsymbol{\mu}^*$ es el peso máximo y $\boldsymbol{\alpha}^i$ son las raíces simples.

Escribiendo $\boldsymbol{\mu}^* = (a, b)$ y usando las raices simples obtenemos

$$\begin{pmatrix} 1 & \sqrt{3} \\ 1 & -\sqrt{3} \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} q^1 \\ q^2 \end{pmatrix}$$

que tiene la solución

$$\boldsymbol{\mu}^* = (a, b) = \left(\frac{q^1 + q^2}{2}, \frac{q^1 - q^2}{2\sqrt{3}} \right)$$

Representaciones irreducibles de $SU(3)$:(q^1, q^2)

$$(0,0)$$

$$(1,0) \quad (0,1)$$

$$(2,0) \quad (1,1) \quad (0,2)$$

$$(3,0) \quad (2,1) \quad (1,2) \quad (0,3)$$

$$(4,0) \quad (3,1) \quad (2,2) \quad (1,3) \quad (0,4)$$

Escribiendo $\boldsymbol{\mu}^* = (a, b)$ y usando las raices simples obtenemos

$$\begin{pmatrix} 1 & \sqrt{3} \\ 1 & -\sqrt{3} \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} q^1 \\ q^2 \end{pmatrix}$$

que tiene la solución

$$\boldsymbol{\mu}^* = (a, b) = \left(\frac{q^1 + q^2}{2}, \frac{q^1 - q^2}{2\sqrt{3}} \right)$$

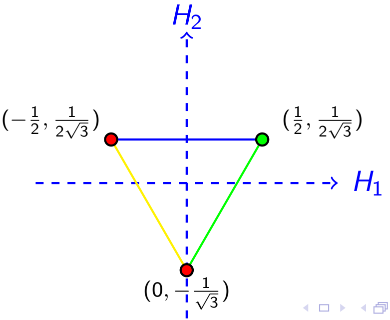
Representaciones irreducibles de $SU(3)$:(q^1, q^2)

Singlete			$(0,0)$		
Quarks & anti-quarks		$(1,0)$		$(0,1)$	
Gluones	$(2,0)$		$(1,1)$		$(0,2)$
	$(3,0)$	$(2,1)$		$(1,2)$	$(0,3)$
	$(4,0)$	$(3,1)$	$(2,2)$	$(1,3)$	$(0,4)$

Reconstruyendo las irreps a partir de los pesos máximos: Representación 3

- Representación $(0,0)$: en este caso $\boldsymbol{\mu}^* = (0,0)$ y hay un único estado.
- Representación $(1,0)$: el peso máximo es $\boldsymbol{\mu}^* = (\frac{1}{2}, \frac{1}{2\sqrt{3}})$.

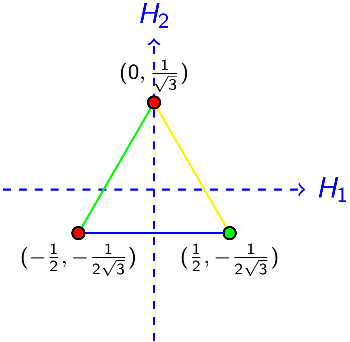
$$\boldsymbol{\beta}^1 \cdot \boldsymbol{\mu}^* = \frac{1}{2}, \qquad \boldsymbol{\beta}^2 \cdot \boldsymbol{\mu}^* = \frac{1}{2}, \qquad \boldsymbol{\beta}^3 \cdot \boldsymbol{\mu}^* = 0.$$



Representación $\bar{\mathbf{3}}$

- Representación $(0,1)$: el peso máximo es $\boldsymbol{\mu}^* = (\frac{1}{2}, -\frac{1}{2\sqrt{3}})$.

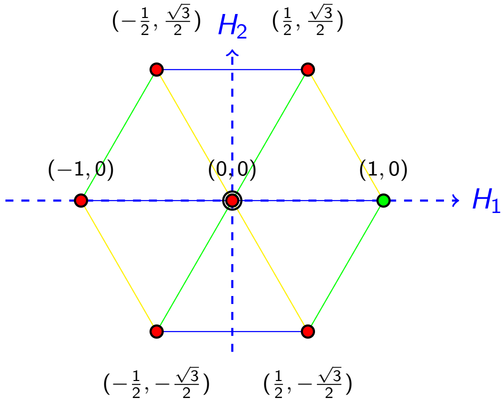
$$\beta^1 \cdot \boldsymbol{\mu}^* = \frac{1}{2}, \qquad \beta^2 \cdot \boldsymbol{\mu}^* = 0, \qquad \beta^3 \cdot \boldsymbol{\mu}^* = \frac{1}{2}.$$



Representación **8**

- Representación $(1,1)$: el peso máximo es $\boldsymbol{\mu}^* = (1,0)$.

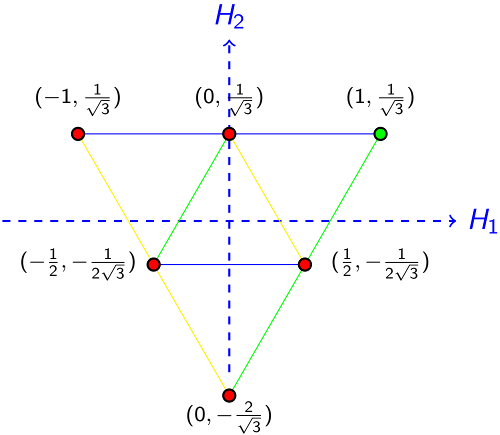
$$\beta^1 \cdot \boldsymbol{\mu}^* = 1, \qquad \beta^2 \cdot \boldsymbol{\mu}^* = \frac{1}{2}, \qquad \beta^3 \cdot \boldsymbol{\mu}^* = \frac{1}{2}.$$



Representación **6**

- Representación $(2,0)$: el peso máximo es $\boldsymbol{\mu}^* = (1, \frac{1}{\sqrt{3}})$.

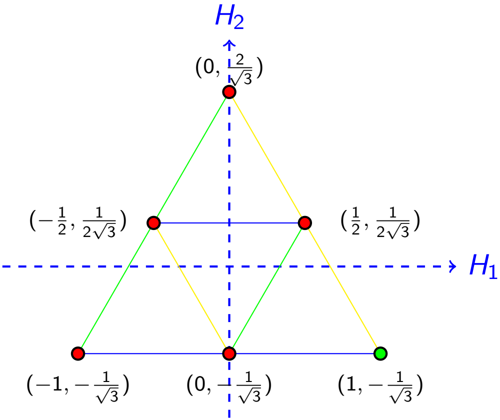
$$\beta^1 \cdot \boldsymbol{\mu}^* = 1, \quad \beta^2 \cdot \boldsymbol{\mu}^* = 1, \quad \beta^3 \cdot \boldsymbol{\mu}^* = 0.$$



Representación $\bar{\mathbf{6}}$

- Representación $(0,2)$: el peso máximo es $\boldsymbol{\mu}^* = (1, -\frac{1}{\sqrt{3}})$.

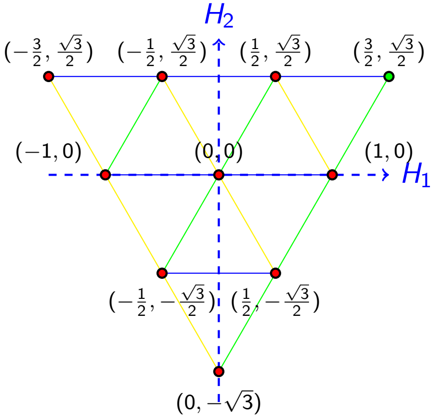
$$\boldsymbol{\beta}^1 \cdot \boldsymbol{\mu}^* = 1, \quad \boldsymbol{\beta}^2 \cdot \boldsymbol{\mu}^* = 0, \quad \boldsymbol{\beta}^3 \cdot \boldsymbol{\mu}^* = 1.$$



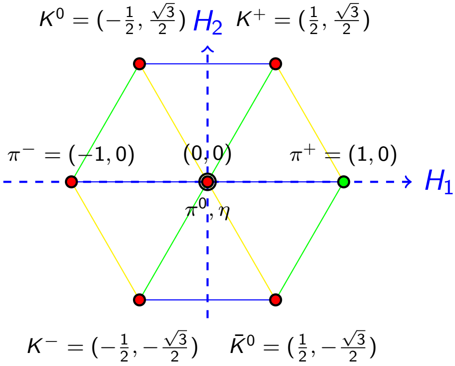
Representación **10**

- Representación $(3,0)$: el peso máximo es $\boldsymbol{\mu}^* = (\frac{3}{2}, \frac{\sqrt{3}}{2})$.

$$\beta^1 \cdot \boldsymbol{\mu}^* = \frac{3}{2}, \quad \beta^2 \cdot \boldsymbol{\mu}^* = \frac{3}{2}, \quad \beta^3 \cdot \boldsymbol{\mu}^* = 0.$$

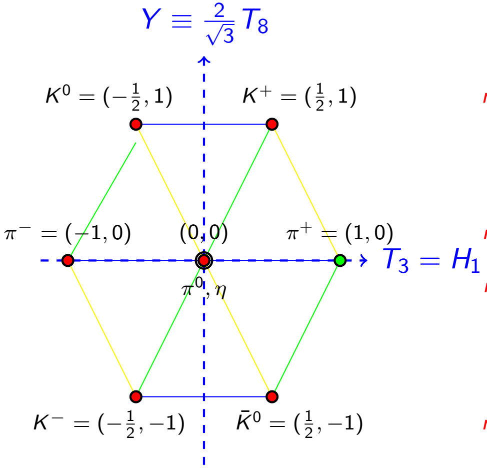


Clasificación de hadrones [1]: Mesones Pseudoescalares ($J^P = 0^-.$)



[1] M. Gell-Mann, California Institute of Technology Synchrotron Laboratory Report No. CTSL—20, 1961 (unpublished); Y.Ne'eman, Nuclear Phys. 26, 222 (1961)

$J^P = 0^-$: Plano Isoespín-Hypercarga



$m_K = 495$ MeV

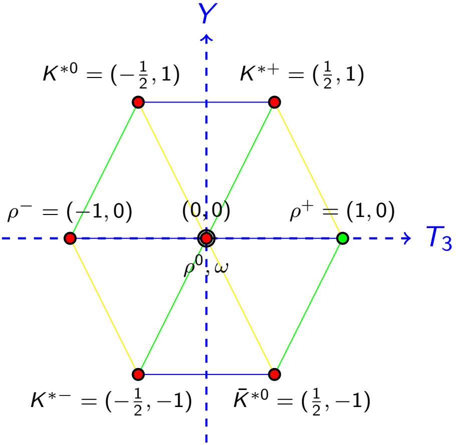
$m_\pi = 137$ MeV

$m_\eta = 550$ MeV

$m_K = 495$ MeV

$$Q = T_3 + \frac{Y}{2}$$

$J^P = 1^-$: Mesones Vectoriales



$m_{K^*} = 892\ MeV$

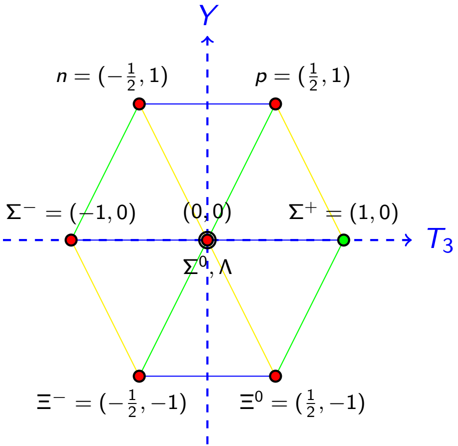
$m_\rho = 770\ MeV$

$m_\omega = 780\ MeV$

$m_{K^*} = 892\ MeV$

$$Q = T_3 + \frac{Y}{2}$$

$J^P = \frac{1}{2}^+$: Bariones de espin 1/2



$m_N = 938\ MeV$

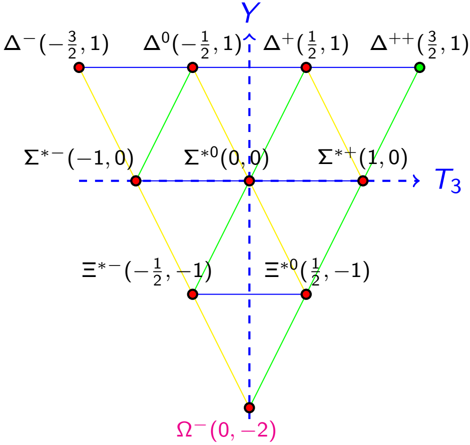
$m_\Sigma = 1190\ MeV$

$m_\Lambda = 1115\ MeV$

$m_\Xi = 1320\ MeV$

$$Q = T_3 + \frac{Y}{2}$$

$J^P = \frac{3}{2}^+$: Bariones de espin 3/2



$m_\Delta = 1320$

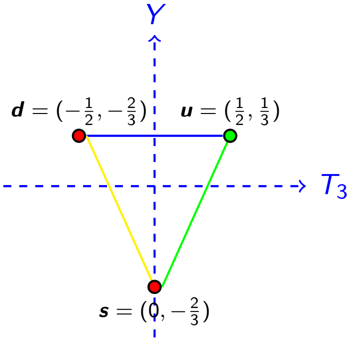
$m_{\Sigma^*} = 1370$

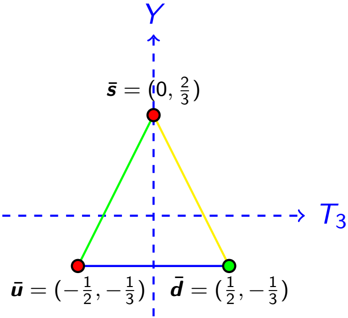
$m_{\Xi^*} = 1450$

$m_\Omega = 1580$

$$Q = T_3 + \frac{Y}{2}$$

Quarks y representaciones fundamentales $\mathbf{3}$ y $\bar{\mathbf{3}}$

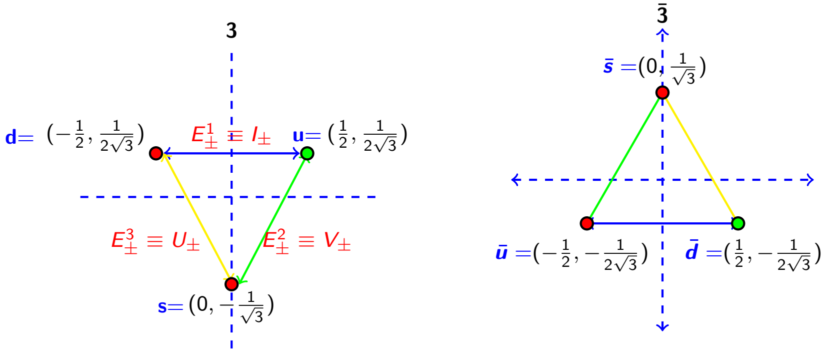




$Q = T_3 + \frac{Y}{2}$

$$Q_u = \frac{2}{3}, \qquad Q_d = -\frac{1}{3}, \qquad Q_s = -\frac{1}{3}$$

Modelo de quarks: Producto tensorial de **3** y $\bar{\mathbf{3}}$

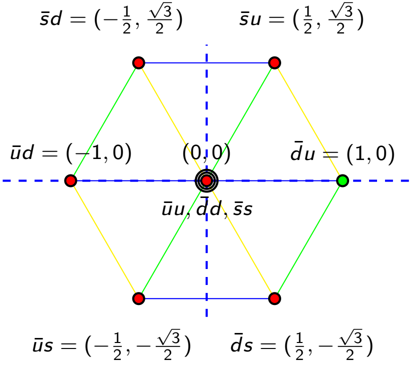


En notación simplificada:

$$I_- u = d,\ I_- \bar{d} = -\bar{u}, \quad V_- u = s,\ V_- \bar{s} = -\bar{u}, \quad U_- s = d,\ U_- \bar{d} = -\bar{s}$$
$$I_+ d = u,\ I_+ \bar{u} = -\bar{d}, \quad V_+ s = u,\ V_+ \bar{u} = -\bar{s}, \quad U_+ d = s,\ U_+ \bar{s} = -\bar{d}.$$

Las otras posibilidades dan un resultado nulo.

- La base del espacio $\mathcal{H}_1 \otimes \mathcal{H}_2$ es $|\bar{q}_i\rangle \otimes |q_j\rangle \equiv \bar{q}_i q_j$, $q_1 = u$, $q_2 = d$, $q_3 = s$.
- Los números cuánticos de los operadores de la SAC son aditivos.
- Hay 9 estados en la base: $\{\bar{u}u, \bar{d}d, \bar{s}s, \bar{u}d, \bar{u}s, \bar{d}u, \bar{d}s, \bar{s}u, \bar{s}d\}$



- Usamos los operadores de escalera para identificar los miembros de las irreps generadas.

- Los estados con $(0,0)$ obtenidos con los operadores de escalera son:

$$\begin{aligned} I_-|\bar{d}u\rangle &= -|\bar{u}u\rangle + |\bar{d}d\rangle, & I_+|\bar{u}d\rangle &= -|\bar{d}d\rangle + |\bar{u}u\rangle \\ V_-|\bar{s}u\rangle &= -|\bar{u}u\rangle + |\bar{s}s\rangle, & V_+|\bar{u}s\rangle &= -|\bar{s}s\rangle + |\bar{u}u\rangle \\ U_-|\bar{d}s\rangle &= -|\bar{s}s\rangle + |\bar{d}d\rangle, & U_+|\bar{s}d\rangle &= -|\bar{d}d\rangle + |\bar{s}s\rangle \end{aligned}$$

- Solo dos de estos estados son independientes.
- El estado normalizado que es parte del triplete de isospin es

$$|\pi^0\rangle = \frac{1}{\sqrt{2}}(|\bar{u}u\rangle - |\bar{d}d\rangle)$$

- El estado ortogonal a éste es

$$|\eta\rangle = \frac{1}{\sqrt{6}}(|\bar{u}u\rangle + |\bar{d}d\rangle - 2|\bar{s}s\rangle)$$

- El otro estado ortogonal no pertenece a esta irrep

$$|\eta'\rangle = \frac{1}{\sqrt{6}}(|\bar{u}u\rangle + |\bar{d}d\rangle + |\bar{s}s\rangle)$$

- Este estado satisface

$$I_+|\eta'\rangle = \frac{1}{\sqrt{6}}(-|\bar{d}u\rangle + |\bar{d}u\rangle) = 0$$

- En forma similar

$$E^j_\pm|\eta'\rangle = 0.$$

- El estado $|\eta'\rangle$ es un singlete de $SU(3)$ (irrep $(0,0)$)
- Conclusión:

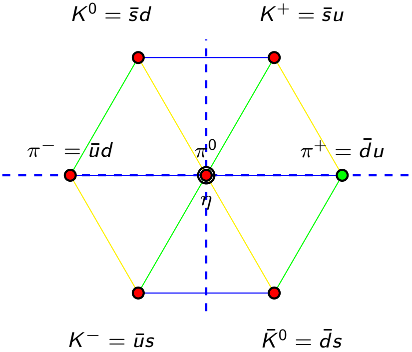
$$\mathbf{\bar{3}} \otimes \mathbf{3} = \mathbf{1} \oplus \mathbf{8}$$

- En forma similar:

$$\mathbf{3} \otimes \mathbf{3} = \mathbf{\bar{3}} \oplus \mathbf{6}$$

$$\mathbf{3} \otimes \mathbf{3} \otimes \mathbf{3} = (\mathbf{\bar{3}} \oplus \mathbf{6}) \otimes \mathbf{3} = \mathbf{1} \oplus \mathbf{8} \oplus \mathbf{8} \oplus \mathbf{10}$$

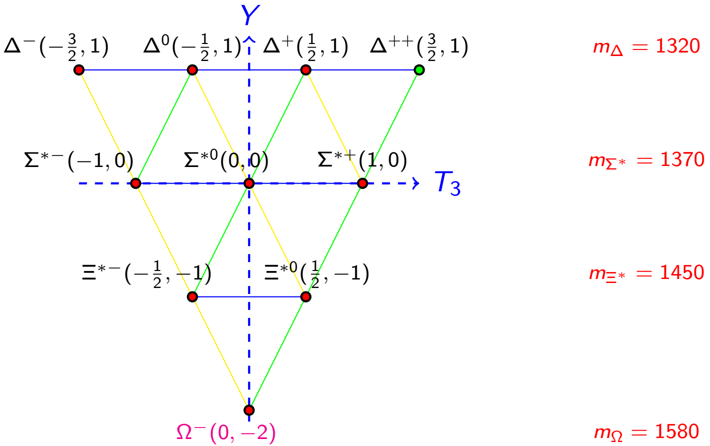
Hadrones y quarks: $\bar{\mathbf{3}} \otimes \mathbf{3} = \mathbf{1} \oplus \mathbf{8}$



$$\pi^0 = \frac{1}{\sqrt{2}}(\bar{u}u - \bar{d}d)$$

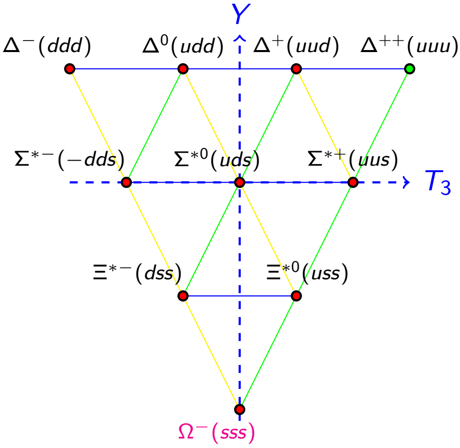
$$\eta = \frac{1}{\sqrt{6}}(\bar{u}u + \bar{d}d - 2\bar{s}s)$$

$SU(3)_F$ quantum numbers of the $J^P = \frac{3}{2}^+$ decuplet



$$Q = T_3 + \frac{Y}{2}$$

Quark content of the $J^P = \frac{3}{2}^+$ decuplet



$m_\Delta = 1320$

$m_{\Sigma^*} = 1370$

$m_{\Xi^*} = 1450$

$m_\Omega = 1580$

- The states Δ^{++}, Δ^- and Ω^- are systems composed of identical fermions.
- Completely symmetric under the exchange of quarks (flavor).
- The spin state $|+++\rangle$ is also symmetric.
- The space state (wave function) for the ground state is also symmetric.
- Systems of identical fermions symmetric under the exchange of particles.
- Something is wrong! (or something is missing!) [2]
- Recall $\mathbf{3} \otimes \mathbf{3} \otimes \mathbf{3} = \mathbf{1} \oplus \mathbf{8} \oplus \bar{\mathbf{8}} \oplus \mathbf{10}$.
- The singlet $\mathbf{1}$ is completely antisymmetric.
- Is there another $SU(3)$ behind and known strong-interacting particles are singlets? color quantum numbers.

$$\Delta^{++}_{color} = \frac{1}{\sqrt{6}}(uuu - uuu + uuu - uuu + uuu - uuu)$$

[2] 1965: Struminsky, Bogolubov-Struminsky-Tavkhelidze, Greenberg, Han-Nambu.

Gauge theory of strong interactions: $SU(3)_c$.

- Hard to paint colors, attach and index for colors $i = 1, 2, 3$.
- Start withe a single flavor $q = u$ and assume that comes in 3 colors u_j, $j = 1, 2, 3$.

$$\mathcal{L}_u = \sum_{j=1}^{3} \bar{u}_j [i\gamma^\mu \partial_\mu - m_{u_j}] u_j = \bar{u}[i\gamma^\mu \partial_\mu - M_u] u$$

with $M_u = Diag(m_{u_1}, m_{u_2}, m_{u_3})$ and $\bar{u} = (\bar{u}_1, \bar{u}_2, \bar{u}_3)$.

- In the case $m_{u_1} = m_{u_2} = m_{u_3} \equiv m_u$, $M_u = m_u \mathbb{1}_{3\times 3}$ the Lagrangian is invariante under the global SU(3)

$$u \to u' = Uu = e^{-iT^a\theta^a} u, \quad U \in SU(3) \qquad a = 1, 2, 3.$$

- Assume now this is a gauge symmetry:

$$u \to u' = U(x)u = e^{-iT^a\theta^a(x)} u, \qquad A'_\mu = U A_\mu U^{-1} + \frac{i}{g}(\partial_\mu U) U^{-1}$$

- The gauge invariant Lagrangian is

$$\mathcal{L}_u = \bar{u}_i[(i\gamma^\mu(\partial_\mu\delta_{ij}+ig_s A^a_\mu T^a_{ij}) - m_u\delta_{ij}]u_j - \frac{1}{4}F^{a\mu\nu}F^a_{\mu\nu}$$

where

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - g_s\ f^a{}_{bc}A^b_\mu A^c_\nu.$$

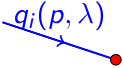
- There are three u_i colored quarks: $i = 1, 2, 3$.
- There are eight colored gauge fields A^a_μ, $a = 1, ..8$: gluon fields.
- The gluon field A^a_μ couples to the quarks u_i and u_j with a strength $g_s T^a_{ij}$: color charge.
- There are many color charges but all of them are related to a single coupling constant g_s by an $SU(3)$ factor.
- Similar results for every flavor $q = u, d, c, s, t, b$.

$$\mathcal{L} = \sum_{q=u,d,c,s,t,b} \bar{q}_i[(i\gamma^\mu(\partial_\mu\delta_{ij}+ig_s A^a_\mu T^a_{ij}) - m_u\delta_{ij}]q_j - \frac{1}{4}F^{a\mu\nu}F^a_{\mu\nu}$$

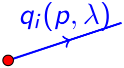
- Very important: QCD allows quark masses but they are forbidden by the chiral structure of weak interactions.
- Quark masses generated by the Higgs mechanism: $m_{q_i} = \lambda_{q_i} v/\sqrt{2}$, where λ_{q_i} is the combination of CKM coefficients arising in the diagonalization of the quark mass matrix (see S12).

$$
\begin{aligned}
-\frac{1}{4}F^{a\mu\nu}F^{a}_{\mu\nu} = & -\frac{1}{4}(\partial^\mu A^{a\nu} - \partial^\nu A^{a\mu} - g_s\ f^{abc} A^{b\mu} A^{c\nu}) \\
& \times (\partial_\mu A^a_\nu - \partial_\nu A^a_\mu - g_s\ f^{ade} A^d_\mu A^e_\nu) \\
= & -\frac{1}{4}(\partial^\mu A^a_\nu - \partial^\nu A^{a\mu})(\partial_\mu A^a_\nu - \partial_\nu A^a_\mu) \\
& + \frac{g_s}{2}\ f^{abc}(\partial_\mu A^a_\nu - \partial_\nu A^a_\mu) A^{b\mu} A^{c\nu} \\
& - \frac{g_s^2}{4}\ f^{abc}\ f^{ade} A^{b\mu} A^{c\nu} A^d_\mu A^e_\nu \\
= & \mathcal{L}_K + \mathcal{L}_{3g} + \mathcal{L}_{4g}.
\end{aligned}
$$

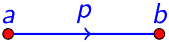
Feynman Rules for QCD



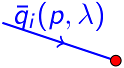
$= u_i(p,\lambda)$



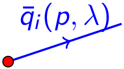
$= \bar{u}_i(p,\lambda)$



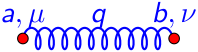
$= i\frac{(\not{p}+m)\delta^{ab}}{p^2-m^2+i\epsilon}$



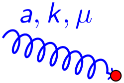
$= \bar{v}_i(p,\lambda)$



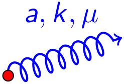
$= v_i(p,\lambda)$



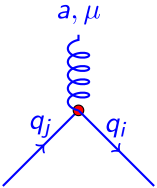
$= i\frac{-g^{\mu\nu}\delta^{ab}}{q^2+i\epsilon}$



$= \varepsilon^a_\mu(k,\lambda)$

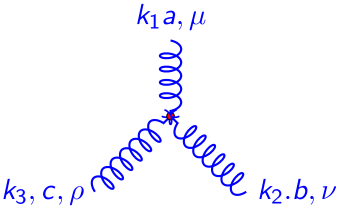


$= \varepsilon^{a*}_\mu(k,\lambda)$

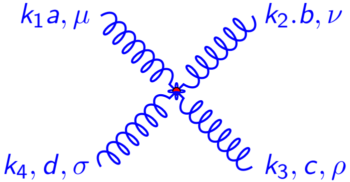


$= ig_s\gamma^\mu T^a_{ij}$

Feynman gauge

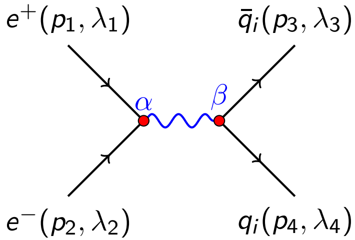


$$= -g_s f^{abc}[g_{\mu\nu}(k_1 - k_2)_\rho + g_{\nu\rho}(k_2 - k_3)_\mu + g_{\rho\mu}(k_3 - k_1)_\nu]$$



$$\begin{aligned} = & - \quad i g_s^2 [f^{eab} f^{ecd}(g_{\mu\rho} g_{\nu\sigma} - g_{\mu\sigma} g_{\nu\rho}) + f^{eac} f^{edb}(g_{\mu\sigma} g_{\nu\rho} - g_{\mu\nu} g_{\rho\sigma}) \\ & + \quad f^{ead} f^{ebc}(g_{\mu\nu} g_{\rho\sigma} - g_{\mu\sigma} g_{\nu\rho})] \end{aligned}$$

Evidence for color: $e^+(p_1)e^-(p_2) \rightarrow \bar{q}_i(p_3)q_i(p_4) \rightarrow$ hadrons



Calculation similar to $e^+(p_1)e^-(p_2) \rightarrow \mu^+(p_3)\mu^-(p_4)$

$$-i\mathcal{M} = \bar{v}_e(p_1,\lambda_1)[ieQ_e\gamma^\alpha]u_e(p_2,\lambda_2)[\frac{-ig_{\alpha\beta}}{(p_1+p_2)^2+i\epsilon}]\bar{u}_\mu(p_4,\lambda_4)[ieQ_i\gamma^\beta]v_\mu(p_3,\lambda_3)$$

$$|\bar{\mathcal{M}}|^2 = \frac{2e^4Q_q^2}{s^2}[(t-m_e^2-m_q^2)^2+(u-m_e^2-m_q^2)^2+2s(m_q^2+m_e^2)]$$

In the high energy limit $s >> 4m_q^2$ we get

$$\sigma(e^+e^- \to \bar{q}q) = \frac{4\pi\alpha^2 N_c Q_q^2}{3s}$$

Recall

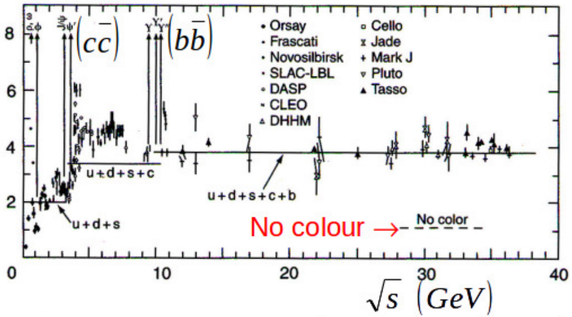
$$\sigma(e^+e^- \to \mu^+\mu^-) = \frac{4\pi\alpha^2}{3s}$$

Quarks eventually produce jets of hadrons

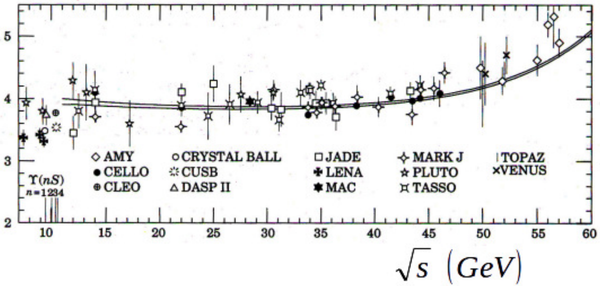
$$R = \frac{\sigma(e^+e^- \to hadrons)}{\sigma(e^+e^- \to \mu^+\mu^-)} = N_c \sum_q Q_q^2$$

This quantity depends on the available CM energy.

$$R = \begin{cases} 3(\frac{4}{9} + \frac{1}{9}) = \frac{5}{3} = 1.66, & \text{for } q = u, d \\ 3(\frac{4}{9} + 2\frac{1}{9}) = 2, & \text{for } q = u, d, s \\ 3(2\frac{4}{9} + 2\frac{1}{9}) = \frac{10}{3} = 3.33, & \text{for } q = u, d, s, c \\ 3(2\frac{4}{9} + 3\frac{1}{9}) = \frac{11}{3} = 3.66, & \text{for } q = u, d, s, c, b \\ 3(3\frac{4}{9} + 3\frac{1}{9}) = 5, & \text{for } q = u, d, s, c, b, t \end{cases}$$

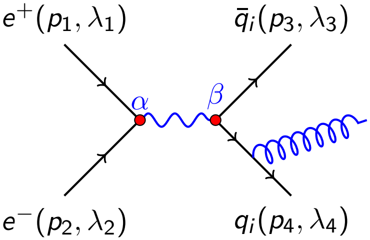


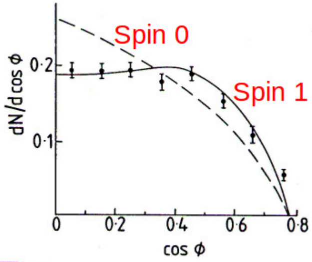
$(c\bar{c})$
$(b\bar{b})$
Orsay
Frascati
Novosilbirsk
SLAC-LBL
DASP
CLEO
DHHM
Cello
Jade
Mark J
Pluto
Tasso
u+d+s+c
u+d+s+c+b
u+d+s
No colour →
No color
0
10
20
30
40
2
4
6
8
$\sqrt{s}\ (GeV)$



AMY
CELLO
CLEO
CRYSTAL BALL
CUSB
DASP II
JADE
LENA
MAC
MARK J
PLUTO
TASSO
TOPAZ
VENUS
$\Upsilon(nS)$
$n=1234$
10
15
20
25
30
35
40
45
50
55
60
2
3
4
5
$\sqrt{s}\ (GeV)$

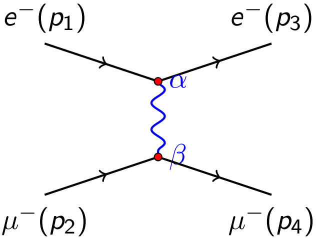
Evidence for gluons: $e^+e^- \to \bar{q}qg$

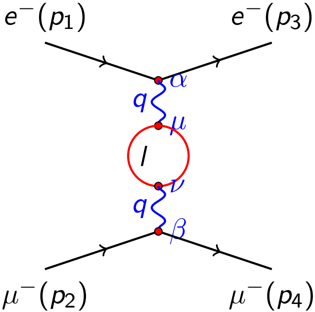




- Signature: three jets of hadrons
- Angular distribution of jets depend on the spin of the particles.
- First three jets events detected by JADE Coll. at DESY in 1978.
- Angular distribution consistent with a gluon with spin 1.

Running couplings: QED





$$-i\mathcal{M}^{(2)} = ie^2 \bar{u}(p_3)\gamma^\alpha u(p_1)\left[\frac{g_{\alpha\beta}}{q^2}\right]\bar{u}(p_4)\gamma^\beta u(p_2)$$

$$-i\mathcal{M}^{(4)} = ie^2 \bar{u}(p_3)\gamma^\alpha u(p_1)\left[\frac{-iT_{\alpha\beta}}{q^4}\right]\bar{u}(p_4)\gamma^\beta u(p_2)$$

$$T_{\alpha\beta} = -e^2 \int \frac{d^4 l}{2\pi^4} \frac{Tr[\gamma_\alpha(\not{l} + m_e)\gamma_\beta(\not{l} + \not{q} + m_e)]}{(l^2 - m_e^2)((l+q)^2 - m_e^2)}$$

- The loop diagram provoques the change

$$\frac{e^2 g_{\alpha\beta}}{q^2} \to \frac{e^2 g_{\alpha\beta}}{q^2} - i\frac{e^2 T_{\alpha\beta}}{q^4}$$

- $T_{\alpha\beta}$ is infinite. Regularize it with a cutoff Λ.

$$T_{\alpha\beta} = - i g_{\alpha\beta} q^2 I(q^2), \qquad I(q^2) = \frac{e^2}{12\pi^2}[\ln\frac{\Lambda^2}{m_e^2} - f(\frac{q^2}{m_e^2})]$$

$$f(\frac{q^2}{m_e^2}) = 6\int_0^1 dzz(1-z)\ln[1 - \frac{q^2}{m_e^2} z(1-z)]], \qquad f(0) = 0.$$

- Modification due to the loop diagram

$$\frac{e^2 g_{\alpha\beta}}{q^2} \to \frac{e^2 g_{\alpha\beta}}{q^2}(1 - I(q^2))$$

or

$$e^2 \to e^2(1 - I(q^2)) = e^2(1 - \frac{e^2}{12\pi}[\ln\frac{\Lambda^2}{m_e^2} - f(\frac{q^2}{m_e^2})]) \equiv e^2(q^2).$$

- At low energies what we measure in an experiment is actually

$$e^2(0) = e^2(1 - \frac{e^2}{12\pi} \ln \frac{\Lambda^2}{m_e^2}).$$

- Using this physical quantity

$$e^2(q^2) = e^2(0) + \frac{e^4}{12\pi} f(\frac{q^2}{m_e^2}) = e^2(0)[1 + \frac{e^4}{12\pi^2 e^2(0)} f(\frac{q^2}{m_e^2})]$$
$$= e^2(0)[1 + \frac{e^2(0)}{12\pi^2} f(\frac{q^2}{m_e^2}) + \mathcal{O}(e^4)]$$

- In terms of the fine structure constant

$$\alpha(q^2) = \alpha(0)[1 + \frac{\alpha(0)}{3\pi} f(\frac{q^2}{m_e^2})] \equiv \alpha(0)[1 + X]$$

- Considering two loops in the propagator yields

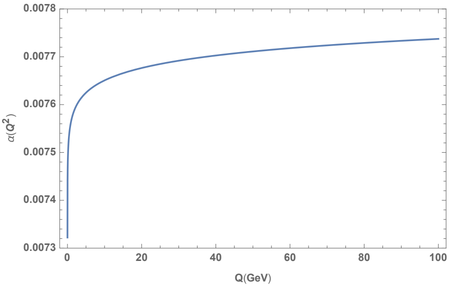
$$\alpha(q^2) = \alpha(0)[1 + X + X^2]$$

- The full sum of loops in the propagator yields

$$\alpha(q^2) = \frac{\alpha(0)}{1 - \frac{\alpha(0)}{3\pi} f(\frac{q^2}{m_e^2})}$$

- For $-q^2 >> m_e^2$ we get $f(\frac{q^2}{m_e^2}) = \ln \frac{-q^2}{m_e^2}$. Define $Q^2 = -q^2$:

$$\alpha(Q^2) = \frac{\alpha(0)}{1 - \frac{\alpha(0)}{3\pi} \ln \frac{Q^2}{m_e^2}}$$



- We have the coupling constant measured at $q^2 = 0$ as a reference. Let us choose a different scale μ^2

$$\frac{1}{\alpha(Q^2)} = \frac{1}{\alpha(0)} - \frac{1}{3\pi}\ln\frac{Q^2}{m_e^2} \Rightarrow \frac{1}{\alpha(\mu^2)} = \frac{1}{\alpha(0)} - \frac{1}{3\pi}\ln\frac{\mu^2}{m_e^2}$$

- Substracting

$$\frac{1}{\alpha(Q^2)} - \frac{1}{\alpha(\mu^2)} = -\frac{1}{3\pi}\ln\frac{Q^2}{\mu^2}$$

- Finally, for $m_e^2 << \mu^2 < Q^2$ we get

$$\alpha(Q^2) = \frac{\alpha(\mu^2)}{1 - \frac{\alpha(\mu^2)}{3\pi}\ln\frac{Q^2}{\mu^2}}$$

Beta function

- Taking the derivative with respect to $t = \ln Q^2$

$$\frac{d}{dt}\frac{1}{\alpha} = -\frac{1}{\alpha^2}\frac{d\alpha}{dt} = -\frac{1}{3\pi}$$

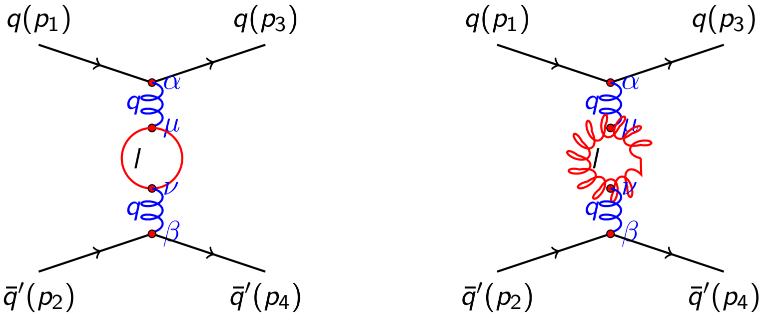
- We define the beta function as

$$\beta(Q^2) \equiv \frac{d\alpha(Q^2)}{d\ln Q^2} = -(\beta_0\alpha^2 + \beta_1\alpha^3 + \beta_2\alpha^4 + \ldots)$$

- Our calculation yields the leading order term $\beta_0 = -\frac{1}{3\pi}$

$$\alpha(Q^2) = \frac{\alpha(\mu^2)}{1 + \beta_0\alpha(\mu^2)\ln\frac{Q^2}{\mu^2}}$$

Running couplings: QCD



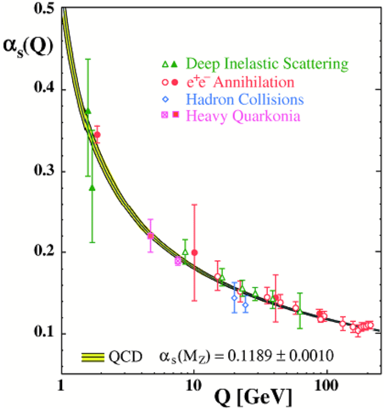
- New contributions from the non-abelian terms.
- Cutoff regularization breaks down gauge symmetry. Use dimensional regularization.

- Adding these contributions to the tree level one yields

$$\alpha_s(Q^2) = \frac{\alpha_s(\mu^2)}{1 + \beta_0 \alpha_s(\mu^2) \ln \frac{Q^2}{\mu^2}}, \quad with \quad \beta_0 = \frac{11N_c - 2N_f}{12\pi}$$

- The factor $\frac{-2N_f}{12\pi} < 0$ comes from the quark loop. Same as the $-\frac{1}{3\pi}$ in QED except for a color factor.
- Causes the raising of the coupling with the increase of Q^2
- The factor $\frac{11N_c}{12\pi} > 0$ comes from the gluon loop. Coupling constant decreases with Q^2. Dominant contribution.
- α_s vanishes at very high energy: **Asymptotic freedom**.
- It increases at low energy reaching high values at $Q \simeq 1\ GeV$: **signals of confinement**.
- Perturbative expansion breaks down at this energy and it is not trustworthy.

QCD Running vs. experiment



- QCD coupling is much larger than e.m. coupling: strong force indeed.
- Also running is stronger.

Standard Model Overview: $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$

- The complete Standard Model Lagrangian is

$$\mathcal{L} = \sum_{a=1}^{3} \bar{L}^a i\gamma^\mu D_\mu L^a + \sum_{i=1}^{6} \bar{R}_i i\gamma^\mu D_\mu R_i - \frac{1}{4} W^a_{\mu\nu} W^{a\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu}$$
$$+ \sum_{a=1}^{3} \bar{q'}_L^a i\gamma^\mu D_\mu q'^a_L + \mathcal{L}_{Yukawa} + (D_\mu \Phi)^\dagger D^\mu \Phi - m^2 \Phi^\dagger \Phi - \lambda(\Phi^\dagger \Phi)^2$$
$$+ \sum_{q=u,d,c,s,t,b} \bar{q}_i [(i\gamma^\mu(\partial_\mu \delta_{ij} + i g_s G^a_\mu T^a_{ij}) - m_q \delta_{ij}] q_j - \frac{1}{4} G^{a\mu\nu} G^a_{\mu\nu}$$

with $m^2 < 0$.

- Covariant derivatives

$$D_\mu L = \left(\partial_\mu - ig\frac{\sigma_i}{2} W^i_\mu - ig'\frac{Y}{2} B_\mu\right) L,$$
$$D_\mu R_i = \left(\partial_\mu - ig'\frac{Y}{2} B_\mu\right) R_i,$$

- Covariant derivatives

$$D_\mu \Phi = \left(\partial_\mu - ig \frac{\sigma_i}{2} W^i_{\ \mu} - ig' \frac{Y}{2} B_\mu \right) \Phi$$
$$D_\mu q'_L = \left(\partial_\mu - ig \frac{\sigma_i}{2} W^i_{\ \mu} - ig' \frac{Y}{2} B_\mu \right) q'_L,$$

- There is a replication of families

$$\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L, \quad \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L, \quad \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L;$$

$$q'^{\,1}_L = \begin{pmatrix} u' \\ d' \end{pmatrix}_L, \quad q'^{\,2}_L = \begin{pmatrix} c' \\ s' \end{pmatrix}_L, \quad q'^{\,3}_L = \begin{pmatrix} t' \\ b' \end{pmatrix}_L.$$

- Quark interaction eigenstates (q') differ from the strong interaction eigenstates (q): C-K-M. *CP Violation*.
- Experimental results show that there is neutrino mixing. Requires neutrinos (tiny) masses: PMNS. *CP Violation*.

G r a c i a s ! ! !