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THE STANDARD MODEL OF ELEMENTARY PARTICLES

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Abstract

It is first reviewed how the concepts of a QFT and of local gauge invariance naturally arise, before they are applied to the electroweak model of leptons. This model is then extended to include quarks and the strong interaction, family replication and quark mixing, CP violation, as well as neutrino mass and mixing. Experimental tests, the determination of fundamental parameters, constraints on and limitations of the minimal model and ideas that go beyond it are also covered.

1.1
Introduction

All known phenomena in the physical world are aspects of only four basic forces: gravity, electromagnetism, the strong force, and the weak force. While these forces appear to have very different characters, all of them are based on *quantum field theories* (QFTs) exhibiting some form of *local gauge symmetry*. Although the historical development was different, with hindsight local gauge invariance is now seen not as a fundamental principle, but as a consequence of the more basic principles of Lorentz invariance and cluster decomposition, as well as the axioms of quantum mechanics. The following Section 1.2 first reviews how the concepts of a QFT and of local gauge symmetry naturally arise before they are applied to the prototype electroweak model of electrons, e^-, positrons, e^+, and electron-neutrinos, ν_e. Section 1.3 discusses a series of extensions of this model to include quarks and the strong interaction, family replication and quark mixing, and CP violation. Experimental tests and constraints on the Standard Model (SM) are the subject of Section 1.4. Finally, the concluding Section 1.5 describes limitations of the minimal model and goes beyond Section 1.3 to incorporate neutrino mass and mixing. It also discusses accidental symmetries and their possible violation by non-renormalizable corrections.

1.2
Principles and Theoretical Foundations of the Standard Model

Rather than reviewing how the SM emerged historically as the correct theory of weak and electromagnetic interactions we first motivate the relevance of QFTs in general (Section 1.2.1) and gauge theories in particular (Section 1.2.2), before specializing to the electroweak gauge theory in Section 1.2.3. Key features like non-Abelian gauge symmetry, spontaneous symmetry breaking and the Higgs mechanism, renormalizability, and anomaly cancellation in chiral gauge theories are covered in turn.

1.2.1
Quantum Field Theories

The mathematical framework to study relativistic particles and their interactions is provided by QFTs. A particle can be defined as an irreducible unitary representation (irrep) of the inhomogeneous Lorentz group (Poincaré group), which is characterized (classified) by its mass, m, and either its spin (the eigenvalue of the square of the angular momentum operator, $\vec{J}$) or in the massless case its helicity[1], h (the eigenvalue of the third component, J_3, of $\vec{J}$). It is convenient to introduce quantum mechanical operators, $a^\dagger$ and a, respectively creating and annihilating particles of a particular species with a specific relativistic four-momentum (energy-momentum) and spin (including its direction). This serves to satisfy (also in non-relativistic theories) the general scientific requirement that distant experiments should yield independent results (locality), *i.e.*, quantum mechanical probabilities factorize (cluster decomposition). The most general Hamiltonian is then a sum of products of any number of $a^\dagger$ and a operators, which are recalled to be defined in momentum space.

However, a relativistic theory requires in a addition to particles transforming according to representations of the Poincaré group also Lorentz-covariant scattering amplitudes (S-matrix). Invariance under Lorentz transformations (boosts) presents an obstacle, because (according to Noether's theorem) there must be a conserved vector related to them, but (unlike linear and angular momentum operators) boost generators fail to commute with the Hamiltonian. This complication introduces an additional condition (with no counterpart in

1) In contrast to the massive case, the commutator algebra of symmetry generators (Lie algebra) does not restrict the possible (real) eigenvalues of J_3, but there is a topological restriction allowing only half-integer of integer values of h.

non-relativistic theories) which often takes the form of a causality requirement, *i.e.*, Hamiltonian densities at space-like separation ought to commute. The important point is that causality conditions are expressed in configuration (position) space.

Therefore one integrates the creation and annihilation operators over all spatial momenta[2] $\vec{p}$ (Fourier transformation),

$$\phi_l(x) = \int \frac{d^3p}{(2\pi)^{3/2}} \frac{1}{\sqrt{2p^0}} (u_l \, a_{\vec{p}} \, e^{-ipx} + v_l \, a^\dagger_{\vec{p}} \, e^{ipx}) \tag{1.1}$$

yielding functions of spacetime with a set of discrete Lorentz and spinor indices (collectively called l) giving rise to finite-dimensional non-unitary (unless a Lorentz scalar) and in general reducible representations of the homogeneous Lorentz group. The Fourier transforms (1.1) are the quantum fields. The Hamiltonian is now a sum of products of quantum fields (and their Hermitian adjoints) and one can construct Lorentz scalars in much the same way as one constructs rotational scalars by the use of Clebsch-Gordon coefficients.

The causality condition on the Hamiltonian translates into a similar commutation relation (or anticommutation relation for fermions) for fields and this will in turn impose constraints on the coefficients, u_l and v_l. In the course of solving these constraints one encounters the fundamental theorems of QFT:

field equations: Every free field must satisfy the second-order differential wave equation, $(\partial^\mu \partial_\mu + m^2)\phi_l(x) = 0$ (Klein-Gordon equation), and many fields must obey other first-order differential equations or algebraic constraint equations, as well.

antiparticles: Antiparticles (1) exist for any particle species (not just for spin-1/2 fermions as in the Dirac formalism) with an additive conserved quantum number (*e.g.*, electric charge, Q).

spin-statistics connection: Fields describing particles with (half)-integer spin are (fermions) bosons.

CPT theorem: Any QFT is invariant under the combined operations of charge (particle–antiparticle) conjugation (C), parity (space) reflexion (P), and time reversal (T).

2) $p^0 = \sqrt{m^2 + \vec{p}^2}$ is the particle energy; x_0 appearing in $px \equiv p^0 x_0 - \vec{p}\vec{x}$ denotes time multiplied by the speed of light.

1.2.2
Local Gauge Symmetries

In some cases it is impossible to find any solution to the constraints on the u_l and v_l. In particular, no four-vector field, A^μ, can be constructed from the annihilation and creation operators for a particle of helicity ± 1 and $m = 0$ (2). Of course, one might use some other Lorentz representation, such as an antisymmetric tensor field, $B^{\mu\nu}$, which does allow a solution. However, the interactions in such a theory will have a faster fall-off at large distances than the inverse-square law of the electrostatic Coulomb force. Such interactions may well exist in nature, but they would be strongly suppressed relative to inverse-square law interactions. But while it may suffice to have a covariantly transforming four-vector field, A^μ, to construct a Lorentz-invariant theory, this is not actually necessary. Indeed, under a general homogeneous Lorentz transformation, Λ, one finds,

$$U(\Lambda)A^\mu(x)U^{-1}(\Lambda) = \Lambda^{\nu\mu}A_\nu(x) + \partial^\mu\Omega(x), \tag{1.2}$$

i.e., the A^μ in this case transforms covariantly up to an extra term, where $\Omega(x)$ is a linear combination of $a^\dagger$ and a operators. Therefore, if for arbitrary functions, $\omega(x)$, we require invariance under local (spacetime-dependent) "gauge" transformations of the form,

$$A^\mu(x) \to A^\mu(x) - \partial^\mu\omega(x), \tag{1.3}$$

then the failure of A^μ to transform covariantly would be immaterial. Thus, local gauge invariance provides us with an additional option to construct long-range forces mediated by particles of $h = \pm 1$ and $m = 0$, and (if present) we should expect gauge-invariant theories to play a dominant role at large distances. This is precisely what is observed and the electromagnetic, strong, and weak forces, are all gauge theories invariant under the replacement (1.3).

This discussion generalizes straightforwardly to the case of gravity. No symmetric rank-two tensor field, $h^{\mu\nu}$, can be constructed from the annihilation and creation operators for a particle of $h = \pm 2$ and $m = 0$ (2). In order to construct an inverse-square law (Newton's law) one still needs to work with $h^{\mu\nu}$, where here the failure to transform covariantly is cured by requiring invariance under general coordinate transformations as in the General Theory of Relativity (3) generalizing the local gauge transformation (1.3).

The mathematics of all this generalizes further to massless particles of higher helicities $h > 2$. Now, Noether's theorem implies that conserved quantities must be associated with these transformations, which are electric charge (and generalizations thereof) for gauge theories and energy-momentum conservation in the case of gravity. There would be associated conserved quantities for the case $h > 2$, but the abundance of conservation laws would rule out the possibility of non-trivial scattering processes of any kind. Thus, the physics of this does not generalize beyond gravity. It is not yet known, whether the only other possible case, $h = \pm 3/2$, is realized in nature[3]. If it is, the corresponding transformations generalizing the local gauge transformation (1.3) are the local "supersymmetry" transformations. Supersymmetry (4; 5) is an active subfield of elementary particle physics and among the leading candidates of possible physics that may modify the SM at higher energies.

The historical development was, of course, rather different. The theory of electromagnetism (6) was not constructed from Lorentz and local gauge invariance but rather gave rise to these concepts (7).

1.2.3
Gauge Group of Electroweak Interaction

The gauge symmetry must and does (8) persist after the field A^μ is coupled to matter. In quantum electrodynamics (QED) one therefore extends the gauge transformation (1.3) to electrons and positrons to undergo the local $U(1)$ (phase) transformation (e is the electromagnetic coupling constant),

$$\Psi(x) \to e^{\pm ieQ\omega(x)}\Psi(x). \tag{1.4}$$

In the minimal electroweak theory one assumes that the electron combines with the electron-neutrino to form a doublet, extending the Abelian $U(1)$ gauge symmetry of QED to some (in general) non-Abelian symmetry group. In non-Abelian gauge theories particles can be transformed into other kinds of particle, making them suitable candidates for describing the weak interaction that can transform, *e.g.*, electrons into neutrinos, or neutrons into protons. Positivity of quantum mechanical probabilities implies that the associated Lie algebra is the direct sum of commuting compact simple and $U(1)$ subalgebras. These have been classified[4] by Cartan (10), leaving only a few possibilities.

To see what these are, a unique feature of the weak interaction (not shared by the other known forces) must now be mentioned, namely that electroweak gauge transformations act distinctly on left- and right-handed particles (chirality). This possibility arises if one postulates invariance under proper Lorentz transformations only (*i.e.*, excluding parity reflections) with respect to which left- and right-handed particles form separate irreps. There is presently no piece of experimental evidence which would incontrovertible require the existence of a right-handed neutrino and it is excluded from the minimal theory. And since right-handed particles are not independent from left-handed antiparticles, we may restrict our attention to the left-handed triplet of fields,

3) The cases $h = 0$ and $h = \pm 1/2$ do not present the kind of complications discussed here.

4) For an extensive discussion with emphasis on the application to particle physics, see Slansky (9).

$(\nu_e, e^-, e^+)_L$, with the understanding that the physics of right-handed particles and antiparticles is determined in terms of the left-handed ones.

Now the largest possible symmetry group that may act on this triplet is $U(3) = SU(3) \times U(1)$, but such a theory would imply lepton number violation at rates that are in conflict with observation. The largest possible subgroup of $U(3)$ consistent with lepton number is $U(2)_L \times U(1)_R = SU(2)_L \times U(1)_L \times U(1)_R$ with $SU(2)_L$ (isospin) acting non-trivially on the doublet of fields, (ν_e, e^-). The diagonal linear combination of $U(1)_L$ and $U(1)_R$ would give rise to a long-range force (unless spontaneously broken) coupled to lepton number, but experiments provide no evidence in its favor and it is dropped. The gauge group for this leptonic model (11; 12) is then,

$$G = SU(2)_L \times U(1)_Y, \tag{1.5}$$

with the remaining combination, $U(1)_Y$, coupling to "hypercharge", Y. The hypercharge assignment must reproduce the observed electric charges, so that (in an appropriate normalization),

$$Y \equiv Q - T_3, \tag{1.6}$$

where T_3 generates the $U(1)$ subgroup of $SU(2)_L$. T_3 must be traceless for each irrep, and one can take $T_3 = \text{diag}(1/2, -1/2)$ ($T_3 = 0$) and consequently $Y = -1/2$ ($Y = +1$) when acting on the doublet (singlet).

1.2.4 Gauge Bosons

Local gauge invariance requires vector bosons to transform in the defining (adjoint) representation of the gauge group, *i.e.*, there must be precisely one gauge boson for each group generator. Thus, there is an (iso)triplet, $\vec{W}_\mu$, transforming under $SU(2)_L$, and an isosinglet, B_μ, for hypercharge. The linear combinations with definite mass and electric charge (shown as superscript) are,

$$W_\mu^\pm = \frac{1}{\sqrt{2}}(W_\mu^1 \mp iW_\mu^2), \tag{1.7}$$

$$Z_\mu^0 = \cos\theta_W W_\mu^3 - \sin\theta_W B_\mu, \tag{1.8}$$

$$A_\mu = \sin\theta_W W_\mu^3 + \cos\theta_W B_\mu, \tag{1.9}$$

where the weak mixing angle, θ_W, is defined so that A_μ can be identified with the photon, *i.e.*, it couples to Q and describes the electromagnetic current. Similarly, the $W^\pm$ bosons will produce charged current processes, such as observed in radioactive β decays, and the doublet-singlet structure of the SM reproduces the vector minus axial-vector ($V - A$) law of the weak interaction. On the other hand, at the time when the SM was under construction

(11; 12) no process indicating the presence of the Z^0 boson was known, and the associated neutral current is a genuine prediction of the model. Denoting the gauge coupling constants for $SU(2)_L$ and $U(1)_Y$, respectively, by g and g', one finds the relations,

$$e = g \sin\theta_W = g' \cos\theta_W = \frac{gg'}{\sqrt{g^2 + g'^2}}. \tag{1.10}$$

An important property of non-Abelian gauge theories (13) (not shared by Abelian ones) is the appearance of non-linear terms in the Lagrangian density,

$$\mathcal{L} = -\frac{1}{4}\left(\partial_\mu \vec{W}_\nu - \partial_\nu \vec{W}_\mu - g\vec{W}_\mu \times \vec{W}_\nu\right)^2 - \frac{1}{4}\left(\partial_\mu B_\nu - \partial_\nu B_\mu\right)^2. \tag{1.11}$$

This implies the existence and precise form and strength of triple and quartic gauge boson self-interaction vertices.

The gauge-invariant interactions of vector bosons with leptons (and other spin-1/2 and spin-0 matter particles) are implemented by replacing the ordinary derivatives acting on the matter fields by gauge-covariant derivatives,

$$D_\mu = \partial_\mu + ig\vec{T}\vec{W}_\mu + ig'YB_\mu. \tag{1.12}$$

1.2.5 Spontaneous Symmetry Breaking

If the short-range weak force is to be described by the $W^\pm$ and Z^0 bosons these would have to be massive. Simply adding a mass term to the Lagrangian (1.11) would break local gauge invariance which (as discussed) is unacceptable. This is a very severe problem which touches upon the basic principles of QFTs as reviewed in Section 1.2.1. Indeed, a massless vector boson has only one degree of freedom (d.o.f.) which is characterized by its helicity, h, and which cannot be altered by a proper Lorentz transformation[5]. On the other hand, a massive vector boson is characterized by the three spin states of an ordinary spatial vector.

The solution to this problem involves the phenomenon usually called *spontaneous symmetry breaking* (SSB), although the symmetry of the Lagrangian remains fully intact. But the lowest energy (vacuum) state of the theory is degenerate, with various possible ground states related by the symmetry of the Lagrangian. Whichever is the actual vacuum state and whatever determines it, the physical consequences will be indistinguishable because of the symmetry but the latter may be obscured or entirely hidden.

The SSB idea applied to *continuous, global* symmetries in QFTs (*i.e.*, excluding local gauge symmetries) reveals the existence of a massless spin-0 field

5) To reach the second d.o.f. characterized by $-h$ one needs to invoke a parity transformation.

(Nambu-Goldstone boson) for each broken symmetry generator (14; 15). This can be proved (16) in general (Goldstone theorem).

For example, consider a doublet of complex scalar fields,

$$\Phi(x) = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_1(x) + i\phi_2(x) \\ \phi_3(x) + i\phi_4(x) \end{pmatrix}, \tag{1.13}$$

with $\phi_i = \phi_i^\dagger$ and with Lagrangian,

$$\begin{aligned} \mathcal{L}_\Phi &= \partial_\mu \Phi^\dagger \partial^\mu \Phi - m_\Phi^2 \Phi^\dagger \Phi - \tfrac{1}{2}\lambda^2 (\Phi^\dagger \Phi)^2 \\ &= \tfrac{1}{2}\partial_\mu \phi_i \partial^\mu \phi_i - \tfrac{1}{2} m_\Phi^2 \phi_i \phi_i - \tfrac{1}{8}\lambda^2 (\phi_i \phi_i)^2, \end{aligned} \tag{1.14}$$

where here and in the following summation over i is understood. $\mathcal{L}_\Phi$ is manifestly invariant under continuous $SO(4) = SU(2) \times SU(2)$ rotations in the space spanned by the ϕ_i. The minimum energy state receives no contribution from the kinetic energy (the first term in $\mathcal{L}_\Phi$). For $m_\Phi^2 > 0$, the minimum of the potential, V (the negative of the last two terms in $\mathcal{L}_\Phi$), is reached for vacuum expectation values (VEVs) $\langle \phi_i \rangle = 0$, which define a unique and trivially $SO(4)$ symmetric vacuum. For $m_\Phi^2 < 0$, however, the ground state condition is

$$|\langle \Phi \rangle| = \sqrt{-\frac{m_\Phi^2}{\lambda^2}} \equiv \frac{v}{\sqrt{2}}, \tag{1.15}$$

which has many solutions and none of them possesses $SO(4)$ symmetry[6]. We may choose, $\langle \phi_i \rangle = 0$ for $i = 1, 2, 4$ and $\langle \phi_3 \rangle = v$. Or we define a new field variable, $\eta \equiv \phi_3 - v$ so that $\langle \eta \rangle = 0$, and write instead,

$$\Phi(x) = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_1(x) + i\phi_2(x) \\ v + \eta(x) + i\phi_4(x) \end{pmatrix}. \tag{1.16}$$

The potential now reads,

$$V = -\frac{\lambda^2 v^4}{8} + \frac{\lambda^2 v^2}{2}\eta^2 + \frac{\lambda^2 v}{2}\eta\phi_i\phi_i + \frac{\lambda^2}{8}(\phi_i\phi_i)^2, \tag{1.17}$$

showing a mass term for η ($m_\eta = \lambda v$), while the ϕ_i are massless in accord with the Goldstone theorem.

6) One may object that there are non-zero matrix elements of the Hamiltonian, H, between different such individual vacuum solutions and that the linear superpositions which one must take to diagonalize H will then be symmetry eigenstates (including the lowest energy state). But for macroscopic systems these off-diagonal matrix elements are much smaller (and in infinite volume theories like QFTs they even vanish) than any imperceptibly small symmetry violating perturbation which would diagonalize H in the individual basis. Moreover, vacuum states should also satisfy the principle of cluster decomposition (see Section 1.2.1) and one can show that general linear combinations do not.

1.2.6 Higgs Mechanism

If the field (1.16) is assumed to transform non-trivially under $SU(2) \times U(1)$ gauge transformations, one can arrange for a particular transformation U to pass to a gauge (unitary gauge) such that,

$$U\Phi(x) = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + H(x) \end{pmatrix}, \tag{1.18}$$

i.e., $\phi_1(x)$, $\phi_2(x)$, and $\phi_4(x)$ can be eliminated. If we apply this to the $SU(2)_L \times U(1)_Y$ electroweak gauge theory, we must replace the derivatives in Eq. (1.14) by the covariant derivatives (1.12), which introduces extra gauge boson dependent terms. The bilinear ones of those are

$$\Delta\mathcal{L} = -\frac{1}{8}v^2 \left[g^2(W_\mu^1)^2 + g^2(W_\mu^2)^2 + (g'B_\mu - gW_\mu^3)^2 \right], \tag{1.19}$$

showing that, as anticipated, the $W_\mu^\pm$ (1.7) and Z_μ^0 (1.8) bosons are indeed mass eigenstates, with mass, respectively, given by

$$M_W = \frac{1}{2}gv, \qquad M_Z = \frac{1}{2}\sqrt{g^2 + g'^2}v, \tag{1.20}$$

while the photon (1.9) remains exactly massless. This shows how the puzzling mismatch of d.o.f. counting mentioned at the beginning of Section 1.2.5 is rectified: the extra (longitudinal) d.o.f. carried by a massive gauge boson is provided by a scalar d.o.f. of what in the context of the breaking of the corresponding *global* symmetry would have been a Goldstone boson (would-be Goldstone boson). This is the Higgs mechanism (17; 18; 19) and the d.o.f. represented by η is the yet to be discovered Higgs boson, H, with mass, $M_H = \lambda v$.

The covariant derivatives will also induce trilinear and quatrilinear terms producing couplings of two gauge bosons to respectively, one and two Higgs bosons. These are fixed in terms of the gauge couplings g and g'.

We have chosen an isodoublet of complex scalars, but the Higgs mechanism would also work with other non-trivial representations. But if in the course of SSB the electron is to acquire a mass term, as well, the only renormalizable possibility (see the following Section 1.2.7) is indeed a non-derivatively coupled $SU(2)_L$ doublet (which can combine with the lepton doublet to form a singlet) with hypercharge $Y = -1/2$ (to cancel the hypercharges of the electron and the positron). This allows to add to the theory a Yukawa term (20),

$$\mathcal{L}_Y = -\sqrt{2}\lambda_e \overline{(\nu_e, e)}_L \Phi e_R + \text{H.c.}, \tag{1.21}$$

where the overline notation is introduced to form a Lorentz-invariant scalar product for fermions and adding the Hermitian conjugate assures a real valued Lagrangian. If again the Higgs field, $\Phi(x)$, is replaced by its VEV (1.15) a mass term for the electron arises,

$$m_e = \lambda_e v. \tag{1.22}$$

1.2.7 Infinities

Quantum mechanical amplitudes for physical processes can be computed in an expansion in powers of the coupling constants (perturbation theory) and are given by a sum of Feynman diagrams (21) translating into concrete mathematical expressions. If one attempts to calculate higher order radiative corrections to these processes (containing closed particle loops) one frequently encounters divergent expressions. One can "renormalize" a theory if the infinities match a set of infinite counterterms that one may add to the Lagrangian density defining the theory. In this case one can absorb all infinities into redefined coupling constants, masses, and fields, provided one works with the most general Lagrangian and the complete set of counterterms allowed by the assumed symmetries.

The number of counterterms may in principle be infinite. There is a stricter sense of renormalizability which refers to field theories that need only a finite number of counterterms and which (for historical reasons) are the ones actually called *renormalizable*. The coefficients of the counterterms in this case are restricted to be either dimensionless[7] or their dimension is a positive power of mass (power counting renormalizability). In gauge theories one still needs to demonstrate that gauge invariance constrains the infinities in the same way as the interactions. This has been achieved first for QED (22) and later for non-Abelian theories (23) including those with spontaneously broken gauge symmetries (24) showing that the SM is renormalizable in the stricter sense.

There is another class of infinities (25; 26) which can arise and for which there are no counterterms to absorb them. They appear in Feynman diagrams containing a closed spin-1/2 fermion triangle with gauge bosons attached (possibly including gravitons) of which at least one couples chirally — just like the $SU_L \times U(1)_Y$ gauge bosons in the SM. Such diagrams can be interpreted as corrections to the three-gauge-boson vertices. But since there are no corresponding divergences in the corrections to the four-gauge-boson vertices or to the vertices generated by the covariant derivatives, the subtle structure required by gauge invariance (*e.g.*, in Eq. (1.11)) breaks down. This phenomenon of symmetry violation by quantum effects (anomaly) does not

7) As is customary among elementary particle physicists, we are using physical units in which, by definition, the fundamental values of the speed of light, c, and of Planck's constant, $h/(2\pi)$, are exactly equal to one. The dimensions of mass, energy, linear momentum, and inverse length are then the same.

necessarily spoil the mathematical consistency of the theory (see Section 1.5.3 for an example), but *chiral gauge anomalies* of the type encountered here do.

It is perhaps surprising that symmetries of the action can be violated by the interactions derived from this very same action. Fujikawa (27) has given an interpretation of chiral anomalies which reveals their fundamental nature. In the spacetime approach (28) to quantum mechanics, amplitudes are given as integrals over all possible path histories (path integral formulation) weighted by a complex phase factor given in terms of the action. The chiral anomaly is here seen as the Jacobian determinant (which can be written as an extra phase factor) arising from a change of integration variables of the form of a local gauge transformation. The non-trivial Jacobian by itself would not affect the perturbation theory because it is field-independent and would pose no problem. But it is infinite and needs regularization in the process of which (gauge) field-dependence is introduced. For this reason, in addition to a gauge-invariant action one also has to require a gauge-invariant path integral measure (more strictly, only the combination of the two needs to be gauge-invariant).

The model of leptons (11; 12) described in this Section is plagued by chiral gauge anomalies and is therefore — as it stands — inconsistent. However, one may add additional fermionic d.o.f. to the model that may precisely cancel the anomalies. These d.o.f. are provided by the quarks discussed in Section 1.3.2.

1.3 The Standard Theory

Section 1.3.1 extends the model to include the muon and tau sectors whose existence is not explained in the SM but which incidentally help to determine the basic coupling parameters, respectively, of the weak and strong interactions, with great precision. The quark sector and the modern theory of the strong interaction (QCD) are added in Section 1.3.2, completing the particle content of the SM. Sections 1.3.3 and 1.3.4 are dedicated to quark mixing and CP violation, respectively.

1.3.1 Lepton Replication and Muon Decay

Besides Yukawa interactions as, *e.g.*, in $\mathcal{L}_Y$, the leptonic model has only four adjustable parameters, namely, the gauge couplings, g and g', as well as from the Higgs Lagrangian (1.14) the Higgs-self coupling, λ, and the only fundamental parameter with dimension of mass in this theory, m_Φ. The combination of the latter two in Eq. (1.15) can be determined experimentally with very good accuracy. This is because of the existence of the muon which forms (with its antiparticle and the left-handed muon-neutrino, ν_μ) the same left-handed $SU(2)_L$ doublet and right-handed singlet structure with the same hypercharges as the electron, providing an exact duplication except for an approximately 200 times larger value for the corresponding Yukawa coupling (1.21), λ_μ as compared to λ_e. Thus, the μ^- is much heavier and can decay into a ν_μ, e^-, and $\overline{\nu}_e$ with an amplitude[8] which to good approximation is proportional to g^2/M_W^2. According to the first Eq. (1.20) this determines the Higgs VEV,

$$v = \frac{2M_W}{g} = \sqrt{\frac{1}{\sqrt{2}G_F}} = 246.22\,\text{GeV}, \tag{1.23}$$

where $G_F = 1.166367 \pm 0.000005 \times 10^{-5}\,\text{GeV}^{-2}$ (4 parts per million precision), is the four-fermion coupling constant appearing in the original Fermi theory (29) of the weak interaction.

It should be mentioned that the extraction of fundamental parameters to such high precision requires the calculation of higher order corrections. These are often difficult and lengthy computations but are feasible within renormalizable field theories. *E.g.*, in the case of the Fermi constant one needs the muon lifetime formula to forth-order (two-loop) precision (30; 31).

The relations (1.10) allow to fix another combination of couplings since the QED fine structure constant,

$$\alpha \equiv \frac{e^2}{4\pi} = \frac{1}{137.0359997 \pm 0.0000001}, \tag{1.24}$$

is known to within 700 parts per trillion. It is extracted from the anomalous magnetic moment of the electron (see Section 1.4.2) and is based on an eighth-order (four-loop) calculation (32).

The gauge boson masses (1.20) are then (to first order) determined by the weak mixing angle alone,

$$M_W = \frac{\sqrt{4\pi\alpha(M_Z)}}{2\sin\theta_W} v \approx \frac{38.59\,\text{GeV}}{\sin\theta_W} \approx 86 \pm 7\,\text{GeV}, \tag{1.25}$$

$$M_Z = \frac{\sqrt{4\pi\alpha(M_Z)}}{2\sin\theta_W\cos\theta_W} v \approx \frac{77.18\,\text{GeV}}{\sin 2\theta_W} \approx 96 \pm 6\,\text{GeV}. \tag{1.26}$$

These equations use the value, $\alpha^{-1}(M_Z) \approx 127.9$, appropriate for high-energy (weak-scale) observables instead of the low energy value in Eq. (1.24). This is because leading radiative corrections — those enhanced by large logarithms

8) The Lagrangian (1.21) and the gauge interactions are such that the total number of e^- and ν_e minus e^+ and $\overline{\nu}_e$ entering and leaving any Feynman diagram must be the same (electron number conservation) and likewise for muons. This is why an antineutrino is emitted together with an electron.

of the form $\ln M_Z/m_f$, where m_f is a fermion mass — have been taken into account. Effectively, these logarithms lead to the concept of the energy scale (μ) dependence[9] (running) of the coupling constants (34; 35), described by the Callan-Symanzik β-function (36; 37),

$$\mu^2 \frac{d}{d\mu^2}\alpha(\mu) = \beta(\mu). \tag{1.27}$$

Note, that α grows with μ (screening), and so $\beta > 0$. The last step in the relations (1.25) and (1.26) shows the predictions for the value $\sin^2\theta_W = 0.20 \pm 0.03$ obtained by the 1978 experiment (38) in electron–deuteron fixed-target scattering (see Section 1.4.2). The $W^\pm$ (39; 40) and Z^0 (41; 42) bosons have been discovered in 1983 in proton–antiproton collisions at the Super Proton Synchrotron (SPS) at the European Organization for Nuclear Research (CERN) near Geneva, Switzerland. The reconstructed masses, $M_W \sim 80$ GeV and $M_Z \sim 92$ GeV, were in perfect agreement with the predictions (1.25) and (1.26).

The most general amplitude for muon decay (43) is described by 19 real parameters (Michel parameters) which reduce to 6 parameters if the outgoing neutrinos are not observed. The Michel parameters have been used to confirm the $V - A$ structure of the SM. The muon sector also played a crucial role in the first observation of a neutral current process. The 1973 discovery (44) at CERN was based on a single event in $\nu_\mu e^-$ elastic scattering.

The lepton sector is completed by yet another replication, comprised of the $\tau^\pm$ and the ν_τ. The $\tau^\pm$ is about 17 times heavier than the $\mu^\pm$, and is the only known lepton heavy enough to decay into hadrons, providing a valuable laboratory for detailed and quantitative studies of the strong interaction (45).

1.3.2 Quarks and Quantum Chromodynamics

It was stressed in Section 1.2.7 that the leptonic model by itself has a gauge anomaly which can be cured by adding additional fermions (quarks). As can be seen from Eq. (1.6), a left-handed quark doublet (u, d) with hypercharge $Y = +1/6$ and two antiquark singlets $\overline{u}$ ($Y = -2/3$) and $\overline{d}$ ($Y = +1/3$) correspond to $Q_u = +2/3$ and $Q_d = -1/3$. These give a contribution to the pure (cubic) hypercharge anomaly of $Y^3 = N_C(2 - 64 + 8)/216 = -N_C/4$. If one chooses $N_C = 3$ (the number of "colors") this cancels the contribution, $Y^3 = -1/4 + 1 = 3/4$, from electron-type leptons. Doublets with other kinds of quarks (flavors) (c, s) and (t, b) along with their antiquarks are needed to

9) There are various definitions arising from different calculational (renormalization) schemes which differ by higher order terms. Values quoted here correspond to what is known as the modified minimal subtraction ($\overline{\text{MS}}$) scheme (33).

cancel the anomaly contributions from the muon and tau sectors. Anomaly cancellation for other gauge boson combinations can also be checked.

The color quantum number has never been observed directly and neither have been free quarks. Qualitatively this can be understood if one assumes that there is another non-Abelian gauge interaction with gauge group $SU(3)_C$, which is acting on color triplets of quarks and color antitriplets of antiquarks[10], and if this new gauge interaction became stronger at large distances[11] so that it would be too costly in energy to isolate a quark (confinement hypothesis). For this, the β-function of quantum chromodynamics (QCD) would have to be negative (antiscreening) in contrast to the case of QED (see Section 1.3.1) and any other renormalizable QFT. Non-Abelian gauge theories (if not coupled to too many matter fields) have indeed this property (47; 48).

The reason that the $SU(3)_C$ gauge bosons (the gluons) have not been observed directly is then not thought to be that they are very massive, but rather massless and confined into colorless hadrons (49; 50). The known hadrons are quark–antiquark bound states (mesons) or bound states of either three quarks or three antiquarks (baryons and antibaryons). Other possibilities such as bound states of four quarks and an antiquark (pentaquarks) or purely gluonic states (glueballs) have been speculated about and in some cases their mass predicted, but they have not yet been uncontroversially observed. Both gluons and quarks can be indirectly observed in high-energy collisions as directionally clustered collections of hadrons (jets). For example, the gluon was discovered (51; 52) in 1979 at the Positron Electron Tandem Ring Accelerator (PETRA) at the German Electron Synchrotron (DESY) in Hamburg as planar three jet events which could be interpreted as the radiation of a gluon by one of the quarks (gluon Bremsstrahlung). This completes the elementary particle spectrum of the SM which is summarized in the Table below.

By precisely measuring the total production rate of jet events one cannot only determine N_C (to which it is proportional to first order) but also

$$\alpha_s \equiv \frac{g_s^2}{4\pi^2}, \tag{1.28}$$

where g_s is the strong gauge coupling associated with $SU(3)_C$ and which enters at second order. Examples for such measurements include hadron pro-

10) Triplets and antitriplets of $SU(3)$ are complex conjugates of each other and form mathematically inequivalent irreps. The only other gauge group in Cartan's classification (10) with a three-dimensional irrep (for $N_C = 3$) is $SU(2)$, but its irreps are (pseudo)-real and therefore would give a hadron spectrum different from what is observed.

11) Conversely, it ought to become weaker at short distances (asymptotic freedom) in agreement with experiments in deep inelastic electron–nucleon scattering (46).

Standard Model Particles							
multiplet			spin	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$	
Higgs			0	**1**	**2**	$-1/2$	
$(\nu_e, e^-)_L$	$(\nu_\mu, \mu^-)_L$	$(\nu_\tau, \tau^-)_L$	1/2	**1**	**2**	$-1/2$	
e_R^-	μ_R^-	τ_R^-	1/2	**1**	**1**	$-\ 1$	plus
$(u, d)_L$	$(c, s)_L$	$(t, b)_L$	1/2	**3**	**2**	$+1/6$	anti-
u_R	c_R	t_R	1/2	**3**	**1**	$+2/3$	particles
d_R	s_R	b_R	1/2	**3**	**1**	$-1/3$	
gluons			1	**8**	**1**	0	
$\vec{W}$			1	**1**	**3**	0	
B			1	**1**	**1**	0	

duction in e^+e^- annihilation at sufficiently high energy, the τ lepton lifetime, and various Z^0 decay observables (see Section 1.4.1).

QCD by itself individually conserves the number of quarks of each flavor (minus the number of each antiflavor). Moreover, QCD perturbation theory also conserves P and C invariance (just like QED), but (unlike in QED) these are generally expected to be broken non-perturbatively (see Section 1.3.4).

1.3.3
Quark Mixing

When one adds to the u (up) and d (down) quarks of the first family the second generation s (strange) and c (charm) quarks, then the Yukawa couplings (1.21) become 2×2 matrices. These complex matrices can be brought to diagonal form with real eigenvalues (the quark masses) by means of bi-unitary[12] transformations (changes of basis). The weak interaction, however, will not be diagonal in this new (mass) basis whenever the left-handed unitary transformation, U_L^u, acting on up- and charm quarks differs from U_L^d, acting on down- and strange quarks. As a result, the linear combinations,

$$d' = \cos\theta_c d + \sin\theta_c s, \qquad s' = -\sin\theta_c d + \cos\theta_c s, \tag{1.29}$$

of mass eigenstates d and s are the ones that actually form the $SU(2)_L$ doublets with u and c, respectively. Here, θ_c is the so-called Cabibbo angle (53), and the right-handed transformation matrices, U_R^u and U_R^d, have been used to arrange that $u' = u$ and $c' = c$ remain unchanged. In the SM, U_R^u and U_R^d have no observable effect which can be traced to the fact that there are only left-handed $SU(2)_L$ doublets. Thus, in the quark sector there is a numerically smaller Fermi constant, $G_F \to G_F \cos^2\theta_c$, effective than in the lepton

12) In general, different matrices are necessary for the left- and right-handed quark fields. Each matrix must be unitary, $U^\dagger = U^{-1}$, so that the quark kinetic energy terms maintain their canonical forms.

sector, with $\theta_c \approx 13^\circ$. Flavor changing charged current transitions between the first two families such as kaon (light mesons containing an s quark) β decays, $K^- \to \pi^0 e^- \overline{\nu}_e$, are possible but suppressed by $\sin^2\theta_c \approx 0.05$.

Flavor changing neutral current (FCNC) transitions, such as K^0–$\overline{K^0}$ oscillations or $K^0 \to \mu^+ + \mu^-$ decays, are also predicted. However, the contributions from two full generations of quarks tend to cancel each other leaving only a small residual effect which can mostly be attributed to the differences in quark masses. Historically, this cancellation (GIM mechanism) was the basis for the prediction (54) of the c quark and an estimate of its mass before the 1974 discovery of the first c–$\overline{c}$ bound state (the J/Ψ) almost simultaneously at the Alternating Gradient Synchrotron (AGS) at the Brookhaven National Laboratory (BNL) (55) and at the Stanford Positron Electron Accelerating Ring (SPEAR) at the Stanford Linear Accelerator Center (SLAC) (56).

1.3.4 CP Violation

Upon inclusion of the t (top) and b (bottom) quarks, 3×3 Yukawa (mass) matrices and the possibility of CP violation arise (57). CP violation typically occurs in the presence of complex phases leading to different quantum mechanical interference effects between charge-conjugate amplitudes. Many phases in the Cabibbo-Kobayashi-Maskawa matrix (53; 57),

$$V_{\mathrm{CKM}} = U_L^u U_L^{d\dagger}, \tag{1.30}$$

can be removed by redefinitions. Indeed, no diagonal phase transformation of the form $U_R = \mathrm{diag}(\alpha_1, \alpha_2, \alpha_3)$, can affect the arrangements, $u' = u$, $c' = c$, and $t' = t$. Any of these (except for one overall phase) can be used to remove phases from the mass matrices, eliminating five observable phases from V_{CKM} (in the three generation case). Being defined as the product (1.30) of two unitary matrices, V_{CKM} is itself unitary, and since it is unconstrained otherwise it has nine (real) parameters. Of these only four are then observable, which can be interpreted as three mixing angles (generalizing θ_c) and one observable (CP violating) phase, $e^{i\delta}$. The same counting reveals $(N-1)^2$ parameters for an N generation V_{CKM} of which $(N-1)(N-2)/2$ are complex phases. Therefore, as long as $e^{i\delta} \neq 1$, the three generation SM predicts CP nonconservation in the electroweak sector. Combining the fact that CP violation occurs for three but not for two generations of quarks with the small observed values of V_{CKM} elements connecting the third with the other two (lighter) generations one expects rather small CP violating effects even for δ not very close to 0 or π. CP violation has been observed in kaons and in mesons containing b-quarks. All results can be understood in terms of a common value of the CKM phase δ.

There is another source of CP (and P) violation from the QCD sector. A term may be added to the QCD Lagrangian which in some sense extends the gauge coupling to complex values, with an imaginary part proportional to a real parameter θ (QCD θ angle). It can be written as a total derivative which renders it harmless in QCD perturbation theory because it will not modify the Feynman rules[13]. Even non-perturbatively it may be absorbed by a chiral phase redefinition of the quark fields but at the expense of having to allow complex quark masses, modifying the phase counting above. The QCD θ angle would give, *e.g.*, a contribution to the CP violating electric dipole moment of the neutron which has not yet been observed. This implies a stringent bound, $\theta \lesssim 10^{-10}$. It is an open question why θ is so small (strong CP problem).

1.4 Experimental Tests

After the discoveries of the weak neutral current (44), the $W^{\pm}$ (39; 40) and Z^0 (41; 42) bosons, and the observation (38) of parity violation in deep inelastic electron–deuteron scattering (see Section 1.4.2), the SM was widely accepted as the correct theory of electroweak interactions, at least to some first approximation. High precision measurements were then needed to establish the SM as a renormalizable field theory and at the level of quantum corrections. Since the $SU(2)_L$ gauge coupling usually enters in the combination $g^2/4\pi^2 \approx 0.01$, one typically has to achieve relative accuracies of better than 1% in genuine electroweak observables.

1.4.1 Z^0 Pole Physics

Z^0 bosons can be produced copiously in e^+e^- annihilation experiments if the center of mass energy is tuned to M_Z (Z resonance). The high statistics achievable on the Z^0 pole was exploited by the Large Electron Positron (LEP) collider at CERN and the Stanford Linear Collider (SLC) at SLAC (Z factories). A joint document (59) combines the results and accounts for common systematic and theoretical uncertainties in the interpretation of the individual experiments.

The LEP program recorded about 17 million Z^0 decays and included an energy scan around the Z^0 pole. The fit to the Z^0 resonance lineshape resulted in a 23 parts per million determination of $M_Z = 91.1876 \pm 0.0021$ GeV. Together with the fine structure constant (1.24) and the Fermi constant appear-

13) The fact that a total derivative term in the Lagrangian can still have a non-trivial physical effect is best understood in the path integral formulation. One points to the existence of extended spacetime-dependent (topological) field configurations (such as the so-called instantons) which one has to include (58).

ing in Eq. (1.23) this fixes three of the four fundamental electroweak input parameters mentioned in Section (1.3.1). Even though the Higgs boson has not been discovered yet, the missing input parameter, M_H, can be constrained by a global fit to all electroweak precision data (see Section 1.4.3). In addition to M_Z, the Z^0 lineshape scan also yielded 0.9 permille determinations of the total Z^0 decay width, $\Gamma_Z = 2.4952 \pm 0.0023\,\text{GeV}$, and the resonance peak cross-section for hadronic final states, $\sigma_{\text{had}} = 41.541 \pm 0.037\,\text{nb}$. The cross-sections for specific quark or lepton flavors, f, are then expressed relative to σ_{had} as ratios, R_f. The observables, Γ_Z, σ_{had}, and R_ℓ ($\ell = e, \mu, \tau$), serve as the theoretically cleanest determinations of α_s. If one subtracts the partial Z^0 decay widths into hadrons and charged leptons from Γ_Z one obtains the "invisible" Z^0 width, Γ_{inv}, which in the SM represents the decay width into neutrinos. Conversely, by computing the SM decay width into one standard neutrino, one can use the measured Γ_{inv} to extract the number of massless neutrinos, $N_\nu = 2.985 \pm 0.009$, which rules out further generations of fermions (unless their properties are qualitatively different from the three known ones). By measuring the cross-section asymmetry, $A_{FB}(f)$, for fermions f going into the forward hemisphere of the detectors relative to the backward direction (forward-backward asymmetry), the LEP Collaborations obtained a number of observables which are proportional to lepton-Z vector couplings, v_ℓ. Since (to leading order) the v_ℓ are proportional to the combination $1 - 4\sin^2\theta_W$ and $\sin^2\theta_W \approx 0.23$ is numerically close to $1/4$, these observables have an enhanced sensitivity to $\sin^2\theta_W$ and allow permille level determinations. This also applies to the final state τ polarization asymmetry, $A_{\text{pol}}(\tau)$, a measurement which included information from the angular distribution, as well.

The SLC operated with a 75% polarized e^- beam and percent level polarimetry permitting competitive measurements of $\sin^2\theta_W$ with a much smaller number of approximately 600,000 Z^0 bosons. This is because left-right (polarization) asymmetries (A_{LR}) are also proportional to v_ℓ, while being much larger than leptonic LEP asymmetries (yielding better data statistics). They are also cleaner (only counting of hadrons or leptons is required) than the quark asymmetries at LEP which need to tag the flavors of heavy quarks and distinguish quarks from antiquarks. The combination of measurements of the weak mixing angle at Z^0 pole energies yields,

$$\sin^2\theta_W(M_Z) = 0.23124 \pm 0.00017. \tag{1.31}$$

The results obtained from the Z factories are in remarkable agreement with the SM predictions. In particular, the decay properties of the Z^0 boson and its couplings to leptons and quarks have been verified at the sub-percent level tightly constraining any possible physics beyond the SM. On the other hand, direct contributions to the scattering amplitudes from outside the SM are suppressed relative to the resonating Z^0 amplitude and are best studied away from the Z^0 pole.

One possibility is to upgrade the colliders to higher energies as was done with LEP (LEP 2) which operated up to 209 GeV (60). Here one can study directly the electroweak energy scale, but high precision measurements are generally difficult due to small data samples. Nevertheless, various key measurements were achieved, most notably the very precise measurement of $M_W = 80.376 \pm 0.033$ GeV. Similarly, the proton–antiproton collider Tevatron at the Fermi National Accelerator Laboratory (FNAL) near Chicago is producing collisions up to 2 TeV. It provided a comparable measurement of $M_W = 80.429 \pm 0.039$ GeV (61; 62), and was also able to discover the very massive t quark (63; 64) and to determine its mass, $m_t = 170.9 \pm 1.8$ GeV (65). The Tevatron experiments are expected to increase their data samples by another order of magnitude before the final shutdown of the collider. But complementary measurements are also possible at much lower energies if sufficient precision can be achieved (see the following Section 1.4.2).

1.4.2
Low Energy Measurements

The 1978 deep inelastic scattering (DIS) experiment (38) of polarized electrons from deuterium at SLAC was crucial in establishing the SM. Rival models based on different gauge groups or with different $SU(2)_L \times U(1)_Y$ irrep assignments could not simultaneously explain contemporary ν-scattering results or were altogether inconsistent with the observed parity nonconservation in eD-DIS. At energy (momentum transfer) scales Q of about 1 GeV the parity violating interference term between the photon and the Z^0 is suppressed by $Q^2/M_Z^2 \sim 10^{-4}$ relative to the parity conserving QED cross-section. In other words, an asymmetry measurement of the order of 10^{-5} corresponds to a 10% determination of the Z^0 mediated amplitude.

The same idea has later been applied (66) to e^-e^- (Møller) scattering at even lower momentum transfer, $Q^2 = 0.026\,\text{GeV}^2$. Just like at the Z^0 pole the parity violating asymmetry, A_{PV}, is here further suppressed by v_ℓ yielding a very small $A_{PV} = (-1.31 \pm 0.17) \times 10^{-7}$. Due to the enhanced sensitivity to the weak mixing angle, this yields $\sin^2\theta_W(Q) = 0.2397 \pm 0.0013$, the most precise value away from the Z resonance. The central value is higher than the one in Eq. (1.31) establishing that $\sin^2\theta_W$ is a running parameter (see Figure 1.1) just like the QED coupling α discussed in Section 1.3.1 and the QCD coupling α_s.

The interference between photon and Z^0 mediated amplitudes can also induce mixing between opposite parity states in atoms. Atomic parity violation (APV) has been seen in various heavy atoms where the effect is larger but it must be enhanced further. For example, it can be isolated as a small modu-

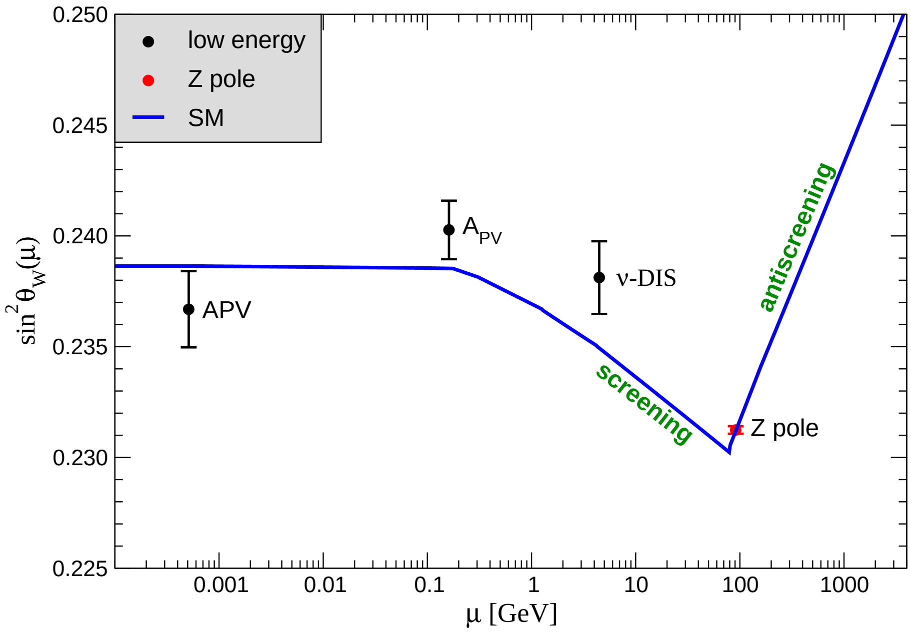

Fig. 1.1 The scale dependence of the weak mixing angle (67; 68) as predicted in the SM, compared to various low energy measurements and the Z^0 pole value (1.31). At $\mu = M_W$, the β-function of $\sin^2\theta_W$ changes sign because only with the inclusion of the $W^\pm$ bosons above M_W does the electroweak theory become a non-Abelian gauge theory with the characteristic antiscreening.

lation of the level mixing induced by an external electric field (Stark-mixing). The most precise measurement has been obtained by a group in Boulder, CO, in cesium (69) and may be interpreted as a precise measurement of $\sin^2\theta_W$ (see Figure 1.1). However, the interpretation requires a solid understanding of the structure of many-electron atoms (70). For cesium the uncertainties associated with atomic wave functions are relatively small and comparable to the experimental ones.

Since neutrinos are electrically neutral they are directly sensitive to the weak interactions, but their cross-sections are proportional to G_F^{-2} and very small. DIS of neutrinos from nuclei have been measured to percent accuracies but here the interpretation of the results is hampered by theoretical uncertainties from the strong interaction. These can be reduced by considering ratios of neutral and charged current cross-sections and by using nuclei with (approximately) equal numbers of protons and neutrons (isoscalars). If a high-intensity and high-energy antineutrino beam is also available one can use the ratio of differences of ν and $\overline{\nu}$ cross-sections (Paschos-Wolfenstein ratio) reducing the theoretical uncertainties even further (71). This was possible by studying neutrinos produced at the Tevatron and resulted in the most precise determination (72) of $\sin^2\theta_W$ from ν-DIS dominating the world average shown in Figure 1.1.

By far the most precise observable testing the SM (as opposed to fixing one of the input parameters) is the anomalous magnetic moment[14] of the muon, a_μ. At the level of the impressive experimental precision achieved at BNL (74),

$$a_\mu \equiv \frac{g_\mu - 2}{2} = (1165920.80 \pm 0.63) \times 10^{-9}, \tag{1.32}$$

receives non-negligible corrections from the weak interaction and is easily affected by new physics contributions. The SM prediction (75) of a_μ is not inconsistent with Eq. (1.32), but there is a tantalizing difference at the level of three standard deviations. Conclusions as to whether this deviation is from new physics at higher energy scales must await further theoretical work because the interpretation of a_μ is complicated by hadronic contributions entering at two-loop order (where it can be constrained by experimental data) and at three-loop order (which are much smaller but very difficult to estimate).

1.4.3
Global Analysis

To fully exploit the information contained in the various experimental results it is best to perform a simultaneous analysis of all data (76; 77; 78). Some of the measurements used for such a study are shown in the following Table.

Standard Model Precision Tests

observable	experimental value	SM prediction	pull	deviation
Γ_Z [GeV]	2.4952 ± 0.0023	2.4968 ± 0.0010	-0.7	-0.5
$\sigma_{\rm had}$ [nb]	41.541 ± 0.037	41.466 ± 0.009	2.0	2.0
R_ℓ	20.767 ± 0.025	20.758 ± 0.011	0.4	0.5
R_b	0.21629 ± 0.00066	0.21584 ± 0.00006	0.7	0.7
R_c	0.1721 ± 0.0030	0.17228 ± 0.00004	-0.1	-0.1
$A_{FB}(\ell)$	0.0171 ± 0.0010	0.01627 ± 0.00023	0.8	1.1
$A_{FB}(b)$	0.0992 ± 0.0016	0.1033 ± 0.0007	-2.5	-2.0
$A_{FB}(c)$	0.0707 ± 0.0035	0.0738 ± 0.0006	-0.9	-0.7
$A_{\rm pol}(\tau)$	0.1465 ± 0.0033	0.1473 ± 0.0011	-0.2	0.1
A_{LR}	0.1514 ± 0.0022	0.1473 ± 0.0011	1.9	2.4
M_W [GeV]	80.398 ± 0.025	80.375 ± 0.015	1.4	1.7
m_t [GeV]	170.9 ± 1.8	171.1 ± 1.9	-0.1	-0.8

The low energy data and other (less precise) observables[15] are not shown but

14) The term anomalous is of historical origin and refers to quantum corrections (73) to the lowest order magnetic moment g of massive particles with spin.

15) For these and many other measurements and tests, including the determination of fermion masses, the parameters in $V_{\rm CKM}$, and lim-

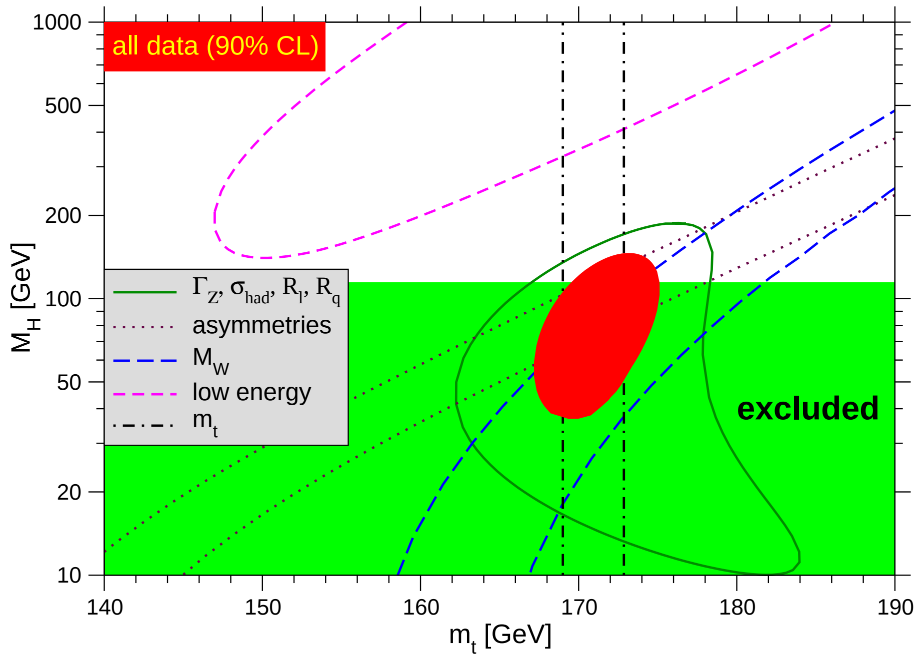

Fig. 1.2 One-standard-deviation (39.35%) uncertainties in M_H as a function of m_t for various inputs, and the 90% confidence level (CL) region allowed by all data. The 95% direct exclusion limit from LEP 2 (79) is also indicated.

included. The column denoted pull shows the deviation (normalized to the total experimental plus theoretical uncertainty added in quadrature) of the experimental values from the SM best fit with M_H allowed as a free parameter, while the deviation column refers to the SM best fit with $M_H = 117$ GeV fixed. The agreement is generally very good and there are no major discrepancies.

The global fit can be used to constrain the SM parameters including the Higgs boson mass,

$$M_H = 77^{+28}_{-22}\,\text{GeV}, \tag{1.33}$$

the weak mixing angle, $\sin^2\theta_W(M_Z) = 0.23119 \pm 0.00014$, and the QCD coupling, $\alpha_s(M_Z) = 0.1217 \pm 0.0017$. The constraints by group of observable in the M_H–m_t plane are shown as contours in Figure 1.2. Good agreement is observed except that the low energy data (driven mostly by the NuTeV (72) result) tend to imply larger M_H. Also, most of the 90% CL ellipse is excluded by the Higgs boson searches at LEP 2, $M_H > 114.4$ GeV (79), and indeed the

its on SM forbidden processes, see the Review of Particle Properties (78). This reference also contains in depth reviews on many SM related topics and is updated biennially.

one-standard-deviation range (1.33) is already ruled out. This may be due to a statistical fluctuation or could be an early indication of loop contributions from new particles beyond the SM affecting the precision observables. Including the results of LEP 2 searches (which include a slight excess of candidate Higgs boson events) in the analysis yields the 95% CL upper bound,

$$M_H \leq 167\,\text{GeV}. \tag{1.34}$$

Thus, if the SM is correct then the Higgs boson is not expected to be much heavier than the massive gauge bosons, and may be producable at the Tevatron and should certainly be detected at the Large Hadron Collider (LHC). The LHC is a soon to be inaugurated accelerator placed in the LEP tunnel at CERN which will mostly study proton–proton collisions at very high (around 14 TeV) energies.

One can also perform a fit to the precision data alone, *i.e.*, excluding the direct m_t from the Tevatron. The result, $m_t = 175^{+10}_{-8}$ GeV, is in perfect agreement with the direct determination and provides an impressive confirmation of the SM at the level of radiative corrections.

1.5 Beyond the Minimal Model

Despite all its successes, the SM is widely believed to be correct only up to small non-renormalizable (see Section 1.2.7) correction terms suppressed by some high energy scale Λ_{new}. This is because the mass parameter, m_Φ, in the Higgs potential generally receives large radiative corrections of the order of Λ_{new}, destabilizing the electroweak scale. Moreover, the gauge group, the irreps, and the values of the input parameters are chosen in an *ad hoc* way to conform with observation, but are not understood at any deeper level.

1.5.1 Accidental Symmetries and non-Renormalizable Corrections

Whatever the underlying theory beyond the SM that cures these problems may be, at the electroweak scale one can organize it in an expansion in inverse powers of Λ_{new}, where each term (operator) contains only SM fields and where the unsuppressed (renormalizable) terms define the SM. The coefficients of the non-renormalizable operators are to be adjusted to reflect the full theory (matching). In other words, extra heavy particles and fields are integrated out (their low energy effects are included as quantum mechanical averages) leaving their footprints in these coefficients. The SM is then considered as the low energy limit (effective field theory) of the more fundamental theory. Since at present one can only speculate as to what this theory might be, one may alternatively take the coefficients of the non-renormalizable operators to parametrize arbitrary extensions of the SM constrained only by the local SM gauge symmetries.

In addition to these non-negotiable local gauge symmetries, the renormalizable SM Lagrangian happens to have additional symmetries which arise simply because any term that would violate them would also violate local gauge invariance. *E.g.*, electron number, muon number[16], and tau number are (at least perturbatively) conserved, and likewise the total quark number and thus baryon number. And as discussed in Section 1.3.4, the two generation SM conserves CP in the electroweak sector, leading to the suppression of CP violation in the full three generation model. Thus, one can understand such symmetries as *accidental* without the need to postulate them as additional physical principals. But by the same token, one would generally expect them to be broken by non-renormalizable terms.

1.5.2
Lepton Number Nonconservation and Neutrino Mass

At order $\Lambda_{\rm new}^{-1}$ baryon number is still accidentally conserved, but one can form Lorentz and locally gauge-invariant terms (80) out of any two lepton doublets and two Higgs doublets. Replacing each Higgs doublet by the VEV (1.15) yields a sum of terms which can be interpreted as mass terms for neutrinos,

$$\mathcal{L}_M = -\sum_{ij} \frac{\lambda_{ij}}{\Lambda_{\rm new}} \overline{\nu_i^c} \nu_j \, v^2. \qquad (1.35)$$

But unlike the mass terms for charged leptons and quarks which originate from the Yukawa terms (1.21) and connect left- and right-handed fields (Dirac mass terms), $\mathcal{L}_M$ connects the left- and right-handed components of conjugate fields (Majorana mass terms).

As a result, Majorana mass terms (81) for neutrinos violate lepton number (the number of leptons minus antileptons) by two units and the same holds true for the corresponding trilinear and cuatrilinear neutrino–Higgs interactions. This implies the important prediction of the possibility of neutrinoless double β decays ($0\nu\beta\beta$ decays), such as $K^- \to \pi^+ e^- e^-$ and more importantly of nuclei. $0\nu\beta\beta$ decays are searched for in experiments with large volume detectors but have not been observed so far.

Another consequence of $\mathcal{L}_M$ is the lepton number conserving but lepton flavor number violating process of neutrino oscillations (the change of one ν flavor into another over long flight paths), provided that the matrix of parameters, λ_{ij}, has non-vanishing off-diagonal entries, in which case the mass

16) These forbid processes such as $\mu \to e\gamma$.

and weak interaction eigenstates of neutrinos are not the same[17]. This mirrors the situation from the quark sector, and indeed a neutrino mixing matrix (82), $V_{\rm MNS}$, analogous to $V_{\rm CKM}$ in Eq. (1.30), arises. The only difference is that left-handed fields are not independent of right-handed antifields so that the counting of observable phases is modified. The result is that there are $N_\nu - 1$ additional (CP violating) phases (Majorana phases) because, in contrast to the quark sector, phase redefinitions of right-handed fields do not enter here. While it appears to be extraordinarily difficult to design an experiment to measure the two Majorana phases (for $N_\nu = 3$), several of the other parameters in $V_{\rm MNS}$ have already been measured in experiments studying neutrinos originating from the sun (83), the earth's atmosphere (84), nuclear reactors (85), and particle accelerators (86). What is observed in these experiments is the disappearance (rate decrease) of neutrinos of a particular flavor from a source with known flavor composition. Conversely, one may detect a neutrino flavor not initially present in the source (appearance experiment). On the other hand, the sum of all neutrino flavors appears to be unchanged (87), as is expected if $V_{\rm MNS}$ is indeed the (only) origin of neutrino oscillations. It is conceivable that the extra phase in $V_{\rm MNS}$ which has its direct analog in $V_{\rm CKM}$ (Dirac phase) may also be accessible in future ν oscillation experiments. In any case, all experimental results are consistent with neutrino mass differences of the order of 10^{-1} eV (from atmospheric neutrinos) and 10^{-2} eV (from solar neutrinos) and generally large mixing angles. Assuming that the λ_{ij} are of $\mathcal{O}(1)$ or smaller[18], one obtains estimates, $\Lambda_{\rm new} \lesssim \mathcal{O}(10^{15}\ {\rm GeV})$ and $\lesssim \mathcal{O}(10^{16}\ {\rm GeV})$, respectively. This is interestingly close to the Planck scale, $M_P \sim \mathcal{O}(10^{18}\ {\rm GeV})$, defined as the scale where quantum corrections to gravity are no longer negligible and where fundamental and far reaching modifications to current theories of elementary particles are expected[19].

The term (1.35) was introduced without reference to any particular model of the physics at higher energy scales. The most important (because very simple) concrete realization is the so-called see-saw mechanism (88; 89; 90) in which right-handed neutrinos, ν_R, are added to the SM. They are singlets under the SM gauge group and one would generally expect them to have very large (Majorana) masses (there is no symmetry reason why they should be of the order of the electroweak scale). The role of $\Lambda_{\rm new}$ is here played by the right-handed neutrino mass scale, and the left-handed neutrino masses are generated by

17) Of course, this also requires that the neutrino masses differ from each other. Hence, neutrino oscillation amplitudes are proportional to neutrino mass differences.

18) The λ_{ij} cannot be much larger than one, since their β-functions (see Section 1.3.1) would drive them back to order unity at the electroweak scale even if they started out much larger at $\Lambda_{\rm new}$.

19) Additional modifications may become necessary at additional scales between M_Z and M_P.

mixing terms analogous to Eq. (1.21). The term (1.35) would now be the result of integrating out the ν_R and one arrives at the SM as an effective field theory as discussed at the beginning of this Section.

Alternatively, since the ν_R do not participate in the SM gauge interactions one may also assume that for some reason they cannot have Majorana masses. In this case, one would be left with the Yukawa terms (1.21) and Dirac neutrino masses, and V_{MNS} would have the same form as V_{CKM}.

1.5.3
Baryon Number Nonconservation and Proton Decay

At order $\Lambda_{\mathrm{new}}^{-2}$ there are interaction terms (80; 91) that violate both baryon and lepton number. These involve one lepton and three quark fields and predict proton decay rates at $\mathcal{O}(\Lambda_{\mathrm{new}}^{-4})$. The proton lifetime, τ_p, can be studied with large water detectors, but so far proton decay has not been observed and a lower (mode-independent) limit, $\tau_p > 2 \times 10^{29}$ years (78), could be set. On dimensional grounds, τ_p is roughly of the order $\Lambda_{\mathrm{new}}^4/m_p^5$ ($m_p = 938$ MeV is the proton mass), so that (assuming that all relevant coupling constants are of order unity) $\Lambda_{\mathrm{new}} \gtrsim \mathcal{O}(10^{15}\ \mathrm{GeV})$, which is not inconsistent with the scales indicated by ν oscillations.

A realization of these model-independent observations are *Grand Unified Theories* (92) in which quarks and leptons are conjectured to inhabit the same irreps of a larger gauge group, the simplest possibility being $SU(5)$. This implies proton decay (*e.g.*, $p \to e^+\pi^0$), with the role of Λ_{new} being played by the masses of the additional gauge bosons predicted by the larger local gauge symmetry (SSB induced by extra Higgs fields would have to be at work).

Baryon number is violated in the SM even without the non-renormalizable operators. As mentioned in Section 1.2.7, anomalies can violate otherwise conserved quantities. In the SM this happens both to baryon and lepton number (but not the difference), and can be understood as an instanton effect (93). However, this is an extremely small (by any standards) quantum mechanical tunneling effect (from one vacuum to a gauge transformed vacuum) and therefore only of academic interest.

Even though proton decay has not yet been observed, baryon number violation is a pressing problem: if baryon number is exactly conserved, then the baryon asymmetry of the universe (BAU), *i.e.*, why the universe contains more baryons than antibaryons (no large concentrations of antimatter are observed) would be difficult to understand. Moreover, baryon number violation is necessary but not sufficient to produce the BAU (94); a fairly strong source of CP violation beyond the CKM phase δ is necessary, as well. Thus, there is a large variety of reasons to strongly suspect that there is physics beyond the SM.

Glossary

Action: The action is defined to generate the classical equations of motions as its stationary "points" (Euler-Lagrange equations). It is particularly useful to construct theories which are invariant under some symmetry such as gauge invariance. A quantity called "action" also enters the weight factor in the path integral formalism, but is not always identical to its classical counterpart.

Anomaly: Symmetry violation by quantum effects. Gauge anomalies may appear in chiral theories in which case they spoil their mathematical consistency. Anomalies of other symmetries are permissable and may lead to interesting physical effects.

Chiral Theory: A theory that fundamentaly distinguishes left-handed and right-handed particles and fields (handedness).

Feynman Diagrams: Effective and intuitive way to visualize the perturbative contributions to quantum mechanical amplitudes. Each line in a Feynman diagram represents a particle. Each diagram translates into a concrete mathematical expression.

Gauge Symmetry: Gauge invariance of the first kind is invariance under global (spacetime independent) phase multiplications and their non-Abelian generalizations. Gauge invariance of the second kind is the generalization to local (spacetime dependent) transformations.

Hamiltonian: Symmetry generator of time translations and thus one of the elements of the Poincaré algebra. It is an operator with energy eigenvalues. In QFTs it usually takes the form of a space integral over a Hamiltonian density, which is also often referred to simply as Hamilitonian.

Lagrangian: The action usually takes the form as a time integral over a function called the Lagrangian. The Hamiltonian (density) can be obtained in terms of the Lagrangian (density).

Loops: Closed line in a Feynman diagram. A loop represents virtual particles of arbitrary energy and momentum over which one has to sum (integrate). Feynman diagrams with loops are often divergent.

Path Integral: Spacetime approach to quantum mechanics in which amplitudes are given as functional integrals over all possible path histories weighted by a complex phase factor given in terms of the action.

Poincaré Group: The fundamental symmetry group of spacetime consisting of translations in space and time, rotations and Lorentz boosts. The

Poincaré group can be generated by *infinitesimal* symmetry transformations. The corresponding symmetry generators form the associated Poincaré algebra.

Regularization: Technique to organize the divergent parts of Feynman diagrams. Physical results should not depend on the regularization method.

Renormalizable Theory (modern sense): Theory in which all divergencies can be absorbed by counterterms.

Renormalizable Theory (strict sense): Theory in which all divergencies can be absorbed by a *finite* number of counterterms.

Renormalization: The process to absorb the divergencies in counterterms. Only the finite sum of the divergent Feynman diagrams and infinite counterterms are physically meaningful.

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