Teoría Electrodébil

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Outline

- Lecture I: Principles and Theoretical Foundations of the Standard Model (SM)
- Lecture II: The Model for Leptons
- Lecture III: The Standard Theory
- Lecture IV: Experimental Tests
- Lecture V: Beyond the Standard Model

Lecture I

Principles and Theoretical Foundations of the SM

- Introduction
- Quantum Mechanics
- Relativistic Quantum Mechanics
- Space Inversion and Time Reversal
- Cluster Decomposition Principle

Lecture I

Principles and Theoretical Foundations of the SM

- Causality
- Free Quantum Fields
- Quantum Field Theory
- Local Gauge Symmetries
- Gauge Theories

Introduction

- The 4 known forces have very different characters.
- But all are based on quantum field theories (QFTs);
- and all exhibit some form of local gauge symmetry.
- Historically: Maxwell's equations → Lorentz invariance and local gauge invariance → current conservation.
- Modern view: Lorentz invariance + quantum mechanics (QM) + cluster decomposition principle → QFT and current conservation → local gauge invariance.

Quantum Mechanics

- Axiom I: physical states are represented by rays in Hilbert space (a complex vector space with scalar product).
- Axiom II: observables are represented by Hermitian operators (linear mappings with adjoints).
- Axiom III: probability: $P(R_1 \rightarrow R_2) = |\langle\Psi_1|\Psi_2\rangle|^2$, $\Psi_i \in R_i$.
- Wigner's symmetry representation theorem: probability conserving ray transformations are represented by unitary and linear or else antiunitary and antilinear (e.g., time inversion symmetry, T) operators.
- $\Rightarrow$ continuous symmetry operators are unitary and linear.

Relativistic Quantum Mechanics

- particles: irreducible unitary representations (irreps) of the inhomogeneous Lorentz (Poincaré) group.

p^2	p_0	standard k^β	little group	comments
> 0	> 0	(M,0,0,0)	SO(3)	massive particle
> 0	< 0	(−M,0,0,0)	SO(3)	E < 0 (unphysical)
= 0	> 0	(k,k,0,0)	ISO(2)	massless particle
= 0	= 0	(0,0,0,0)	SO(3,1)	vacuum (no particles)
= 0	< 0	(−k,k,0,0)	ISO(2)	E < 0 (unphysical)
< 0	any	(0,M,0,0)	SO(2,1)	tachyon (|v| > c)

- spin of massive particle: algebra as in QM.
- helicity of massless particle: topology of SO(3,1) = $SL(2,\mathbb{C})/Z_2$ is that $\mathbb{R}^3 \times S^3/Z_2$ and is doubly connected

Space Inversion and Time Reversal

- Photons (gravitons) with $h = +1$ ($+2$) and $h = -1$ (-2) belong to different irreps of the proper, orthochronous, inhomogeneous Lorentz group.

- But $\{P, P\} = [P, J] = 0 \Rightarrow h \to -h$ under $P \Rightarrow$ photons (gravitons) with $h = \pm 1$ (± 2) belong to the same irrep if P is included.

- But ν ($h = +½$) and $\bar{\nu}$ ($h = -½$) are distinguished.

- $T^2\, \Psi = -\Psi$ if Ψ is a state with an odd # of ½-integer particles; if $T\, \Psi = \zeta\, \Psi \Rightarrow T^2\, \Psi = T\, \zeta\, \Psi = \zeta^*\, T\, \Psi = \zeta^* \zeta\, \Psi = \Psi \neq -\Psi \Rightarrow$ Kramers degeneracy $\Rightarrow$ EDMs and GDMs forbidden by T.

Cluster Decomposition Principle

- Introduce creation & annihilation operators, $a^\dagger$ and a.
- Theorem: free Hamiltonian can always be written as

$$H = \sum_{N,M=0}^{\infty} \int \prod_{i=1}^{N} \prod_{j=1}^{M} dp_i dq_j a^\dagger(p_i) a(q_j) h_{NM}(p_i, q_j)$$

- Cluster decomposition principle: Distant experiments yield unrelated results, i.e. S-matrix elements (scattering amplitudes) factorize.
- Theorem: satisfied if h^{nm} contains only one δ-function.
- Note: $a^\dagger$ and a are defined in momentum space.
- 2 identical particles: |...p...p'...> = α |...p'...p...>; α can not depend on other particles in |...>, J, P, path (D>2).

Causality

- The S-matrix must also be Lorentz-covariant.
- Proper Lorentz-transformations $\Rightarrow$ (Noether's theorem) conserved charges K^a; but $[H,K^a] \neq 0$ while $[H,P^a] = [H,J^a] = 0$.

➡ Complication with no counterpart in non-rel. theories.

➡ Lorentz-invariance requires causality, $[\mathcal{H}(x),\mathcal{H}(y)] = 0$ for $(x-y)^2 \leq 0$.

- Note: this condition is formulated in configuration space.

Free Quantum Fields

- Introduce creation & annihilation fields,

$$\psi_-^\beta(x) = \frac{1}{(2\pi)^3}\sum_{\sigma n}\int \frac{d^3p}{2p^0} v^\beta(\vec{p},\sigma,n)a^\dagger(\vec{p},\sigma,n)\; e^{ipx}$$

$$\psi_+^\beta(x) = \frac{1}{(2\pi)^3}\sum_{\sigma n}\int \frac{d^3p}{2p^0} u^\beta(\vec{p},\sigma,n)a(\vec{p},\sigma,n)\; e^{-ipx}$$

- $\psi(x) \equiv \kappa\psi^+ + \lambda\psi^-$ (scalar), $D(x) \equiv \dfrac{1}{(2\pi)^3}\displaystyle\int \frac{d^3p}{2p^0} e^{-ip(x)} \Rightarrow$
- $[\psi(x),\psi(y)]_{+-} = \kappa\lambda\,(1 \pm 1)\; D(x-y) \Rightarrow$ lower sign

  $[\psi(x),\psi^\dagger(y)]_{+-} = (|\kappa|^2 \pm |\lambda|^2)\; D(x-y) \Rightarrow \kappa = \lambda$

- $\Rightarrow$ scalar fields are bosons.

Quantum Field Theory

- index l → fields are finite-dimensional, non-unitary irreps of the Lorentz group.
- $\mathcal{H}$ is now a sum of products of quantum fields (incl. derivatives), where Lorentz scalars are constructed using technique of Clebsch-Gordon coefficients.
- free field equations: Klein-Gordon equation, $(\Box + m^2)\phi_i = 0$ and first order differential or algebraic constraint equation, e.g. $\partial^\beta V_\beta(x)=0$ or $[i\gamma_\beta\partial^\beta - m]\,\Psi(x)=0$.
- antiparticles (not just Dirac fermions).
- spin-statistics connection
- CPT theorem

Local Gauge Symmetries

- Sometimes there is no solution for u and v.
- Theorem: No 4-vector field, A^β, can be constructed from the a and $a^\dagger$ for a particle of $h = \pm 1$ and $m = 0$!
- Could use $B^{\beta\gamma} = -B^{\gamma\beta}$, but uniqueness theorem ⇒ $B^{\beta\gamma} = F^{\beta\gamma} = \partial^\beta A^\gamma - \partial^\gamma A^\beta$; possible, but gives no $1/r^2$-law.
- $U(\Lambda) A^\beta U^{-1}(\Lambda) = \Lambda^{\gamma\beta} A_\gamma(x) + \partial^\beta \Omega(x)$ ⇒ Lorentz-invariance is restored if we require invariance under $A^\beta(x) \rightarrow A^\beta(x) - \partial^\beta \omega(x)$ ⇒ $\mathcal{L}(A) = A^\beta J_\beta$ with $\partial^\beta J_\beta = 0$.
- Generalizes to gravity: $h = \pm 2$, $m = 0$ and $h^{\beta\gamma} = h^{\gamma\beta}$.
- $h = \pm\, 3/2$, $m = 0$ and gravitino ⇒ supersymmetry.

Gauge Theories

- Extend to matter (QED): $\psi(x) \to \exp[\pm ieQ\omega(x)]\,\psi(x)$

- Non-Abelian gauge symmetry:

$$\delta\psi^a(x) = i\omega^r(x) t_r{}^{ab}\psi(x)^b, \quad \delta A^r{}_\beta = C^{rst}\omega^t(x) A^s{}_\beta - \partial_\beta\omega^r(x)$$

- Positivity of quantum mechanical scalar product $\Rightarrow$ direct sum of U(1) and compact simple Lie subalgebras SU(N), SO(N), USp(2N), G_2, F_4, E_6, E_7, E_8 (Cartan).

- QED conserves parity (P) and P connects h and $-$h $\Rightarrow$ although different irreps, both are called "photons".

- Neutrinos, $h = +1/2$, $m = 0$, differ from antineutrinos, $h = -1/2 \Rightarrow$ chiral gauge symmetry.

Lecture II

The Model for Leptons

- Gauge Group of Electroweak Interactions
- Gauge Bosons
- Gauge Couplings
- Spontaneous Symmetry Breaking (SSB)
- Scalar Doublet

Lecture II

The Model for Leptons

- Goldstone Theorem
- Higgs Mechanism
- Gauge Boson Masses
- Infinities
- Renormalizability
- Anomalies

Gauge Group of Electroweak Interactions

- $(\nu^e, e^-, e^+)^L$ → maximum choice is $U(3) \equiv SU(3)\times U(1)$, but lepton # violation at unacceptable rates.
- $(\nu^e, e^-)^L + (e^-)^R$ → $U(2)^L\times U(1)^R \equiv SU(2)^L\times U(1)^L\times U(1)^R$
- $U(1)^{L+R}$: long range force (unless broken) coupled to lepton #; but there is no evidence for it ⇒
- $G = SU(2)^L\times U(1)^y$ with $y \equiv Q - T_3$, and T_3 traceless.
- $T_3{}^L = \mathrm{diag}(½, -½)$, $T_3{}^R = 0 \Rightarrow y(\nu^e, e^-) = -½$, $y(e^-) = +1$.

Gauge Bosons

- adjoint irrep → 1 gauge boson per group generator

$$W^\pm_\mu = \frac{1}{\sqrt{2}}(W^1_\mu \mp i W^2_\mu),$$

$$Z^0_\mu = \cos\theta_W W^3_\mu - \sin\theta_W B_\mu,$$

$$A_\mu = \sin\theta_W W^3_\mu + \cos\theta_W B_\mu,$$

- W: weak charged current processes (e.g., β-decays).
- doublet-singlet structure → (V−A)-law of weak force.
- Z^0: neutral current (predicted).

Gauge Couplings

$$e = g\sin\theta_W = g'\cos\theta_W = \frac{gg'}{\sqrt{g^2+{g'}^2}}.$$

$$\mathcal{L} = -\frac{1}{4}\left(\partial_\mu\vec{W}_\nu - \partial_\nu\vec{W}_\mu - g\vec{W}_\mu\times\vec{W}_\nu\right)^2 - \frac{1}{4}\left(\partial_\mu B_\nu - \partial_\nu B_\mu\right)^2.$$

- → triple and quartic gauge boson self-interactions
- interaction with leptons (and other spin-1/2 and spin-0 matter particles) through covariant derivatives

$$D_\mu = \partial_\mu + ig\vec{T}\vec{W}_\mu + ig'YB_\mu.$$

Spontaneous Symmetry Breaking (SSB)

- Short-range weak force $\Rightarrow$ W and Z must be massive; but adding mass terms breaks gauge invariance.
- E.g., 2 vs. 3 physical degrees of freedom (d.o.f.).
- SSB: symmetries of $\mathcal{L}$ remain fully intact, but lowest energy (vacuum) state of the theory is degenerate.
- Symmetry makes physical consequences of various vacua indistinguishable but is itself obscured (hidden).
- For continuous, global symmetries in QFTs, SSB yields massless spin-0 fields (Nambu-Goldstone) bosons.

Scalar Doublet

$$\Phi(x) = \frac{1}{\sqrt{2}} \left(\begin{array}{c} \phi_1(x) + i\phi_2(x) \\ \phi_3(x) + i\phi_4(x) \end{array} \right), \qquad \phi_i = \phi_i^\dagger$$

$$\begin{aligned} \mathcal{L}_\Phi &= \partial_\mu \Phi^\dagger \partial^\mu \Phi - m_\Phi^2 \Phi^\dagger \Phi - \tfrac{1}{2}\lambda^2 (\Phi^\dagger \Phi)^2 \\ &= \tfrac{1}{2}\partial_\mu \phi_i \partial^\mu \phi_i - \tfrac{1}{2} m_\Phi^2 \phi_i \phi_i - \tfrac{1}{8}\lambda^2 (\phi_i \phi_i)^2. \end{aligned}$$

$m_\Phi^2 > 0 :$ $\quad \langle \phi_i \rangle = 0,$ $\qquad SO(4) = SU(2) \times SU(2).$

$m_\Phi^2 < 0 :$ $\quad |\langle \Phi \rangle| = \sqrt{-\frac{m_\Phi^2}{\lambda^2}} \equiv \frac{v}{\sqrt{2}},$ $\qquad SO(3) = SU(2).$

$$\Phi(x) = \frac{1}{\sqrt{2}} \left(\begin{array}{c} \phi_1(x) + i\phi_2(x) \\ v + \eta(x) + i\phi_4(x) \end{array} \right) \Rightarrow$$

$$V = -\frac{\lambda^2 v^4}{8} + \frac{\lambda^2 v^2}{2}\eta^2 + \frac{\lambda^2 v}{2}\eta \phi_i \phi_i + \frac{\lambda^2}{8}(\phi_i \phi_i)^2.$$

Goldstone Theorem

- O(N): N−1 Goldstone bosons.
- O(N) → O(N−1): ½ N (N−1) − ½ (N−1) (N−2) = N−1.
- A spontaneoulsy broken continuos symmetry requires the existence of a particle with m = s = 0 and the same parity and quantum numbers as J^0 (current).
- If the symmetry is only approximate: pseudo-Goldstone bosons.
- If the symmetry is explicitly broken to O(N−1) then by virtue of a vacuum alignment condition there is no further breaking to O(N-2).

Higgs Mechanism

- If Φ(x) transforms non-trivially under SU(2)×U(1) gauge transformations, go to unitary gauge,

$$U\Phi(x) = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + H(x) \end{pmatrix}.$$

- The bilinear terms in the covariant derivative of Φ ⟶

$$\Delta\mathcal{L} = -\frac{1}{8}v^2 \left[g^2(W_\mu^1)^2 + g^2(W_\mu^2)^2 + (g'B_\mu - gW_\mu^3)^2\right] \;\Rightarrow$$

$$M_W = \frac{1}{2}gv, \qquad M_Z = \frac{1}{2}\sqrt{g^2 + g'^2}v, \qquad M_\gamma = 0.$$

- ⇒ puzzling d.o.f. counting rectified.
- Notice that we can now write $\sin^2\theta_W = 1 - \frac{M_W^2}{M_Z^2}$.

Higgs Mechanism

- D.o.f. represented by η: Higgs boson H with $M_H = \lambda v$.
- Trilinear and quatrilinear Higgs-gauge boson couplings.
- Can use other Higgs irreps than doublets, but a doublet allows Yukawa terms. E.g.,

$$\mathcal{L}_Y = -\sqrt{2}\lambda_e \overline{(\nu_e, e)}_L \Phi e_R + \text{H.c.} \Rightarrow m_e = \lambda_e v.$$

- Summary of parameters: g, g', λ, λ^e, v.
- Custodial SU(2) X SU(2).

Gauge Boson Masses

$$M_W = \frac{\sqrt{4\pi\alpha(M_Z)}}{2\sin\theta_W} v \approx \frac{38.59\ \text{GeV}}{\sin\theta_W} \approx 86 \pm 7\ \text{GeV},$$

$$M_Z = \frac{\sqrt{4\pi\alpha(M_Z)}}{2\sin\theta_W\cos\theta_W} v \approx \frac{77.18\ \text{GeV}}{\sin 2\theta_W} \approx 96 \pm 6\ \text{GeV}.$$

- Use $\alpha^{-1}(M_Z) \approx 127.9$ resumming $\ln M_Z/m_f$ terms.
- Callan-Symanzik β-function $\mu^2\, d/d\mu^2\, \alpha(\mu) \equiv \beta(\mu)$.
- α grows with energy (screening) $\Leftrightarrow$ $\beta > 0$.
- Also use $\sin^2\theta^W = 0.20 \pm 0.03$ from 1978 experiment (Prescott et al.) on eD fixed-target scattering.

Infinities

- Perturbation theory $\longrightarrow$ Feynman diagrams.
- Closed particle loops $\longrightarrow$ divergent expressions.
- A theory can be renormalized if the infinities match set of infinite counterterms that one may "add" to $\mathcal{L}$
- → can absorb infinities into redefined coupling constants, masses and fields if all counterterms are included.
- Note, however, that renormalization has nothing directly to do with infinities.

Renormalizability

- # of counterterms may in principle be $\infty \Rightarrow$ "non-renormalizable" theories are renormalizable.
- Examples: Gauge theories with HDOs and gravity.
- Stricter (Dyson) sense of renormalizability: finite # of counterterms sufficient.
- Dimension of all interaction coefficients, $\Delta_i = 4 - d_i - \Sigma_i^a\, n_i^a\, (s^a + 1) \geq 0$.
- Example: Standard Model (SM)
- Still need to show that gauge invariance constrains the infinities in the same way as the counterterms.
- QED: Dyson, SM: 't Hooft, Veltman; Lee, Zinn-Justin

Anomalies

- Anomalies: symmetry violation by quantum effects.
- Gauge anomalies: unacceptable (no counterterms).
- Arise from chiral loops with (D+2)/2 gauge-bosons (possibly including gravitons) attached.
- In D=4: triangle anomalies → like triple-boson vertex divergence, but without corresponding ∞ for 4 bosons.
- Path-integral (spacetime approach to QM) interpretation: ∞ in Jacobian determinant; field-independent but regularization introduces gauge-field dependence.
- The model for leptons has gauge anomalies ⇒ modify!

Lecture III

The Standard Theory

- Lepton Replication and Muon Decay
- Experimental Milestones
- Quarks
- Gluons
- Asymptotic Freedom
- Quantum Chromodynamics
- SM Particle Summary

Lecture III

The Standard Theory

- Quark Mixing
- Flavor Changing Transitions
- Electroweak CP Violation
- New Type of CPV Discovered?
- Strong CP Violation
- CKM-Matrix
- SM Parameter Summary

Lepton Replication & μ Decay

- Who ordered the muon? But good thing someone did!
- Amplitude of $\mu \to \nu_\mu e^- \bar{\nu}_e$ proportional to $g^2/M_W^2 \Rightarrow$
$$v = \frac{2M_W}{g} = \sqrt{\frac{1}{\sqrt{2}G_F}} = 246.22\ \text{GeV}.$$
- Fermi constant
(FAST, μLan): $G_F = 1.166367 \pm 0.000005 \times 10^{-5}\ \text{GeV}^{-2}$.
- Extraction from muon lifetime requires forth-order (two-loop) corrections (van Ritbergen, Stuart).
- $\alpha = e^2/4\pi = 1/(137.0359997 \pm 0.0000001)$ from electron four-loop anomalous magnetic moment formula.
- Tau: only known hadronically decaying lepton.

Experimental Milestones

- ✓ W and Z discovered in p anti-p collisions at SPS.
- ✓ $M_W \sim 80$ GeV and $M_Z \sim 92$ GeV reconstructed.
- Most general amplitude for μ-decay: 19 (6) real Michel parameters if outgoing ν is (not) observed.
- ✓ ⇒ V−A structure confirmed.
- ✓ Neutral current discovery at CERN (1973) in a single event in $\nu_\mu e^-$ elastic scattering.
- ✓ ν and e^--scattering experiments in 1978 proved $\not{P}$ and singled out gauge groups of irreps.

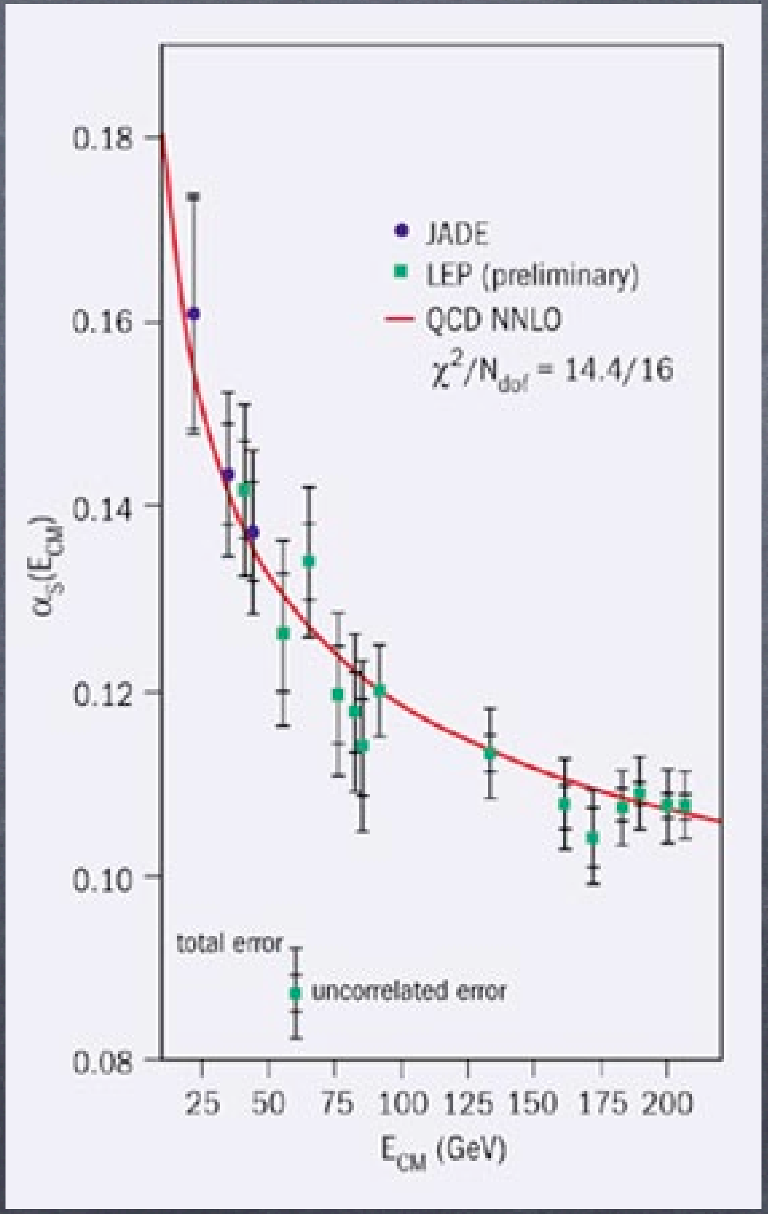
Quarks

- Left-handed quark doublet (u,d) with $y = +1/6$ and two antiquark singlets $\bar{u}$ ($y = -2/3$) and d ($y = +1/3$) $\Rightarrow Q^u = +2/3$ and $Q^d = -1/3 \Rightarrow$
- $y^3 = N\,(2 - 64 + 8)/216 = -N/4 \Rightarrow$ # of colors $N = 3$ to cancel $y^3 = -1/4 + 1 = 3/4$ from $(\nu^e, e^-)^L + (e^-)^R$.
- Likewise, need (c,b) and (t,b) for μ and τ sectors.
- All other gauge anomalies also cancel.

Gluons

- The color quantum # or free quarks have never been observed →
- assume non-Abelian color gauge group SU(3) with a coupling becoming stronger at large distances ↔ confinement hypothesis ⇒ need $\beta < 0$ (antiscreening).
- Non-Abelian gauge theories only QFTs with $\beta < 0$.
- ⇒ SU(3) gauge bosons (gluons) massless and confined into colorless hadrons (mesons, baryons, antibaryons, pentaquarks (?), glueballs (?), etc.).

Asymptotic Freedom



Quantum Chromodynamics

- Gluons and quarks can be indirectly observed in high-energy collisions as directionally clustered collections of hadrons (jets).

✓ E.g., gluon discovery at PETRA (DESY): planar 3-jet events (gluon Bremstrahlung by one of a quark pair).

- Jet event rates ⇒ N and $\alpha_s \equiv \frac{g_s^2}{4\pi}$, the latter also in
- high energy e^+e^- annihilation, τ lifetime, Z^0 decays.

SM Particle Summary

multiplet			spin	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$
Higgs			0	$\mathbf{1}$	$\mathbf{2}$	$-1/2$
$(\nu_e, e^-)_L$	$(\nu_\mu, \mu^-)_L$	$(\nu_\tau, \tau^-)_L$	$1/2$	$\mathbf{1}$	$\mathbf{2}$	$-1/2$
e_R^-	μ_R^-	τ_R^-	$1/2$	$\mathbf{1}$	$\mathbf{1}$	$-\ 1$
$(u, d)_L$	$(c, s)_L$	$(t, b)_L$	$1/2$	$\mathbf{3}$	$\mathbf{2}$	$+1/6$
u_R	c_R	t_R	$1/2$	$\mathbf{3}$	$\mathbf{1}$	$+2/3$
d_R	s_R	b_R	$1/2$	$\mathbf{3}$	$\mathbf{1}$	$-1/3$
gluons			1	$\mathbf{8}$	$\mathbf{1}$	0
$\vec{W}$			1	$\mathbf{1}$	$\mathbf{3}$	0
B			1	$\mathbf{1}$	$\mathbf{1}$	0

(plus antiparticles)

Quark Mixing

- Including (c,s) ⇒ Yukawa couplings ⟶ 2×2-matrices.
- Diagonalize with bi-unitary trafo (change of basis).
- Weak interaction eigenstates (d', s') not diagonal in new (mass) basis (d, s) when U^{Lu} differs from U^{Ld}: $d' = \cos\theta\, d + \sin\theta\, s$, $s' = -\sin\theta\, d + \cos\theta\, s$.
- $\theta \approx 13°$: Cabibbo angle
- U^{Ru} and U^{Ld} have been used to arrange u' = u, d' = d, and are unobservable (only left-handed doublets).

➡ In quark sector $G_F \to G_F \cos^2\theta_c$ (intra-generation).

Flavor Changing Transitions

- Flavor changing charged current (FCCC) transitions between first two families; e.g., $K^- \rightarrow \pi^0 e^- \overline{\nu}_e$, possible but suppressed by $\sin^2\theta \approx 0.05$
- Flavor changing neutral current (FCNC) transitions are also predicted; e.g., K^0–$\overline{K^0}$ oscillations or $K^0 \rightarrow \mu^+\mu^-$.
- But contributions from two full generations tend to cancel each other leaving only a small residual effect mostly from quark mass differences.

➡ GIM mechanism (Glashow, Iliopoulos, Maiani) $\Rightarrow$ charm prediction.

✓ J/Ψ bound state discovered at AGS and SPEAR (1974)

Electroweak CP Violation

- Including (t,b) $\Rightarrow$ 3×3-Yukawa (mass) matrices.
- CP violation (CPV) typically occurs in the presence of complex phases leading to different interference effects between charge-conjugate amplitudes.
- Many phases in (Cabibbo-Kobayashi-Maskawa) matrix,

  $V_{\rm CKM} = U^u_L U^{d\,\dagger}_L = V^{\dagger -1}_{\rm CKM}$, removable by redefinitions.
- Phase-transformations, $U^R = U^L = \mathrm{diag}(\alpha_1,\alpha_2,\alpha_3)$, keep masses unchanged; use to remove 6−1 = 5 phases.
- Remain 3 mixing angles and 1 observable (CP violating) phase, $e^{i\delta}$.

Electroweak CP Violation

- For N families, $N^2-(2N-1) = (N-1)^2$ parameters $\Rightarrow$ $(N-1)^2 - N(N-1)/2 = (N-1)(N-2)/2$ CP phases.
- Since $N \geq 3$ is needed and observed values of CKM-matrix connecting third with lighter two families $\Rightarrow$ electroweak CPV predicted but small for any δ.

✓ CPV observed in kaons and B-mesons.

✓ All results consistent with one common value for δ.

– But not with baryon asymmetry of the universe (BAU)

– And what about direct CPV in $B \to K\pi$ decays?

New Type of CPV Discovered?

$$A^0 = \frac{\Gamma(B^0 \to K^+\pi^-) - \Gamma(B^0 \to K^-\pi^+)}{\Gamma(B^0 \to K^+\pi^-) + \Gamma(B^0 \to K^-\pi^+)} > 0$$

$$A^\pm = \frac{\Gamma(B^+ \to K^+\pi^0) - \Gamma(B^- \to K^-\pi^0)}{\Gamma(B^+ \to K^+\pi^0) + \Gamma(B^- \to K^-\pi^0)} < 0$$

Belle, BaBar (2008)

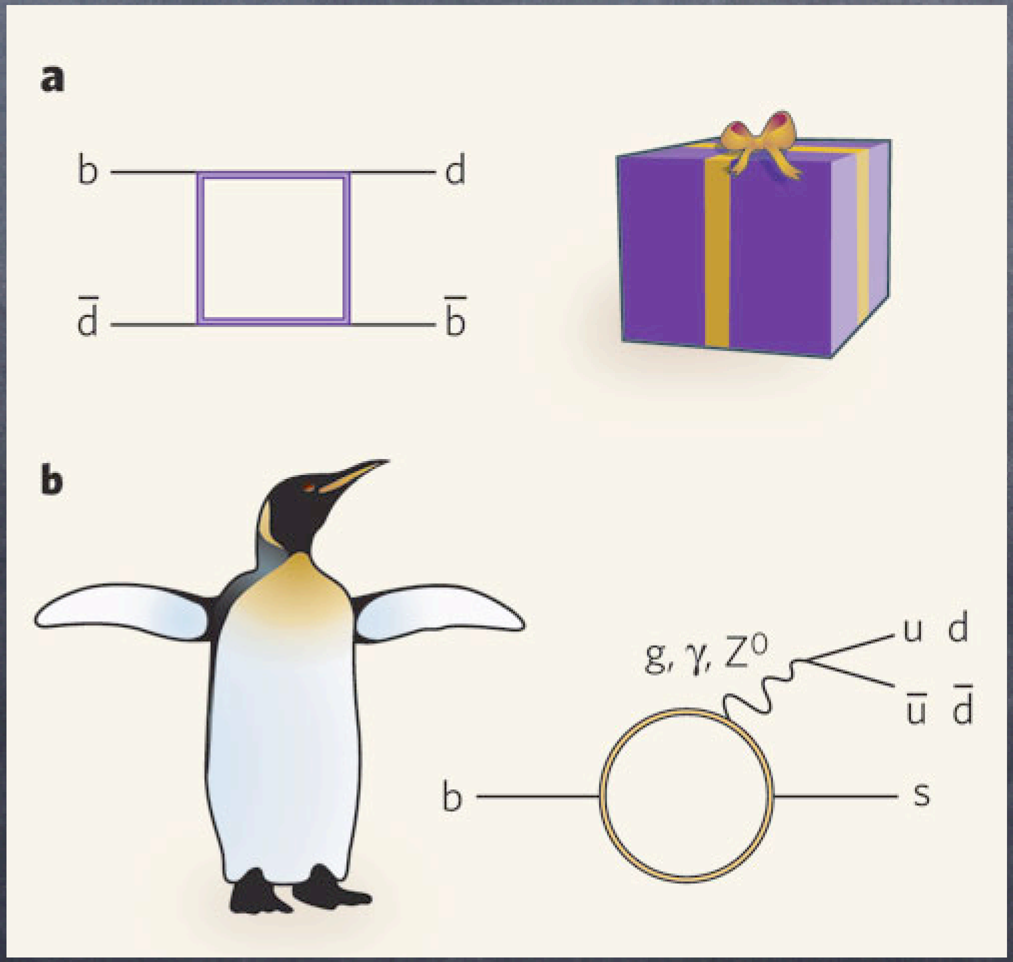


illustration from Peskins' Nature article

Strong CP Violation

- θ-angle: $\mathcal{L}(\theta) = -\theta/64\pi^2\ \epsilon^{\beta\gamma\varphi\chi} F^a_{\beta\gamma} F^a_{\varphi\chi}$ (breaks CP & P)
- Total derivative ⇒ harmless in perturbation theory.
- But non-trivial effect through extended spacetime-dependent (topological) field configurations (instantons)
- May be absorbed by chiral phase redefinition, $\psi_i \rightarrow \exp(i\gamma_5\alpha_i)\psi_i$ which is equivalent to $\theta \rightarrow \theta + 2\Sigma_i\alpha_i$ but this would introduce complex masses.

➡ But OK if any one quark mass was zero.

- $\theta \lesssim 10^{-10}$ from neutron electric dipole moment (EDM)

→ strong CP problem.

CKM-Matrix

- Jarlskog invariant J: 2×area of any unitarity triangle.

$V_{\rm CKM} =$

$$\begin{pmatrix} 0.97383^{+0.00024}_{-0.00023} & 0.2272^{+0.0010}_{-0.0010} & (3.96^{+0.09}_{-0.09}) \times 10^{-3} \\ 0.2271^{+0.0010}_{-0.0010} & 0.97296^{+0.00024}_{-0.00024} & (42.21^{+0.10}_{-0.80}) \times 10^{-3} \\ (8.14^{+0.32}_{-0.64}) \times 10^{-3} & (41.61^{+0.12}_{-0.78}) \times 10^{-3} & 0.999100^{+0.000034}_{-0.000004} \end{pmatrix},$$

and the Jarlskog invariant is $J = (3.08^{+0.16}_{-0.18}) \times 10^{-5}$.

- from Particle Data Group (2006)

SM Parameter Summary

- 3 gauge couplings: g, g', and g^s or α, $\sin^2\theta^W$, and α^s
- 2 Higgs potential parameters: λ and m_Φ^2 or M^H and v
- 9 fermion masses: e, μ, τ, u, d, s, c, b, and t
- 3 CKM mixing angles
- 1 CKM phase
- 1 QCD θ-angle (does not enter Feynman rules)

➡ Total: 19 arbitrary real parameters

Lecture IV
Experimental Tests

- The Status 25 Years Ago
- Z^0 Pole Physics
- Z^0 Pole Formulas
- LEP
- Z^0 lineshape
- SLC

Lecture IV

Experimental Tests

- Mass Determinations
- Master Equations
- $\sigma(e^+e^- \to \text{hadrons})$
- Polarized Electron Scattering
- Atomic Parity Violation
- Running Weak Mixing Angle
- SM Parameters: Results

The Status 25 Years Ago

- ✓ Weak neutral currents (1973)
- ✓ P-violation in e^--D deep inelastic scattering (1978)
- ✓ Gauge bosons (1983)
- ➡ SM correct at least to first approximation
- Need high precision experiments to establish the SM as a renormalizable QFT at level of quantum effects
- $g^2/4\pi^2 \approx 0.01 \Rightarrow$ need better than 1% accuracies
- ➡ Z factories LEP (CERN) and SLC (SLAC)

Z^0 Pole Physics

- Z^0 lineshape at LEP (3)
- Leptonic BRs and FB asymmetries at LEP (6)
- Leptonic LR (LR-FB) asymmetries at SLC (4)
- Tau polarization at LEP (2)
- Charge asymmetries (2)
- Strange quarks (3)
- Heavy flavor BR and asymmetries (6)

Z^0 Pole Formulas

$$\Gamma_W(W^+ \to e^+\nu_e) = \frac{G_F M_W^3}{6\sqrt{2}\pi}$$

$$v_f = t_f^{3L} - 2Q_f \sin^2\theta_W$$

$$\Gamma_W(W^+ \to u_i\overline{d}_j) = \frac{C G_F M_W^3}{6\sqrt{2}\pi}|V_{ij}|^2$$

$$\sin^2\theta_W \approx 0.23 \sim 1/4$$

$$\Gamma_Z(Z \to \psi_f\overline{\psi}_f) = \frac{C G_F M_Z^3}{6\sqrt{2}\pi}(v_f^2 + a_f^2)$$

$$A_f \equiv \frac{2v_f a_f}{v_f^2 + a_f^2}$$

$$A^0_{FB}(f) \equiv \frac{\sigma_F - \sigma_B}{\sigma_F + \sigma_B} = \frac{3}{4}A_e A_f$$

$$A^0_{LR} \equiv \frac{\sigma_L - \sigma_R}{\sigma_L + \sigma_R} = A_e$$

$$A^0_{LR,FB}(f) \equiv \frac{\sigma^f_{LF} - \sigma^f_{LB} - \sigma^f_{RF} + \sigma^f_{RB}}{\sigma^f_{LF} + \sigma^f_{LB} + \sigma^f_{RF} + \sigma^f_{RB}} = \frac{3}{4}A_f$$

$$a_f = t_f^{3L}$$

LEP

- 17 million Z^0 decays including Z^0 pole energy scan

$$\Rightarrow \quad M_Z = 91.1876 \pm 0.0021 \text{ GeV}$$
$$\Gamma_Z = 2.4952 \pm 0.0023 \text{ GeV}$$
$$\sigma_{had} = 41.541 \pm 0.037 \text{ nb}$$

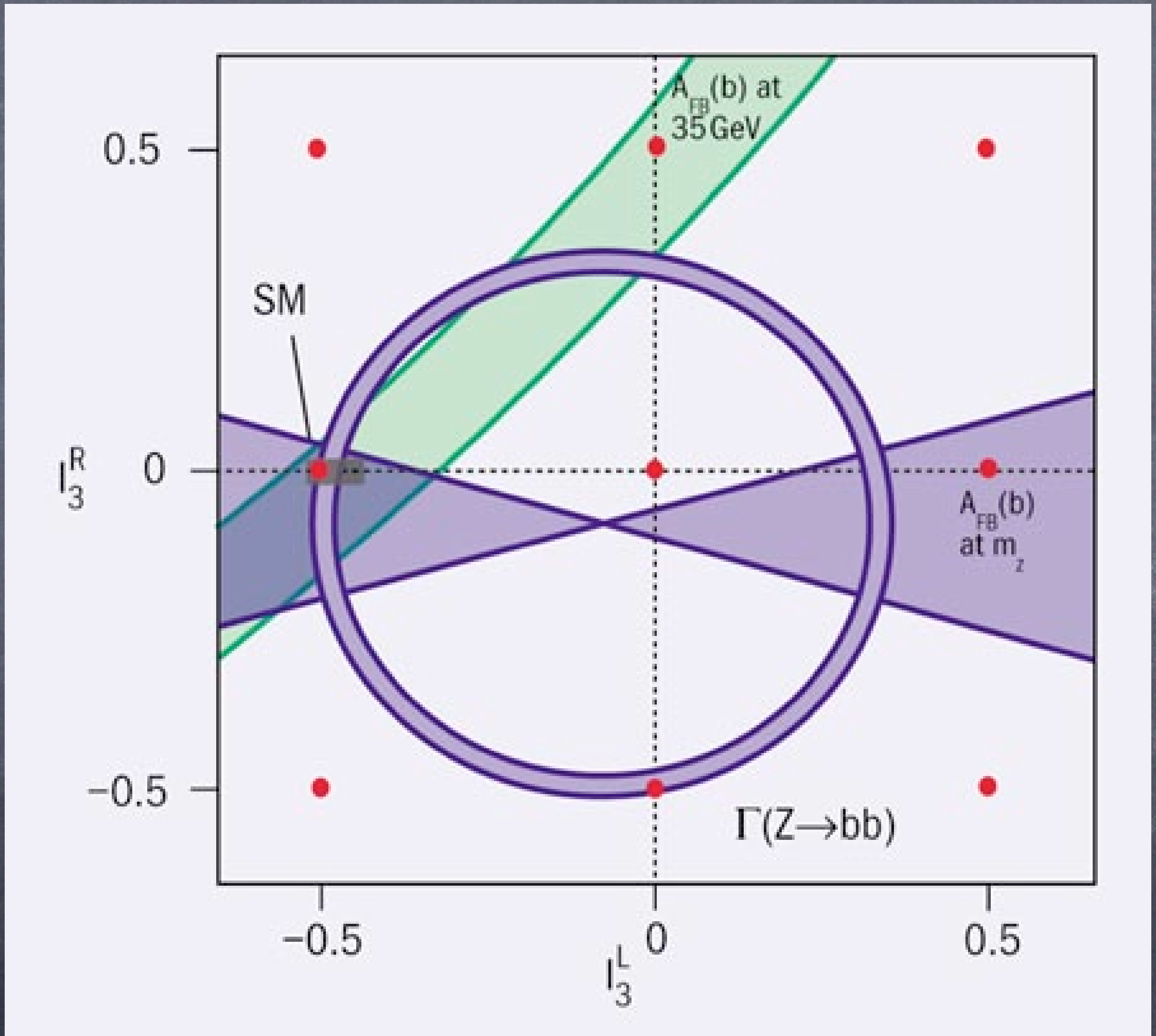
$$\Gamma_Z, \sigma_{\text{had}}, R_\ell(\ell = e, \mu, \tau) \Rightarrow \alpha_s(M_Z) = 0.1213 \pm 0.0030$$

$$\Gamma_{\text{inv}} = \Gamma_Z - \Gamma_{\text{had}} - \Gamma_l \Rightarrow N_\nu = 2.985 \pm 0.007$$

- $v^e \propto 1-4\sin^2\theta^W \approx 0.075 \ll 1 \Rightarrow$ sensitivity increase

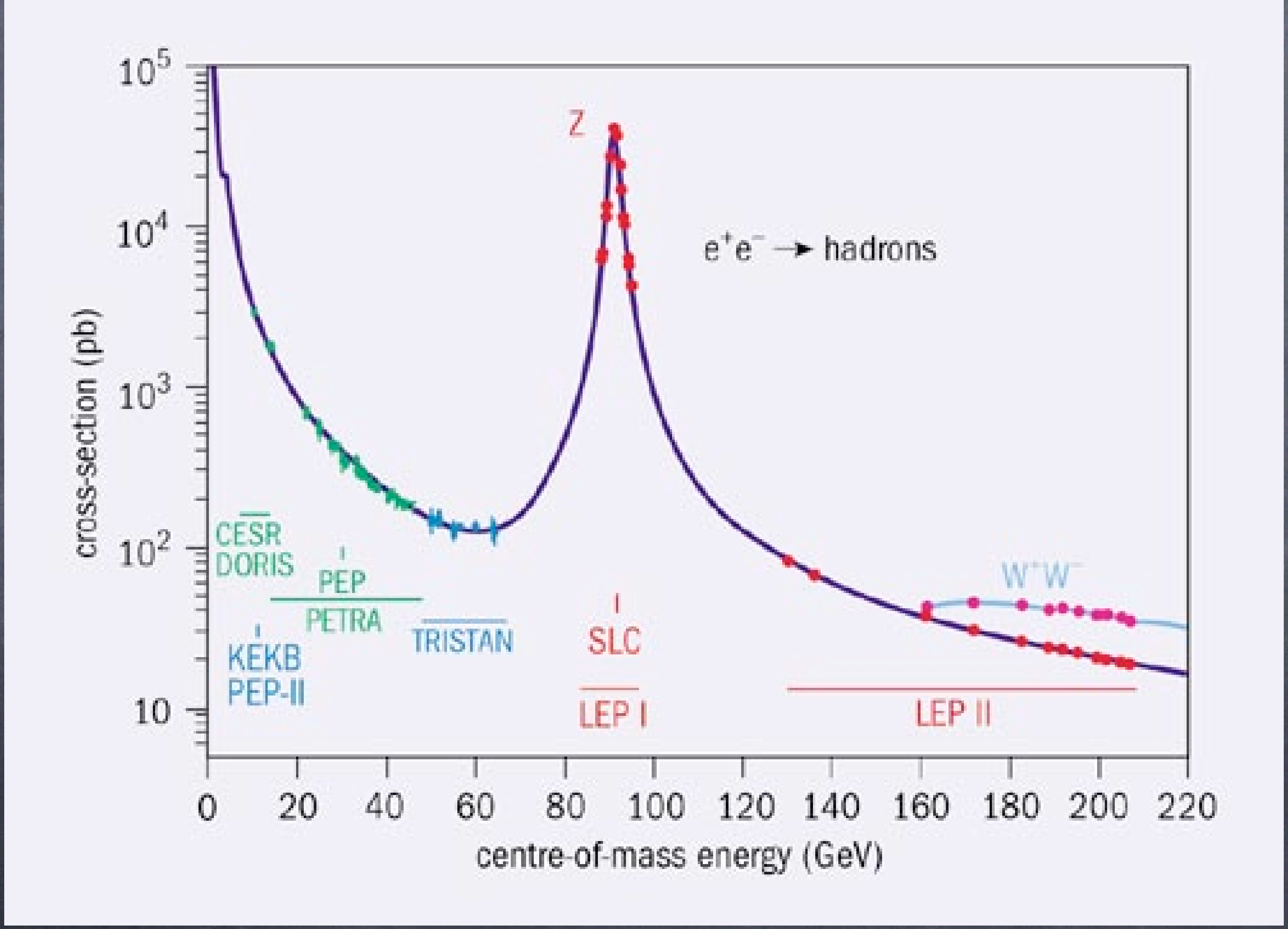
$$\frac{\sin^2\theta_W}{v_e}\frac{\partial v_e}{\partial \sin^2\theta_W} \approx 12.3$$

The Weak Isospin of the Bottom Quark



⇒ top quark exists

Z^0 lineshape



SLC

- 600,000 Z^0 bosons with a 75% polarized e^- beam.
- Polarimetry: $\mathcal{O}(1\%) \longrightarrow$ correlation of systematic errors
- A^{LR} linear in $v^e \longrightarrow A^{LR}$ larger $\longrightarrow$ better statistics
- No need to tag quark flavor or distinguish quark from antiquark (only counting of hadrons/leptons) $\longrightarrow$ clean.
- LEP and SLC (combined): $\sin^2\theta^W = 023124 \pm 0.00017$.
- Quark and lepton couplings to Z^0 boson verified to better than 1% accuracy.
- But non-standard amplitudes would be hide under Z^0.

Mass Determinations

- Z mass and width from LEP 1
- W mass (and width) from LEP 2 and Tevatron (FNAL)
- Top quark mass from Tevatron & (before) from global fit
- Charm and bottom quark masses (QCD sum rules)
- Light quark masses (chiral perturbation theory)
- Higgs boson from global fit & (later) from LHC (CERN)

Master Equations

$$\sin^2\hat{\theta}_W(M_Z) \equiv \hat{s}^2 = \frac{A^2}{M_W^2(1-\Delta\hat{r}_W)},$$

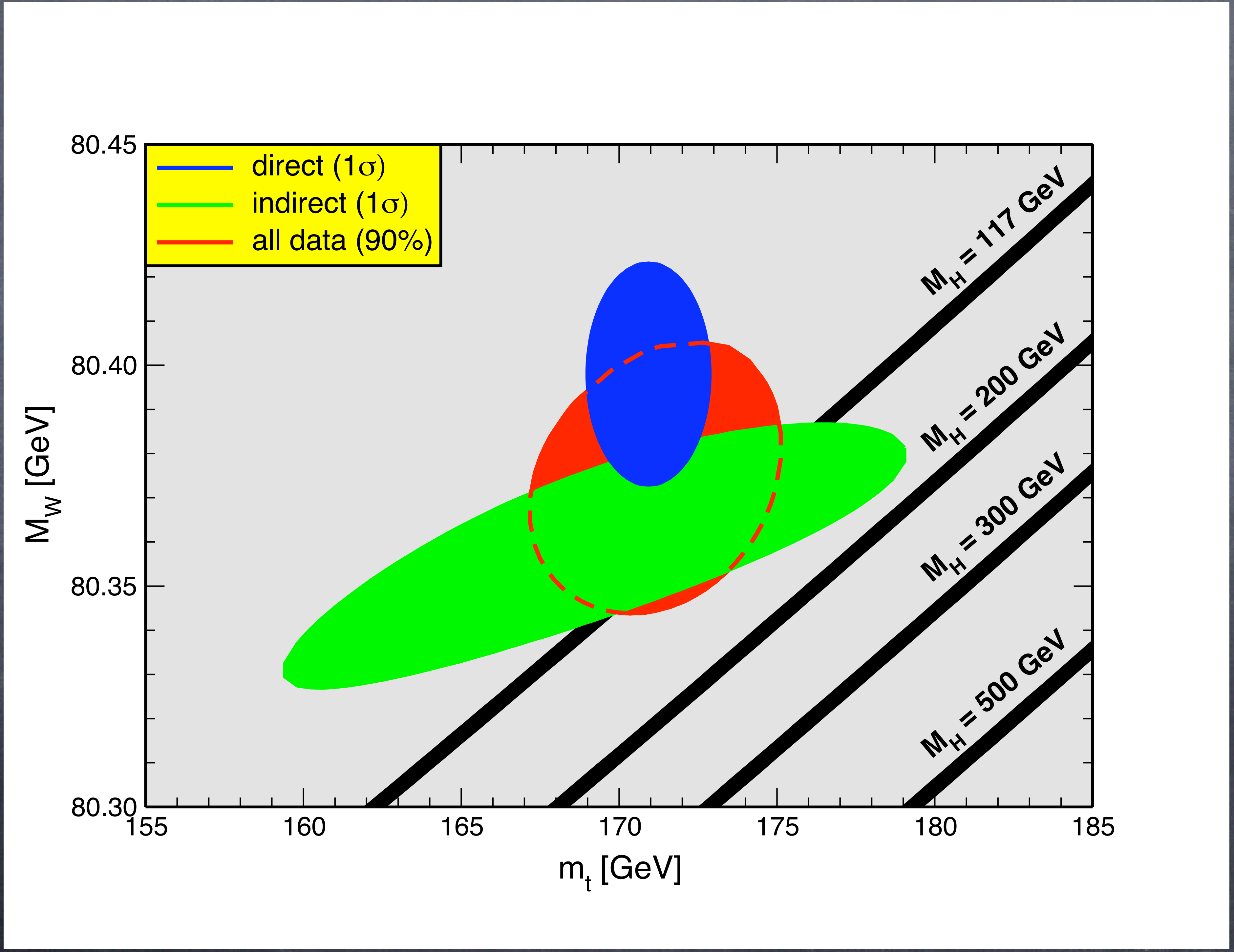
$$\sin^2\hat{\theta}_W(M_Z)\cos^2\hat{\theta}_W = \frac{A^2}{M_Z^2(1-\Delta\hat{r}_Z)}$$

$$A = \left[\frac{\pi\alpha}{\sqrt{2}G_F}\right]^{1/2}$$

$$\Delta\hat{r}_W = \frac{\alpha}{\pi}\hat{\Delta}_\gamma + \frac{\hat{\Pi}_{WW}(M_W^2) - \hat{\Pi}_{WW}(0)}{M_W^2} + V + B$$

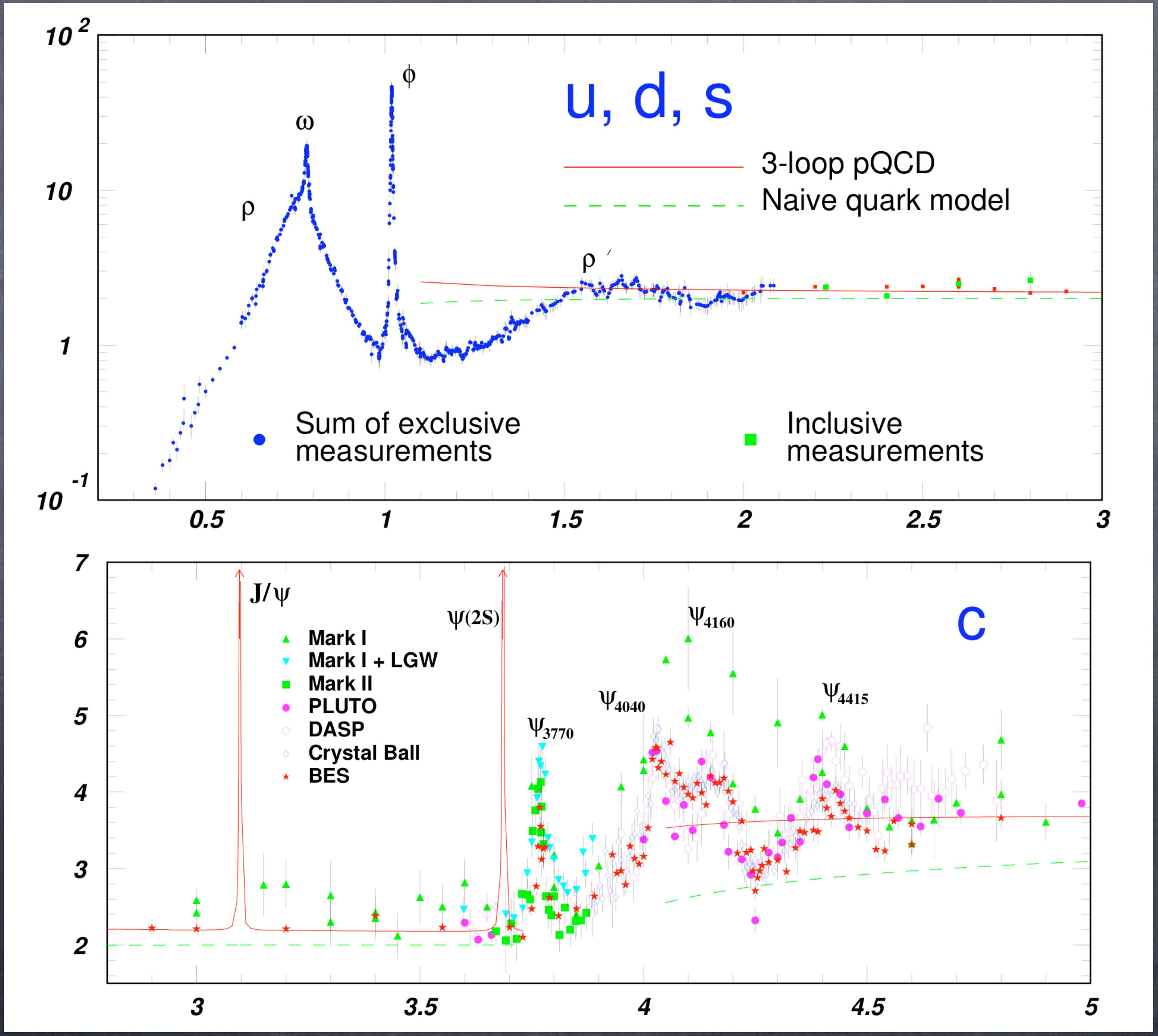
$$\Delta\hat{r}_Z = \Delta\hat{r}_W + (1-\Delta\hat{r}_W)\frac{\hat{\Pi}_{ZZ}(M_Z^2) - \frac{\hat{\Pi}_{WW}(M_W^2)}{\cos^2\hat{\theta}_W}}{M_Z^2}$$

W vs. Top Mass

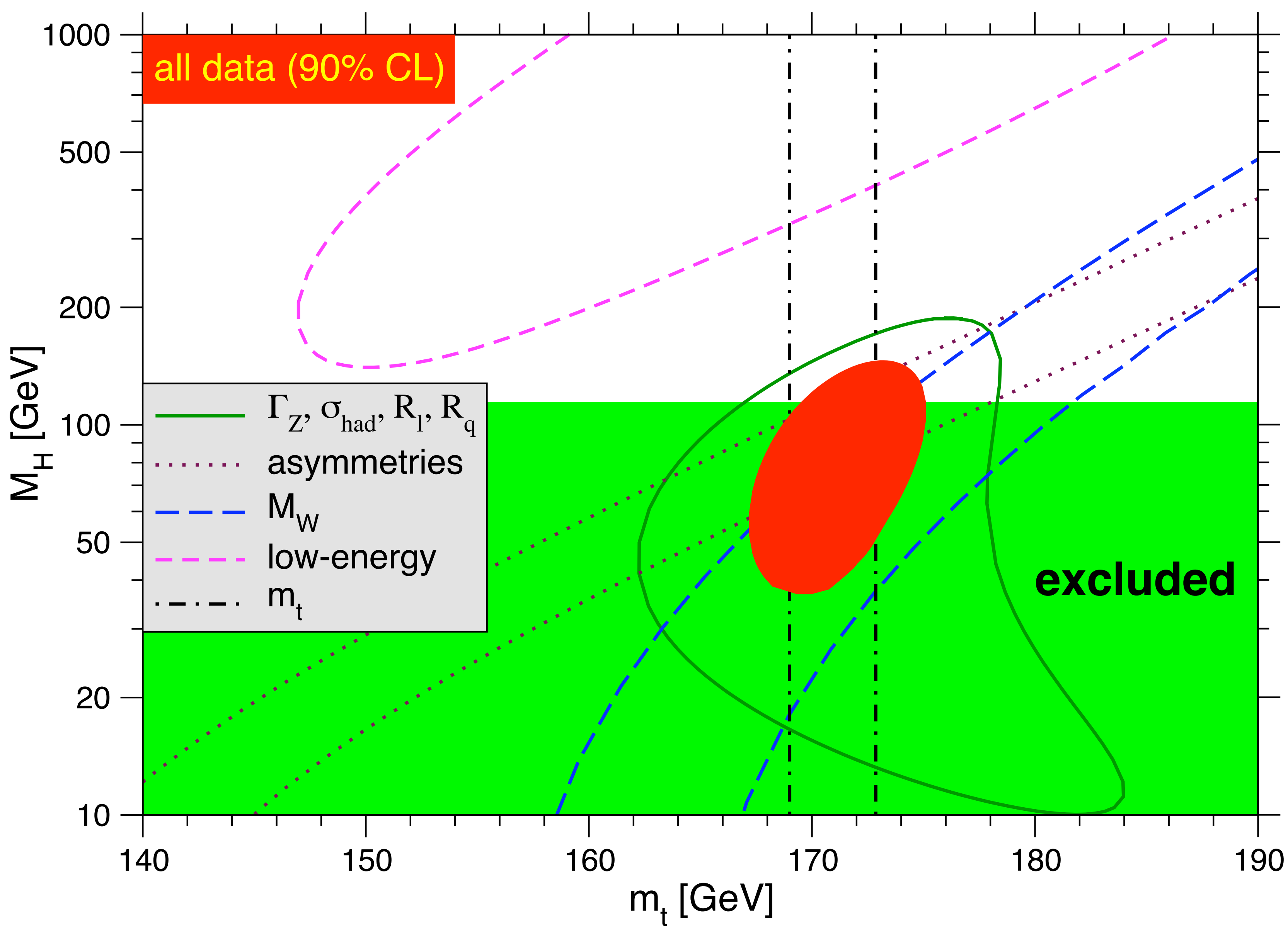


direct (1σ)
indirect (1σ)
all data (90%)
M_H = 117 GeV
M_H = 200 GeV
M_H = 300 GeV
M_H = 500 GeV
80.45
80.40
80.35
80.30
155
160
165
170
175
180
185
M_W [GeV]
m_t [GeV]

$\sigma(e^+e^- \longrightarrow \text{hadrons})$



Higgs vs. Top Mass



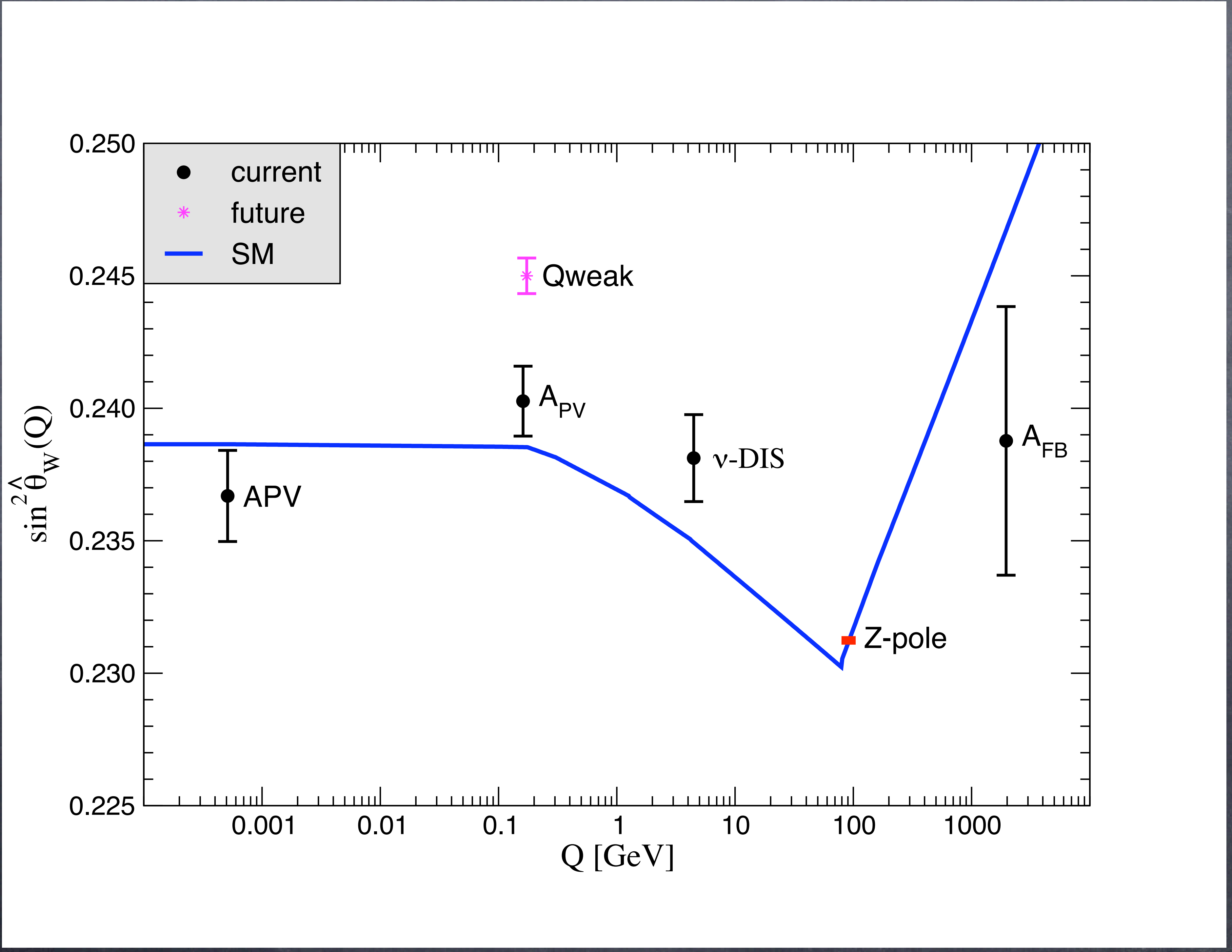
Polarized Electron Scattering

- LR cross-section asymmetry: Interference between P conserving γ amplitude and $\not{P}$ Z^0 mediated amplitude.
- eD-DIS (1978): $Q^2/M^2 \sim 10^{-4} \Rightarrow 10^{-5}$ uncertainty $\leftrightarrow$ 10% determination of Z^0 amplitude (SLAC)
- Polarized e^-e^- (Møller)-scattering: $Q^2 = 0.026$ GeV^2; $A^{pv} = (-1.31 \pm 0.17)\times 10^{-7} \Rightarrow$ (enhanced sensitivity) $\sin^2\theta^W(Q^2) = 0.2397 \pm 0.0013$ (SLAC).
- Qweak (2011): polarized e^-p-scattering (JLab) $\Rightarrow$ weak charge of the proton ($\propto 1-4\sin^2\theta^W$) to $\pm 4\%$.

Atomic Parity Violation

- Atomic Parity Violation → mixing between opposite parity states.
- Effect extremely small; use small modulation of level mixing by external electric field (Stark-mixing).
- Effect $\propto Z^3 \Rightarrow$ use heavy atoms.
- Comparison of hyperfine levels ⇒ weak charges and anapole moment.
- Complication: atomic structure calculations.
- Most precise: $^7s \rightarrow {}^6s$ transition in Cs (Boulder) ⇒ $Q^W(Cs) = 72.62 \pm 0.46 \Rightarrow \sin^2\theta^W = 0.2291 \pm 0.0019$.

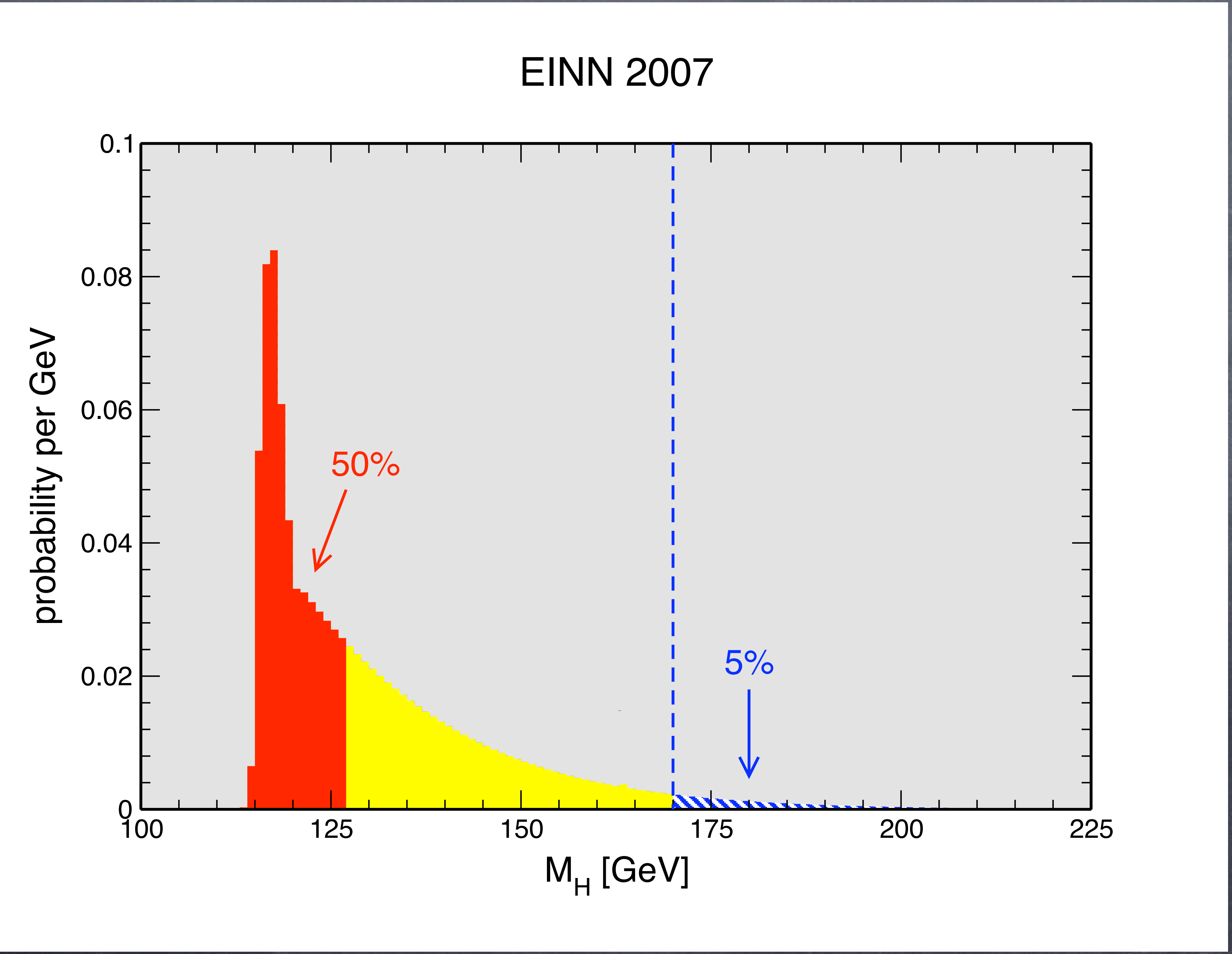
Running Weak Mixing Angle



SM Parameters: Fit Results

parameter	central value	uncertainty
$1/\hat{\alpha}(M_Z)$	127.909	± 0.019
$\sin^2\hat{\theta}_W(M_Z)$	0.23119	± 0.00014
$\hat{\alpha}_s(M_Z)$	0.1217	± 0.0017
M_W	80.375 GeV	± 15 MeV
M_Z	91.1876 GeV	± 2.1 MeV
M_H	77 GeV	+28 -22 GeV
$\hat{m}_c(\hat{m}_c)$	1.274 GeV	+36 -45 MeV
$\hat{m}_b(\hat{m}_b)$	4.196 GeV	± 28 MeV
M_t	171.1 GeV	± 1.9 GeV

Higgs Boson Mass



Lecture V

Beyond the Standard Model

- SM Limitations
- Other Low Energy Tests
- Muon $g-2$
- Accidental Symmetries and non-Renormalizable Terms
- Lepton Number Nonconservation and ν Mass
- ν Oscillations

Lecture V

Beyond the Standard Model

- Baryon Number Nonconservation and p Decay
- Running Gauge Couplings (RG)
- SM RG
- MSSM RG
- Oblique Parameters
- Conclusions

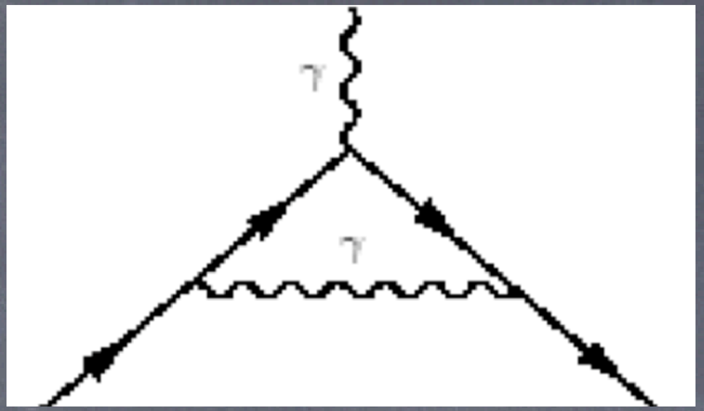
SM Limitations

- Hierarchy problem (quadratic Higgs mass corrections).
- Cosmological constant problem.
- Strong CP problem.
- Gauge group, irreps, and parameters *ad hoc*.
- ν oscillations.
- Gravity non-renormalizable.
- Baryon asymmetry of the universe.
- Dark matter.

Other Low Energy Tests

- τ lifetime and leptonic BRs (LEP, CLEO)
- ν-DIS (NuTeV, CCFR, CHARM, CDHS)
- $b \to s\,\gamma$ (BaBar, Belle, CLEO)
- Michel parameters (TWIST)
- Electric Dipole Momonts (EDMs)
- Lepton Flavor Violation (LFV)
- CKM-unitarity
- Anomalous magnetic moment of the muon ($g-2$)

Muon g−2



- Measuring with small uncertainties yields sensitivities to high energy scales:

$$\left(\frac{\Lambda}{g}\right)_{\text{new}} \sim \frac{1}{\sqrt{\Delta \mathcal{O}}}$$

- Anomalous magnetic moment of the muon

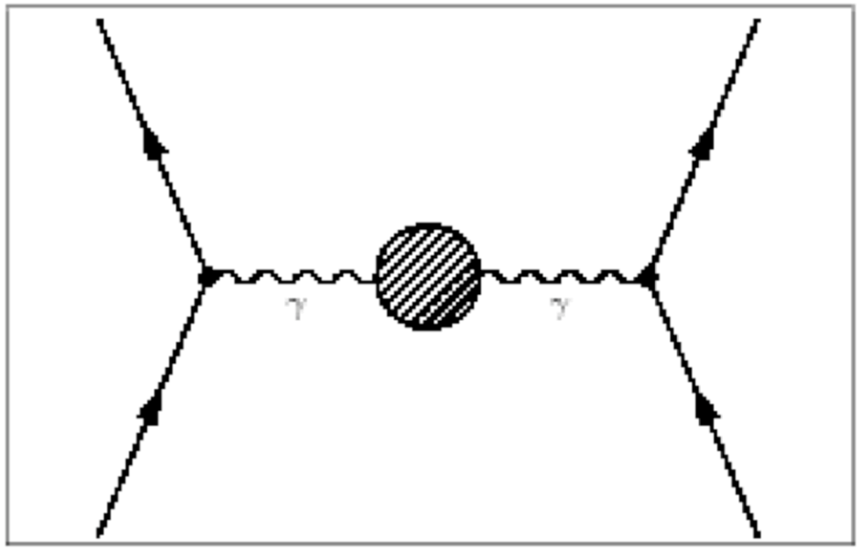
$$a_\mu \equiv \frac{g_\mu - 2}{2} = (1165920.80 \pm 0.63) \times 10^{-9}$$

$$\Lambda_{\text{new}} \sim \frac{m_\mu}{\sqrt{0.63 \times 10^{-9}}} \approx 4.2 \text{ TeV}$$

➡ Sensitivity for physics beyond the SM.

- Discrepancy of 2.7 σ (standard deviations).

- Complication: Hadronic loop effects.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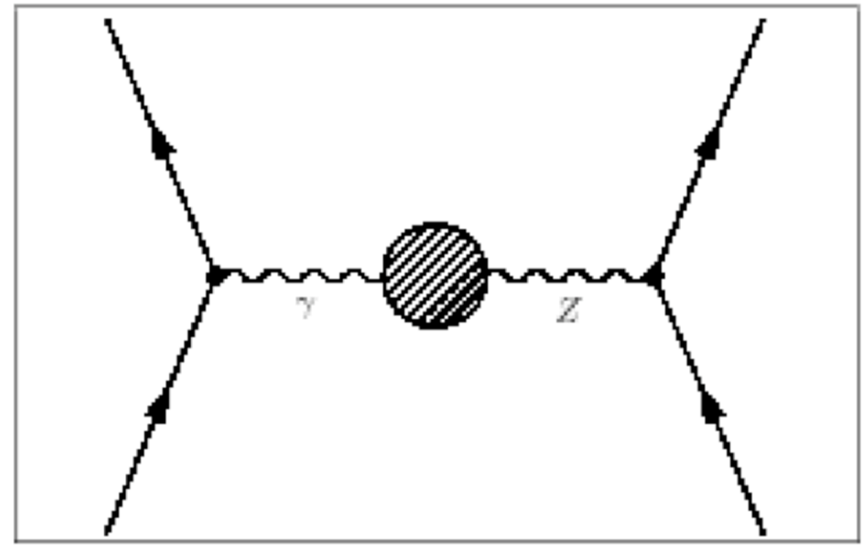
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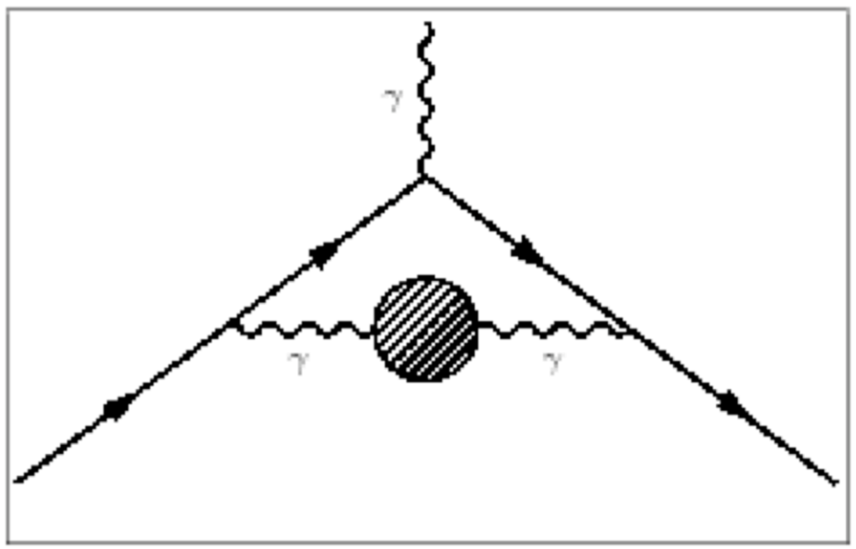
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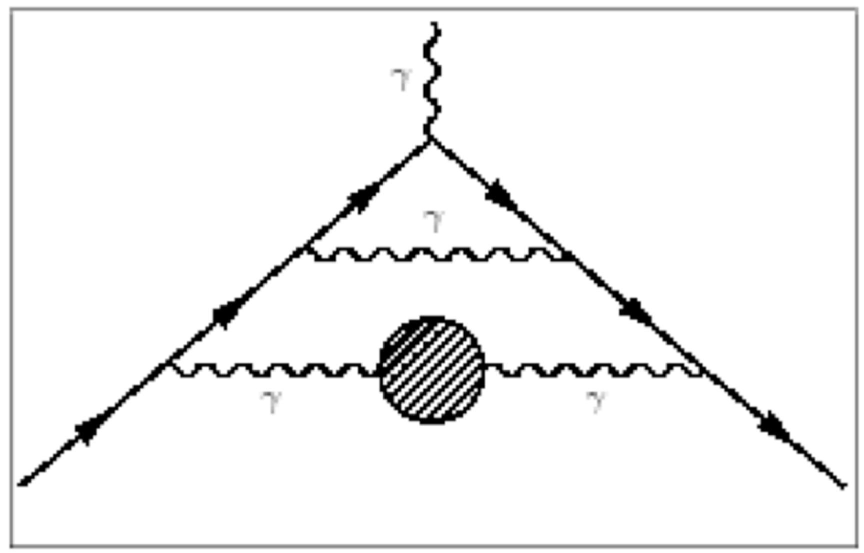
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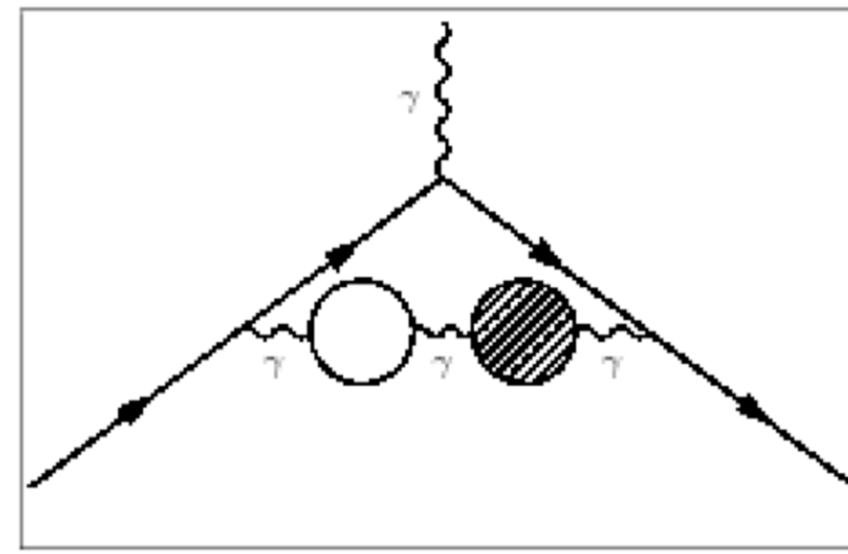
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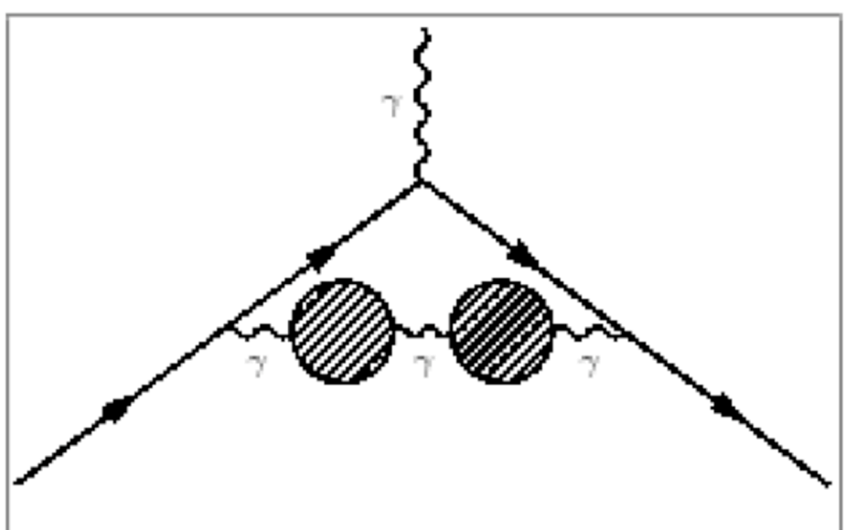
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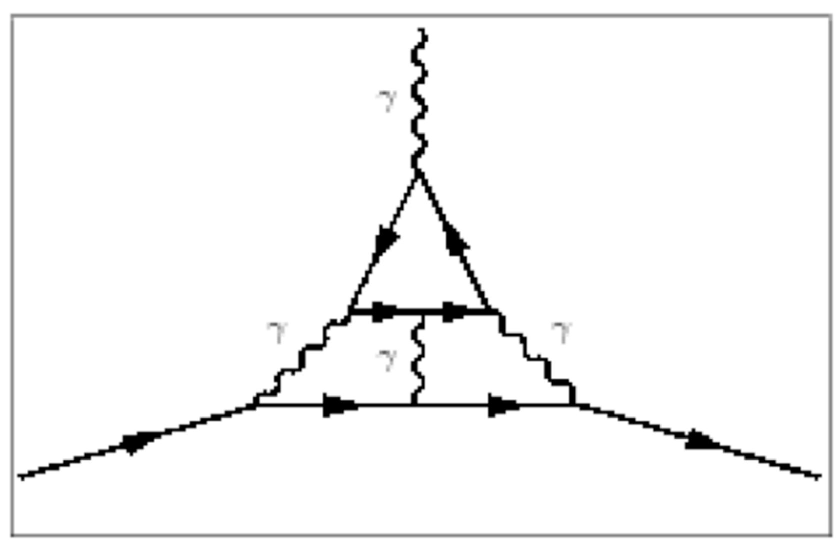
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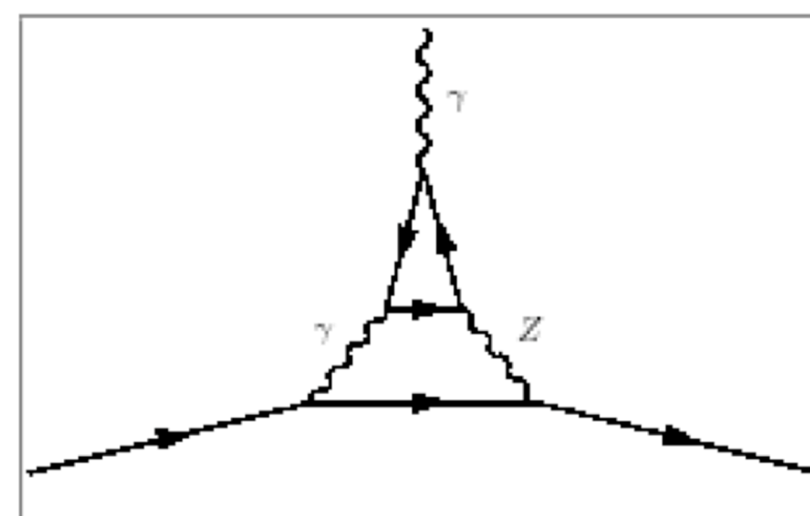

γ
γ
Z

Accidental Symmetries and non-Renormalizable Terms

- Power expansion in Λ^{-1} with each (gauge and Lorentz invariant) "operator" containing only SM fields.
- Unsuppressed terms ≡ SM (effective field theory).
- Non-renormalizable terms "match" full theory; can alternatively be taken as new adjustable parameters.
- In SM: accidental electron #, muon #, tau #, and baryon # conservation (e.g., $\mu \nrightarrow e\,\gamma$).
- Also, accidental approximate CP conservation.
- Expect these to be violated by higher dimensional operators (HDOs).

Lepton Number Nonconservation and ν Mass

- At $\mathcal{O}(\Lambda^{-1})$ baryon # still accidentally conserved.
- But one can form invariant terms out of 2 lepton and 2 Higgs doublets. H → v ⇒

$$L_M = -\sum_{ij} \frac{\lambda_{ij}}{\Lambda_{new}} \overline{\nu_i^c} \nu_j \, v^2 .$$

- Majorana mass term: connects left-handed and right-handed components of conjugate fields. ⟶
- Lepton # violated by 2 units (same for Hν interaction)

➡ 0νββ-decays (e.g., $K^- \to \pi^+ e^- e^-$ or nuclei decays)

- Lepton # violation not yet observed.

ν Oscillations

- Needs $\lambda^{ab} \neq 0$, for $a \neq b$, and $m^a \neq m^b$.

➡ Mass eigenstates ≠ weak interaction eigenstates.

➡ Lepton # conserving but lepton flavor # violating.

➡ Maki, Nakagawa, Sakata ν mixing-matrix (cf. CKM).

- LH fields ↔ RH antifields ⇒ phase counting different.
- Dirac phase plus $N_\nu - 1$ additional (CP violating) Majorana phases (not yet observed).
- ν oscillations observed in νs from the sun, earth's atmosphere, nuclear reactors & particle accelerators.

ν Oscillations

- Disappearance experiments: rate decrease of νs from source with known flavor composition.
- Appearance: detection of ν flavor not initially present.

✓ Σ of all ν flavors appears unchanged (SNO).

- $\Delta m^2 \sim \mathcal{O}(10^{-1}\text{ eV})$ [atmospheric] and $\mathcal{O}(10^{-2}\text{ eV})$ [solar].
- Generally large mixing angles (except θ_{13}).
- $\lambda^{ab} \lesssim \mathcal{O}(1) \Rightarrow \Lambda \lesssim \mathcal{O}(10^{15}\text{ GeV})$.
- Realization: see-saw mechanism (integrate out very heavy right-handed Majorana ν).
- If right-handed ν has no Majorana mass: Dirac νs.

Baryon Number Nonconservation and p Decay

- At $\mathcal{O}(\Lambda^{-2})$ baryon and lepton # violation possible.
- Invariant terms made of 1 lepton and 3 quark fields.

➡ Proton decay rate ~ $\mathcal{O}(\Lambda^{-4})$; $\tau(p)$ ~ $\mathcal{O}(\Lambda^4/m(p)^5)$.

- Experiments: $\tau(p) > 2\times10^{29}$ years $\Rightarrow \Lambda \gtrsim \mathcal{O}(10^{15}$ GeV).
- Realization: Grand Unfied Theories (GUTs) with very heavy gauge bosons (SSB!) producing Λ; e.g. SU(5).

➡ $p \to e^+\pi^0$, etc.

- Only academic interest: p decay by instantons in SM.
- Baryon # violation necessary for BAU.

Running Gauge Couplings (RG)

- β-function $\mu^2\, d/d\mu^2\, \alpha(\mu) \equiv \beta(\mu) = \beta_0 \alpha^2/\pi + \mathcal{O}(\alpha^3) \Rightarrow$

$$\alpha(\mu) = \frac{\alpha(\mu_0)}{1 - \frac{\alpha(\mu_0)}{\pi}\beta_0 \ln\frac{\mu^2}{\mu_0^2}} \Rightarrow \alpha^{-1}(\mu) = \alpha^{-1}(\mu_0) - \frac{\beta_0}{\pi}\ln\frac{\mu^2}{\mu_0^2}$$

$$\mu_U = M_Z \exp\left[\frac{\pi}{2}\frac{\alpha_1^{-1}(M_Z) - \alpha_2^{-1}(M_Z)}{\beta_0^{(1)} - \beta_0^{(2)}}\right]$$

$$\alpha_3^{-1}(M_Z) = \frac{\beta_0^{(3)} - \beta_0^{(2)}}{\beta_0^{(1)} - \beta_0^{(2)}}\alpha_1^{-1}(M_Z) + \frac{\beta_0^{(3)} - \beta_0^{(1)}}{\beta_0^{(2)} - \beta_0^{(1)}}\alpha_2^{-1}(M_Z)$$

$$\alpha^{-1}(\mu_U) = \frac{\alpha_2^{-1}\beta_0^{(1)} - \alpha_1^{-1}\beta_0^{(2)}}{\beta_0^{(1)} - \beta_0^{(2)}}$$

SM RG

$$\frac{1}{\hat{\alpha}_1(M_Z)} = \frac{3}{5}\frac{\cos^2\hat{\theta}(M_Z)}{\hat{\alpha}(M_Z)} = 59.003 \pm 0.014,$$

$$\frac{1}{\hat{\alpha}_2(M_Z)} = \frac{\sin^2\hat{\theta}(M_Z)}{\hat{\alpha}(M_Z)} = 29.571 \pm 0.018,$$

$$\frac{1}{\hat{\alpha}_3(M_Z)} = \frac{1}{\hat{\alpha}_s(M_Z)} = 8.217 \pm 0.115.$$

$$\beta_0^{(1)} = \frac{3}{5}\left[\frac{1}{6}\sum_f y_f^2 + \frac{1}{24}\sum_s y_s^2\right] = \frac{3}{5}\left[\frac{n_F}{6}\frac{10}{3} + \frac{n_H}{24}\right] = \frac{41}{40},$$

$$\beta_0^{(2)} = \frac{1}{6}\sum_f T_f + \sum_s \frac{1}{12}T_s - \frac{11}{12}C_A = \frac{n_F}{3} + \frac{n_H}{24} - \frac{11}{6} = -\frac{19}{24},$$

$$\beta_0^{(3)} = \frac{1}{3}\sum_q T_q - \frac{11}{12}C_A = \frac{n_F}{3} - \frac{11}{4} = -\frac{7}{4}$$

MSSM RG

$$\beta_0^{(1)} = \frac{3}{5} \times \frac{1}{4} \sum_\Phi y_\Phi^2 = \frac{3}{5}\left[\frac{n_F}{4}\frac{10}{3} + \frac{n_H}{8}\right] = \frac{33}{20},$$

$$\beta_0^{(2)} = \frac{1}{4}\sum_\Phi T_\Phi - \frac{3}{4}C_A = \frac{n_F}{2} + \frac{n_H}{8} - \frac{3}{2} = \frac{1}{4},$$

$$\beta_0^{(3)} = \frac{1}{2}\sum_q T_q - \frac{3}{4}C_A = \frac{n_F}{2} - \frac{9}{4} = -\frac{3}{4}.$$

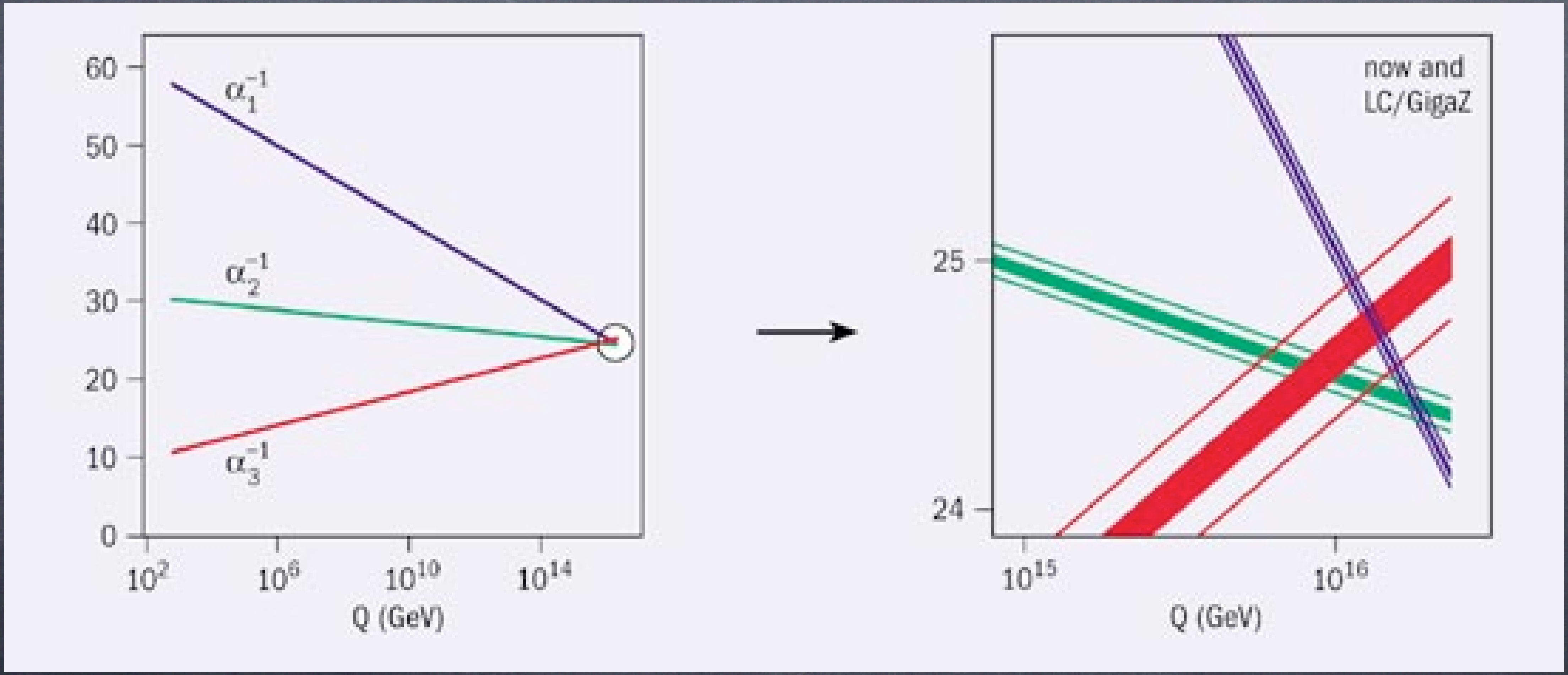
$\mu_U^{\rm SM}(\text{1-loop}) = 1.0 \times 10^{13}\ \text{GeV}$ $\mu_U^{\rm MSSM}(\text{1-loop}) = 2.0 \times 10^{16}\ \text{GeV}$

$\alpha_s(M_Z)^{\rm SM}(\text{1-loop prediction}) = 0.071$

$\alpha_s(M_Z)^{\rm MSSM}(\text{1-loop prediction}) = 0.117$

$\alpha^{-1}(\mu_U)^{\rm SM} = 42.4$ $\alpha^{-1}(\mu_U)^{\rm MSSM} = 24.3$

MSSM RG



60
50
40
30
20
10
0
α_1^{-1}
α_2^{-1}
α_3^{-1}
10^2
10^6
10^{10}
10^{14}
Q (GeV)
now and
LC/GigaZ
25
24
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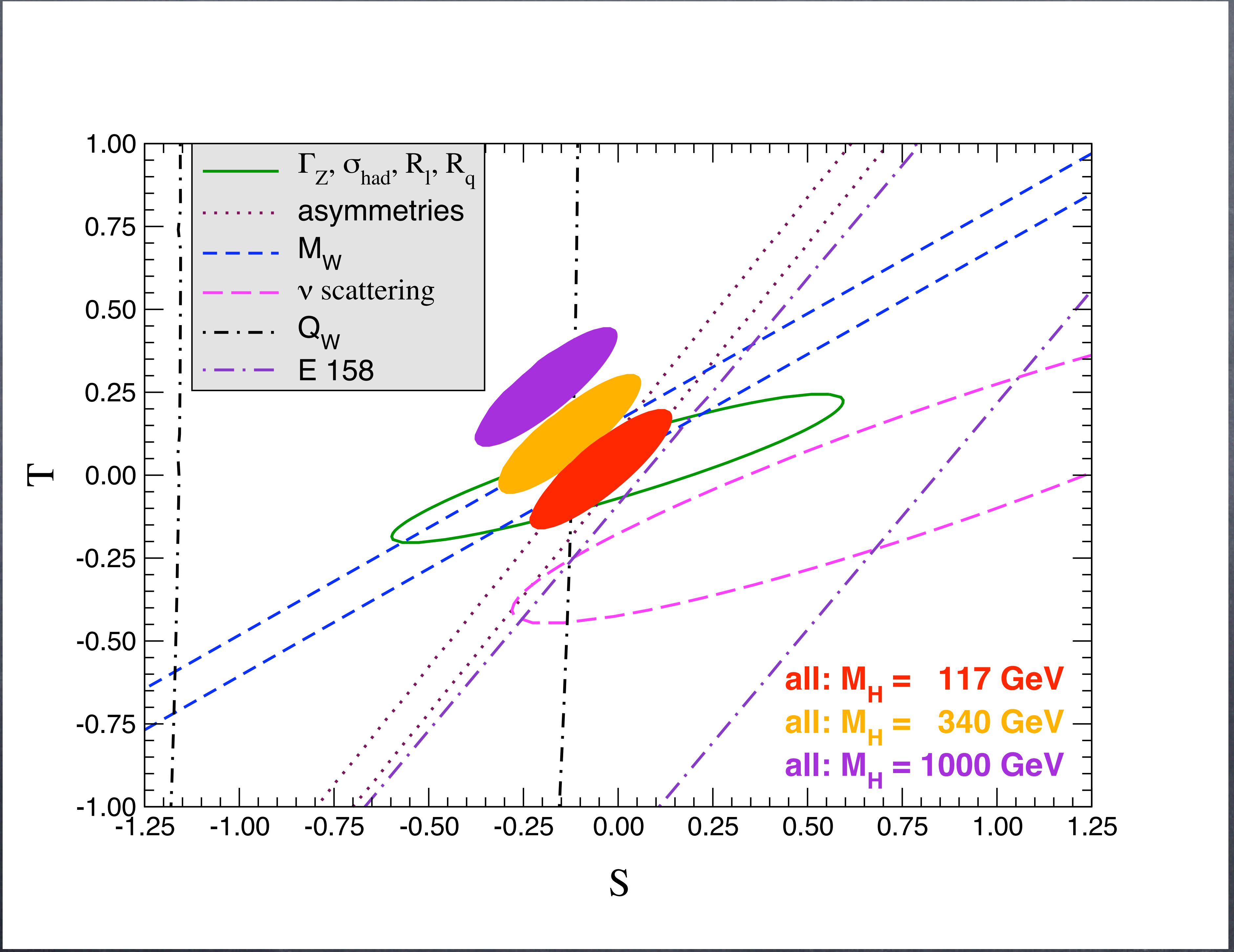
10^{16}
Q (GeV)

Oblique Parameters



Conclusions

- Structure of SM follows basically from QM and Lorentz invariance.

✓ Experimentally extremely well tested and correct.

✓ Most SM parameters well measured.

- Still need to discover the Higgs boson.
- Some smaller (inconclusive) but interesting deviations.
- Naturalness and fine-tuning problems.

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