

Concepts and Applications of Effective Field Theories: Flavor Physics and Beyond (Matthias Neubert)

BCVSPIN – MSPF – MITCHELL Joint School, December 2014

1. Matching of Wilson coefficients in the effective weak Hamiltonian

Assume that, in addition to its standard interactions, the Z^0 boson has a small flavor-changing coupling to left-handed b and s quarks:

$$\mathcal{L}_Z = \frac{g_2}{\cos \theta_W} Z^\mu \left\{ \sum_f \bar{f} \gamma_\mu \left(T_f^3 \frac{1 - \gamma_5}{2} - Q_f \sin^2 \theta_W \right) f + \left(\varepsilon_{bs} \bar{s} \gamma_\mu \frac{1 - \gamma_5}{2} b + \text{h.c.} \right) \right\},$$

where $|\varepsilon_{bs}| \ll 1$. The sum in the first term extends over all Standard Model fermions. T_f^3 is the third component of weak isospin, Q_f the electric charge in units of e , g_2 the SU(2) gauge coupling, and θ_W the weak mixing angle.

Calculate the contributions to the Wilson coefficients C_{3-10} in the effective weak Hamiltonian for $b \rightarrow s\bar{q}q$ transitions arising from tree-level Z -boson exchange, working to first order in ε_{bs} . Recall that $m_Z \cos \theta_W = m_W$ and $G_F/\sqrt{2} = g_2^2/8m_W^2$. Use the fact that $T_f^3 = 0$ for right-handed quarks, while $T_f^3 = Q_f - Y$ with $Y = 1/6$ for left-handed quarks.

2. Effective Lagrangian for light-by-light scattering

In the Standard Model, the elastic scattering of two photons $\gamma\gamma \rightarrow \gamma\gamma$ (light-by-light scattering) is forbidden at tree level, because the photon is a neutral particle. This process can, however, be induced at one-loop order via the exchange of virtual particles carrying electric charge.

- a) Draw the one-loop diagrams contributing to the $\gamma\gamma \rightarrow \gamma\gamma$ process in the Standard Model.
- b) At very low energy ($E_\gamma \ll m_e$), the light-by-light scattering amplitude can be obtained from an effective low-energy Lagrangian. Construct the leading (lowest-dimensional) gauge-invariant operators that can appear in this Lagrangian. What is the mass dimension of these operators?
- c) Estimate the corresponding contributions to the scattering amplitude by dimensional analysis, taking into account factors of gauge couplings, loop factors, powers of external momenta, and masses of heavy virtual particles. Which loop graphs give the dominant contributions at very low energy?