Concepts and Applications of Effective Field Theories: Flavor Physics and Beyond

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JOHANNES GUTENBERG UNIVERSITÄT MAINZ

BCVSPIN-MSPF-MITCHELL Joint School 2014
Las Hadas Golf Resort & Marina
Manzanillo, Colima, Mexico, 8-13 December 2014

Mainz Institute for Theoretical Physics

Effective Field Theory

Effective field theories are a very powerful tool in quantum field theories:

- systematic formalism for the analysis of **multi-scale problems**
- simplifies **practical calculations**, often makes them feasible ("Taylor expansion of Feynman graphs")
- particularly important in QCD, where short-distance effects are calculable perturbatively, but long-distance effects are not
- provides new perspective on **renormalization**
- basis of **factorization** (i.e. scale separation) and **resummation** of large logarithmic terms

Effective Field Theory

Useful reviews:

- E. Witten, Nucl. Phys. B 122 (1977) 109
- S. Weinberg, Phys. Lett. B 91 (1980) 51
- L. Hall, Nucl. Phys. B 178 (1981) 75
- J. Polchinsky, hep-th/9210046
- A. Buras, hep-ph/9806471
- M. Neubert, hep-ph/0512222

Effective Field Theory

Lecture I:

- General concepts of EFTs
- Scale separation, integrating out high-energy modes, low-energy effective Lagrangian
- Modern view of QFTs and general principles

Lecture II:

- Applications
- The Standard Model as an effective field theory
- Interesting insights


Lecture I: Concepts of Effective Field Theory

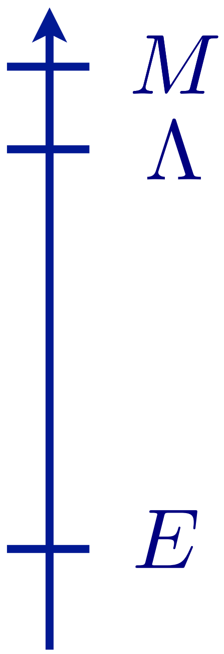
Derivation of the effective Lagrangian

Consider a QFT with a characteristic (fundamental) high-energy scale *M*

We are interested in performing experiments at energies $E \ll M$

Step 1: Choose a cutoff $\Lambda < M$ and divide all quantum fields into high- and low-frequency components ($\omega > \Lambda$ and $\omega < \Lambda$):

$$\boxed{\phi = \phi_L + \phi_H}$$



Recall:

$$\phi(x) = \int \frac{d^3k}{(2\pi)^3\, 2E_k} \left(a_k\, e^{-ik\cdot x} + a_k^\dagger\, e^{ik\cdot x} \right)$$

Derivation of the effective Lagrangian

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Physics (any Green function) at low energies $E \ll \Lambda$ is entirely described in terms of the fields ϕ_L; Green functions of these fields can be derived from the generating functional:

$$Z[J_L] = \int \mathcal{D}\phi_L \, \mathcal{D}\phi_H \, e^{iS(\phi_L,\phi_H) + i\int d^Dx \, J_L(x)\, \phi_L(x)}$$

$$\langle 0|\, T\{\phi_L(x_1)\ldots\phi_L(x_n)\}\, 0\rangle = \frac{1}{Z[0]} \left(-i\, \frac{\delta}{\delta J_L(x_1)} \right) \ldots \left(-i\, \frac{\delta}{\delta J_L(x_n)} \right) Z[J_L] \Big|_{J_L=0}$$

Derivation of the effective Lagrangian

<u>Step 2:</u> Since the high-frequency fields ϕ_H do not appear in the generating functional, we can **"integrate them out"** in the path integral:

$$Z[J_L] \equiv \int \mathcal{D}\phi_L \, e^{iS_\Lambda(\phi_L)+i\int d^Dx \, J_L(x)\, \phi_L(x)}$$

where

$$\boxed{e^{iS_\Lambda(\phi_L)} = \int \mathcal{D}\phi_H \, e^{iS(\phi_L,\phi_H)}}$$

and $S_\Lambda(\phi_L)$ is called the **Wilsonian effective action**

Dependence on the cutoff Λ enters via the condition on the frequencies of the fields

Derivation of the effective Lagrangian

Step 3: Effective action is **non-local** on the scale $\Delta t \sim 1/\omega$, corresponding to the propagation of high-energy modes that have been removed from the Lagrangian

Since the remaining fields have energies $\omega < \Lambda$, the non-local effective action can be expanded in an **infinite series of local operators:**

$$S_\Lambda(\phi_L) = \int d^D x \, \mathcal{L}_\Lambda^{\rm eff}(x)$$

where:

$$\boxed{\mathcal{L}_\Lambda^{\rm eff}(x) = \sum_i g_i \, Q_i(\phi_L(x))}$$

coupling constants (Wilson coefficients)

local operators built out of fields ϕ_L and their derivatives

Dimensional analysis

Does a Lagrangian consisting of an infinite number of interactions and hence an infinite number of (renormalized) coupling constants give us any predictive power?

- Not if one adopt an old-fashioned view about renormalization and renormalizable QFTs
- But not all is lost...

Can use **naive dimensional analysis** to estimate the size of individual terms in the infinite sum to a given matrix element

Dimensional analysis

As is common practice in particle physics, we adopt units where $\hbar = c = 1$, such that $[m] = [E] = [p] = [x^{-1}] = [t^{-1}]$ are all measured in the same units (mass units)

Denote by $[g_i] = -\gamma_i$ the mass dimension of the coupling constants in the effective Lagrangian

Since by assumption the theory has only a single fundamental scale *M*, it follows that:

$$g_i = C_i \, M^{-\gamma_i}$$

where by **naturalness** we expect that $C_i = O(1)$

Dimensional analysis

At low energy, it follows that the contribution of a given term $g_i\, Q_i$ to an observable (which for simplicity we assume to be dimensionless) scales like:

$$\boxed{C_i \left(\frac{E}{M}\right)^{\gamma_i} = \begin{cases} O(1)\,; & \text{if } \gamma_i = 0 \\ \ll 1\,; & \text{if } \gamma_i > 0 \\ \gg 1\,; & \text{if } \gamma_i < 0 \end{cases}}$$

Therefore, only operators with $\gamma_i \leq 0$ are important for $E \ll M$

This is what makes the effective Lagrangian useful!

Depending on the precision goal, one can truncate the infinite sum over terms by only retaining operators whose γ_i value is smaller than a certain value

Dimensional analysis

Since the Lagrangian has mass dimension *D* = dimensionality of spacetime (the action is dimensionless), it follows that

$$\delta_i = [Q_i] = D + \gamma_i$$

Hence we can summarize:

Dimension	Importance for $E \to 0$	Terminology
$\delta_i < D,\ \gamma_i < 0$	grows	relevant operators (super-renormalizable)
$\delta_i = D,\ \gamma_i = 0$	constant	marginal operators (renormalizable)
$\delta_i > D,\ \gamma_i > 0$	falls	irrelevant operators (non-renormalizable)

Only a **finite number** of relevant and marginal operators exist!

Dimensional analysis

Comments:

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$\delta_i > D,\ \gamma_i > 0$	falls	irrelevant operators (non-renormalizable)

- **“relevant” operators** are usually unimportant, since they are forbidden by some symmetry (otherwise they give rise to a hierarchy problem)
- **“marginal” operators** are all there is in renormalizable QFTs
- **“irrelevant” operators** are the most interesting ones, since they tell us something about the fundamental scale M

Example: ϕ^4- theory at weak coupling

Use the free Lagrangian to derive the mass dimension of all fields and couplings, assuming the theory is weakly coupled:

$$S = \int d^D x \left(\frac{1}{2}\,\partial_\mu \phi\,\partial^\mu \phi - \frac{m^2}{2}\,\phi^2 - \frac{\lambda}{4!}\,\phi^4 \right)$$

In *D* dimensions, it follows that:

$$[\phi] = \frac{D}{2} - 1\,, \quad [m] = 1\,, \quad [\lambda] = 4 - D$$

Hence:

- The mass term is a relevant operator
- The interaction term is marginal in $D=4$ (relevant in $D<4$)

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In D dimensions, it follows that:

$$[\phi] = \frac{D}{2} - 1 \,, \quad [m] = 1 \,, \quad [\lambda] = 4 - D$$

Hence:

- An operator containing n_1 fields ϕ and n_2 derivatives has dimension:

$$\delta_i = n_1 \left(\frac{D}{2} - 1 \right) + n_2 \,, \qquad \gamma_i = (n_1 - 2) \left(\frac{D}{2} - 1 \right) + (n_2 - 2)$$

- For *D*>2, adding fields or derivatives increases the dimension!

Comments

Examples of effective field theories:

High-energy theory	Fundamental scale	Low-energy theory
Standard Model	$M_W \sim 80\,\mathrm{GeV}$	Fermi theory
GUT	$M_{\mathrm{GUT}} \sim 10^{16}\,\mathrm{GeV}$	Standard Model
String theory	$M_S \sim 10^{18}\,\mathrm{GeV}$	QFT
11-dim. M theory	...	String theory
...	...	...
QCD	$m_b \sim 5$ GeV	HQET, NRQCD
	$M_{\mathrm{ChSM}} \sim 1$ GeV	ChPT

- SM and GUTs are perturbative QFTs
- Fermi theory contains only irrelevant operators (4 fermions)
- String/M theory: fundamental theory is non-local and even spacetime breaks down at short distances

Comments

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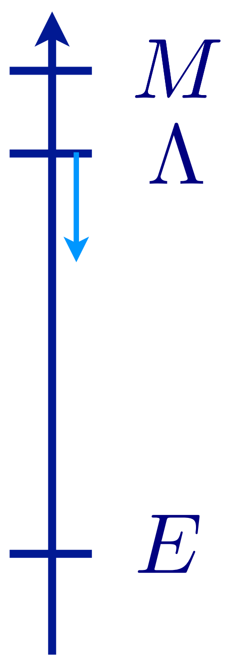
	$M_{ChSM} \sim 1$ GeV	ChPT

- QCD at low energy: example with strong coupling, where the relevant degrees of freedom at low energy (hadrons) are different from the degrees of freedom of QCD
- Low-energy theory is strongly coupled, yet ChPT is useful

Running couplings / Wilson coefficients

Often the fields ϕ_H correspond to heavy particles, whose effects become unimportant at low energies

But the frequency decomposition implies that **high-energy excitations of massless particles** (such as gauge bosons) are also integrated out from the low-energy effective theory



Consider now the situation where we lower the cutoff Λ without crossing the threshold for a heavy particle that could be integrated out:

- the structure of the operators Q_i in the effective Lagrangian remains the same
- hence, the effect of lowering the cutoff must be entirely absorbed into the values of the coupling constants g_i

Follows that $g_i = g_i(\Lambda)$ are **running,** Λ-dependent parameters!

Modern quantum field theory

“Theorem of modesty”:

- no QFT ever is complete on all length and energy scales
- all QFTs are low-energy effective theories valid in some energy range, up to some cutoff Λ

Giving up renormalizability as a construction criterion for “decent” QFTs:

- at low energy, any effective theory will automatically reduce to a “renormalizable” QFT, meaning that “non-renormalizable” interactions give rise to small contributions $\sim(E/M)^n$
- this does not make renormalization irrelevant, but it provides a different point of view (Wilsonian picture of the RG)

Modern quantum field theory

We should forget the folklore about “cancellations of infinities”

Adopt the more physical viewpoint that:

- low-energy physics depends on the **short-distance dynamics** of the fundamental theory only through a small number of **relevant and marginal couplings,** and possibly through some irrelevant couplings if our measurements are sufficiently precise
- this finite number of couplings can be renormalized (i.e., infinities can be removed consistently) using a finite number of experimental data
- the criterion of “renormalizability” is **automatically fulfilled** (approximately) by any effective field theory

Modern quantum field theory

We should forget the folklore about "cancellations of infinities"

Adopt the more physical viewpoint that:

- contrary to the old paradigm of strictly forbidding irrelevant interactions, we always expect them to be present and give rise to small effects, which may or may not be observable at a given level of accuracy
- this provides an **"indirect way"** to search for hints of physics beyond the (current) Standard Model:

low-energy, high-precision measurements

- e.g.: flavor physics, neutrino physics, $(g-2)_\mu$, EDMs, dark-photon searches, …

Modern quantum field theory

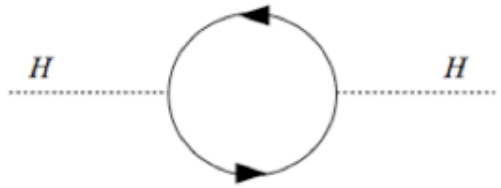
Instead, relevant ("super-renormalizable") interactions cause problems!

Consider, e.g., the mass term $m^2\phi^2$ in scalar field theory

Dimensional analysis suggests that $m^2 \sim M^2 \sim \Lambda_{\mathrm{UV}}^2$

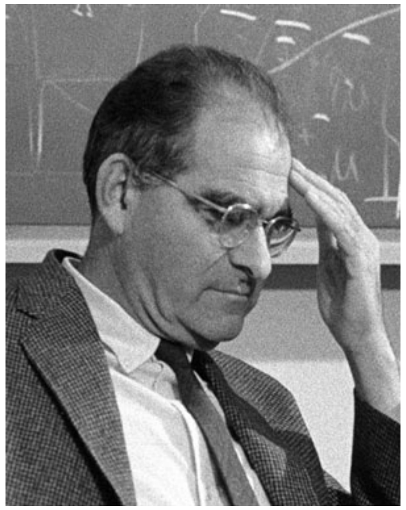
But then a light scalar particle should not be present in the low-energy effective theory!

Hierarchy problem!



The same argument applies for all mass terms in any QFT!

And likewise for the cosmological constant!



Victor Weisskopf

Modern quantum field theory

New paradigm: EFTs must be **natural** in the sense that **all mass terms should be forbidden** by (exact or broken) symmetries!

Indeed:

- **gauge invariance:** forbids mass terms for gauge fields (photons and gluons in the Standard Model)
- **chiral symmetry:** forbids mass terms for fermions (all matter fields in the Standard Model)

Explains why the SM is a (broken) chiral gauge theory!

- But the **Higgs boson** exists and causes a naturalness problem!
- **Supersymmetry:** would link the masses of scalars and fermions and hence, in combination with chiral symmetry, forbid mass terms for scalar fields (solves hierarchy problem)

Is nature supersymmetric?

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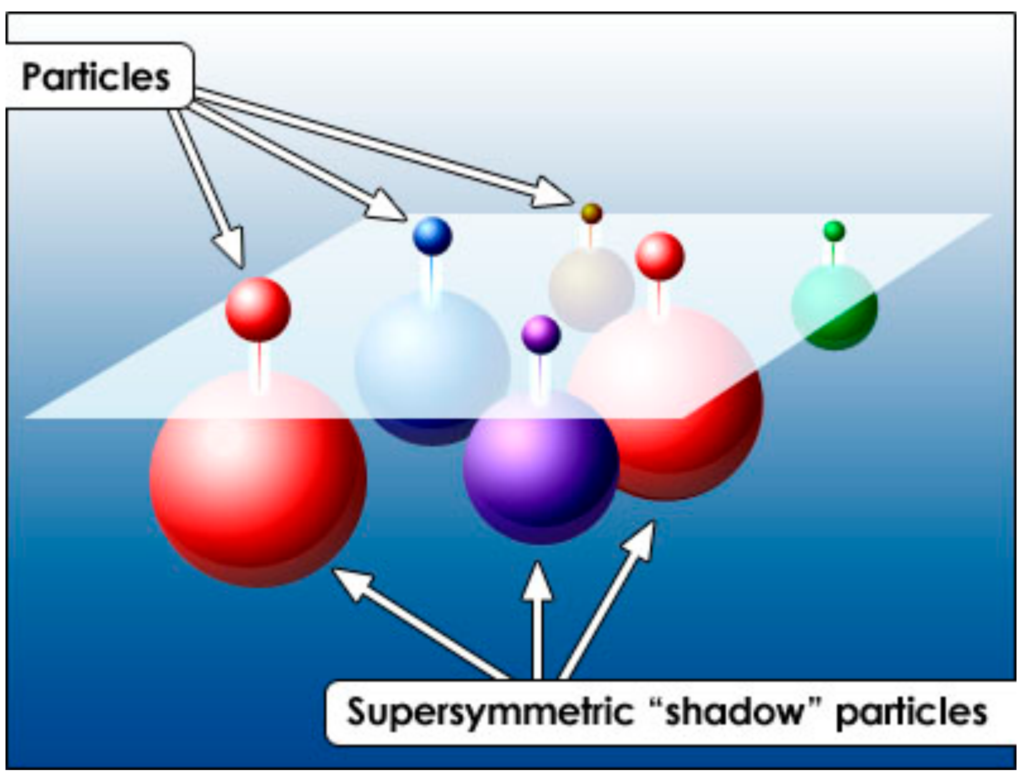
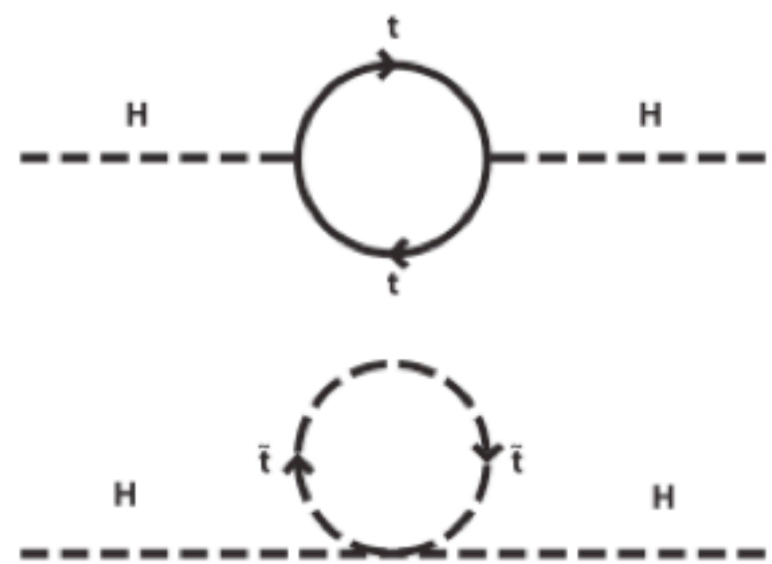
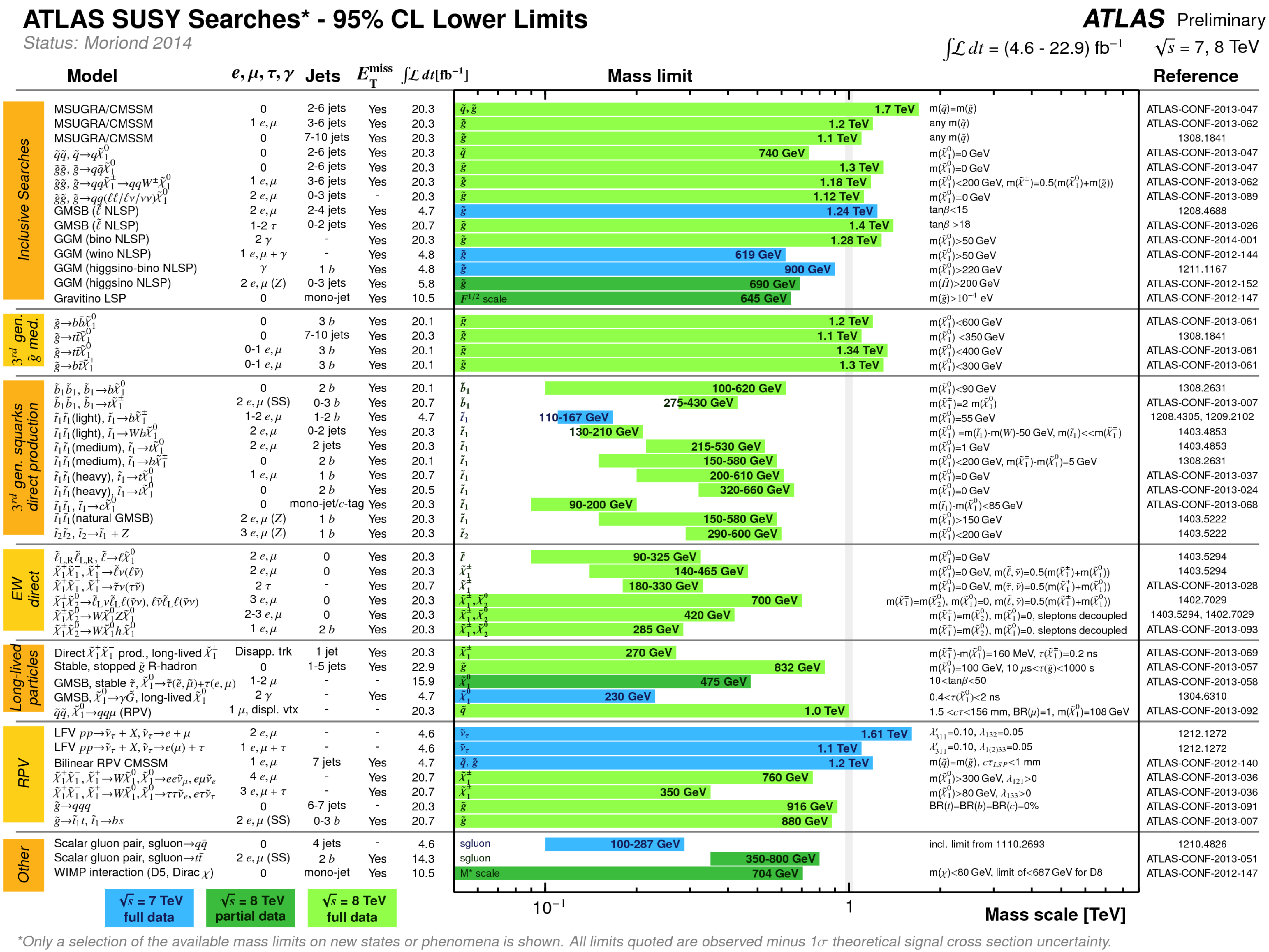




Is nature supersymmetric?



Lecture II: Some applications of EFTs

Standard Model as an effective field theory

Some interesting insights can be gained by considering the Standard Model (SM) as a low-energy effective theory of some more fundamental theory (supersymmetry, extra dimensions, new strongly coupled physics, GUT, ...)

We will denote the **scale of New Physics** by *M*; this could be as large as 10^{16} GeV for some applications, but as small as 10^3 GeV (= 1 TeV) for others

The SM Lagrangian should then be extended to an effective Lagrangian, which besides the SM terms contains **additional, irrelevant operators**

These operators must **respect the symmetries of the SM** (gauge invariance, Lorentz symmetry, CPT) but are otherwise unrestricted

Standard Model as an effective field theory

The effective Lagrangian up to operator dimension D=6 reads:

$$\mathcal{L}_{\rm SM} = \mathcal{L}_{\rm SM}^{(4)} + \frac{1}{\Lambda}\sum_k C_k^{(5)} Q_k^{(5)} + \frac{1}{\Lambda^2}\sum_k C_k^{(6)} Q_k^{(6)} + \mathcal{O}\left(\frac{1}{\Lambda^3}\right)$$

unique operator (neutrino masses):

$$Q_{\nu\nu} = (\widetilde{\varphi}^\dagger l_p)^T C(\widetilde{\varphi}^\dagger l_r)$$

Weinberg (1979)

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59 operator (2499 incl. flavor q. numbers)

X^3		φ^6 and $\varphi^4 D^2$		$\psi^2\varphi^3$	
Q_G	$f^{ABC} G_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$	Q_φ	$(\varphi^\dagger\varphi)^3$	$Q_{e\varphi}$	$(\varphi^\dagger\varphi)(\bar l_p e_r \varphi)$
$Q_{\widetilde G}$	$f^{ABC} \widetilde G_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$	$Q_{\varphi\Box}$	$(\varphi^\dagger\varphi)\Box(\varphi^\dagger\varphi)$	$Q_{u\varphi}$	$(\varphi^\dagger\varphi)(\bar q_p u_r \widetilde\varphi)$
Q_W	$\varepsilon^{IJK} W_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$	$Q_{\varphi D}$	$\left(\varphi^\dagger D^\mu\varphi\right)^\star \left(\varphi^\dagger D_\mu\varphi\right)$	$Q_{d\varphi}$	$(\varphi^\dagger\varphi)(\bar q_p d_r \varphi)$
$Q_{\widetilde W}$	$\varepsilon^{IJK} \widetilde W_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$				

$X^2\varphi^2$		$\psi^2 X\varphi$		$\psi^2\varphi^2 D$	
$Q_{\varphi G}$	$\varphi^\dagger\varphi\, G_{\mu\nu}^A G^{A\mu\nu}$	Q_{eW}	$(\bar l_p \sigma^{\mu\nu} e_r)\tau^I \varphi W_{\mu\nu}^I$	$Q_{\varphi l}^{(1)}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu \varphi)(\bar l_p \gamma^\mu l_r)$
$Q_{\varphi\widetilde G}$	$\varphi^\dagger\varphi\, \widetilde G_{\mu\nu}^A G^{A\mu\nu}$	Q_{eB}	$(\bar l_p \sigma^{\mu\nu} e_r)\varphi B_{\mu\nu}$	$Q_{\varphi l}^{(3)}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu^I \varphi)(\bar l_p \tau^I \gamma^\mu l_r)$
$Q_{\varphi W}$	$\varphi^\dagger\varphi\, W_{\mu\nu}^I W^{I\mu\nu}$	Q_{uG}	$(\bar q_p \sigma^{\mu\nu} T^A u_r)\widetilde\varphi\, G_{\mu\nu}^A$	$Q_{\varphi e}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu \varphi)(\bar e_p \gamma^\mu e_r)$
$Q_{\varphi\widetilde W}$	$\varphi^\dagger\varphi\, \widetilde W_{\mu\nu}^I W^{I\mu\nu}$	Q_{uW}	$(\bar q_p \sigma^{\mu\nu} u_r)\tau^I \widetilde\varphi\, W_{\mu\nu}^I$	$Q_{\varphi q}^{(1)}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu \varphi)(\bar q_p \gamma^\mu q_r)$
$Q_{\varphi B}$	$\varphi^\dagger\varphi\, B_{\mu\nu} B^{\mu\nu}$	Q_{uB}	$(\bar q_p \sigma^{\mu\nu} u_r)\widetilde\varphi\, B_{\mu\nu}$	$Q_{\varphi q}^{(3)}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu^I \varphi)(\bar q_p \tau^I \gamma^\mu q_r)$
$Q_{\varphi\widetilde B}$	$\varphi^\dagger\varphi\, \widetilde B_{\mu\nu} B^{\mu\nu}$	Q_{dG}	$(\bar q_p \sigma^{\mu\nu} T^A d_r)\varphi\, G_{\mu\nu}^A$	$Q_{\varphi u}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu \varphi)(\bar u_p \gamma^\mu u_r)$
$Q_{\varphi WB}$	$\varphi^\dagger\tau^I\varphi\, W_{\mu\nu}^I B^{\mu\nu}$	Q_{dW}	$(\bar q_p \sigma^{\mu\nu} d_r)\tau^I \varphi\, W_{\mu\nu}^I$	$Q_{\varphi d}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu \varphi)(\bar d_p \gamma^\mu d_r)$
$Q_{\varphi\widetilde WB}$	$\varphi^\dagger\tau^I\varphi\, \widetilde W_{\mu\nu}^I B^{\mu\nu}$	Q_{dB}	$(\bar q_p \sigma^{\mu\nu} d_r)\varphi\, B_{\mu\nu}$	$Q_{\varphi ud}$	$i(\widetilde\varphi^\dagger D_\mu\varphi)(\bar u_p \gamma^\mu d_r)$

Operators other than four-fermion operators

Buchmüller, Wyler (1986)
Hagiwara et al. (1987 & 1993)
Grzadkowski, Iskrzynski, Misiak, Rosiek (2010)

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The effective Lagrangian up to operator dimension *D*=6 reads:

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$(\bar{L}L)(\bar{L}L)$		$(\bar{R}R)(\bar{R}R)$		$(\bar{L}L)(\bar{R}R)$	
Q_{ll}	$(\bar{l}_p\gamma_\mu l_r)(\bar{l}_s\gamma^\mu l_t)$	Q_{ee}	$(\bar{e}_p\gamma_\mu e_r)(\bar{e}_s\gamma^\mu e_t)$	Q_{le}	$(\bar{l}_p\gamma_\mu l_r)(\bar{e}_s\gamma^\mu e_t)$
$Q_{qq}^{(1)}$	$(\bar{q}_p\gamma_\mu q_r)(\bar{q}_s\gamma^\mu q_t)$	Q_{uu}	$(\bar{u}_p\gamma_\mu u_r)(\bar{u}_s\gamma^\mu u_t)$	Q_{lu}	$(\bar{l}_p\gamma_\mu l_r)(\bar{u}_s\gamma^\mu u_t)$
$Q_{qq}^{(3)}$	$(\bar{q}_p\gamma_\mu \tau^I q_r)(\bar{q}_s\gamma^\mu \tau^I q_t)$	Q_{dd}	$(\bar{d}_p\gamma_\mu d_r)(\bar{d}_s\gamma^\mu d_t)$	Q_{ld}	$(\bar{l}_p\gamma_\mu l_r)(\bar{d}_s\gamma^\mu d_t)$
$Q_{lq}^{(1)}$	$(\bar{l}_p\gamma_\mu l_r)(\bar{q}_s\gamma^\mu q_t)$	Q_{eu}	$(\bar{e}_p\gamma_\mu e_r)(\bar{u}_s\gamma^\mu u_t)$	Q_{qe}	$(\bar{q}_p\gamma_\mu q_r)(\bar{e}_s\gamma^\mu e_t)$
$Q_{lq}^{(3)}$	$(\bar{l}_p\gamma_\mu \tau^I l_r)(\bar{q}_s\gamma^\mu \tau^I q_t)$	Q_{ed}	$(\bar{e}_p\gamma_\mu e_r)(\bar{d}_s\gamma^\mu d_t)$	$Q_{qu}^{(1)}$	$(\bar{q}_p\gamma_\mu q_r)(\bar{u}_s\gamma^\mu u_t)$
		$Q_{ud}^{(1)}$	$(\bar{u}_p\gamma_\mu u_r)(\bar{d}_s\gamma^\mu d_t)$	$Q_{qu}^{(8)}$	$(\bar{q}_p\gamma_\mu T^A q_r)(\bar{u}_s\gamma^\mu T^A u_t)$
		$Q_{ud}^{(8)}$	$(\bar{u}_p\gamma_\mu T^A u_r)(\bar{d}_s\gamma^\mu T^A d_t)$	$Q_{qd}^{(1)}$	$(\bar{q}_p\gamma_\mu q_r)(\bar{d}_s\gamma^\mu d_t)$
				$Q_{qd}^{(8)}$	$(\bar{q}_p\gamma_\mu T^A q_r)(\bar{d}_s\gamma^\mu T^A d_t)$

$(\bar{L}R)(\bar{R}L)$ and $(\bar{L}R)(\bar{L}R)$		B-violating	
Q_{ledq}	$(\bar{l}_p^j e_r)(\bar{d}_s q_t^j)$	Q_{duq}	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jk}\left[(d_p^\alpha)^T C u_r^\beta\right]\left[(q_s^{\gamma j})^T C l_t^k\right]$
$Q_{quqd}^{(1)}$	$(\bar{q}_p^j u_r)\varepsilon_{jk}(\bar{q}_s^k d_t)$	Q_{qqu}	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jk}\left[(q_p^{\alpha j})^T C q_r^{\beta k}\right]\left[(u_s^\gamma)^T C e_t\right]$
$Q_{quqd}^{(8)}$	$(\bar{q}_p^j T^A u_r)\varepsilon_{jk}(\bar{q}_s^k T^A d_t)$	$Q_{qqq}^{(1)}$	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jk}\varepsilon_{mn}\left[(q_p^{\alpha j})^T C q_r^{\beta k}\right]\left[(q_s^{\gamma m})^T C l_t^n\right]$
$Q_{lequ}^{(1)}$	$(\bar{l}_p^j e_r)\varepsilon_{jk}(\bar{q}_s^k u_t)$	$Q_{qqq}^{(3)}$	$\varepsilon^{\alpha\beta\gamma}(\tau^I\varepsilon)_{jk}(\tau^I\varepsilon)_{mn}\left[(q_p^{\alpha j})^T C q_r^{\beta k}\right]\left[(q_s^{\gamma m})^T C l_t^n\right]$
$Q_{lequ}^{(3)}$	$(\bar{l}_p^j \sigma_{\mu\nu} e_r)\varepsilon_{jk}(\bar{q}_s^k \sigma^{\mu\nu} u_t)$	Q_{duu}	$\varepsilon^{\alpha\beta\gamma}\left[(d_p^\alpha)^T C u_r^\beta\right]\left[(u_s^\gamma)^T C e_t\right]$

Four-fermion operators

Flavor observables are crucial in order to explore this enormous parameter space!

Standard Model as an effective field theory

We will discuss a couple of interesting aspects of SM physics from the perspective of this construction:

- neutrino masses and the see-saw mechanism
- effective weak interactions in the quark sector
- anomalous magnetic moment of the muon
- proton decay
- conservation of baryon and lepton numbers (accidental symmetries)
- Higgs production at the LHC

Neutrino masses and the see-saw mechanism

The discovery of **non-zero neutrino masses** is often described as a departure from the SM

But this is no longer true if we consider the SM as an effective low-energy theory

Without a right-handed neutrino (which indeed is not part of the SM), it is impossible to write a neutrino mass term at the level of relevant or marginal operators

However, it **is** possible to write a gauge-invariant **neutrino mass term** at the level of **irrelevant operators** of dimension ≥5:

$$\boxed{\mathcal{L}_{\text{neutrino mass}} = \frac{g}{M}\left(\tilde{l}_L^T \Phi^*\right) C \left(\tilde{\Phi} l_L\right)}$$

Neutrino masses and the see-saw mechanism

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After electroweak symmetry breaking, this gives rise to a Majorana mass term of the form:

$$\boxed{\mathcal{L}_{\text{neutrino mass}} = -\frac{v^2 g}{2M}\,\tilde{\nu}_L^T C\,\nu_L}$$

The SM as an effective field theory **predicts** that neutrinos should be massive, with $m_\nu \sim v^2/M$ suppressed by the fundamental scale of some BSM physics

Neutrino masses and the see-saw mechanism

Experiments hints at the fact that the fundamental scale relevant for the generation of neutrino masses is very heavy,

$$\boxed{M \sim 10^{14}\,\mathrm{GeV}}$$

which is not far from the scale of grand unification

Extensions of the SM containing **heavy, right-handed neutrinos** (with masses that are naturally of order *M*) provide explicit examples of fundamental theories which yield such a Majorana mass term when the heavy, right-handed neutrinos are integrated out **(see-saw mechanism)**

Weak interactions at low energies (flavor physics)

Fermi's description of the weak interactions at low energy is a prime example of an effective field theory, which has provided first evidence for the **scale of electroweak symmetry breaking**

At the low energies relevant for neutron β-decay, kaon physics, charm physics or B-meson physics (few MeV - few GeV), we can integrate out the heavy W and Z bosons as well as the top-quark and Higgs boson from the SM

This gives rise to a low-energy effective theory containing **4-fermion interactions** (Fermi theory) and **dipole interactions** between fermions and the photon and gluon

This effective Lagrangian successfully describes the huge phenomenology of **flavor-changing processes**

Weak interactions at low energies (flavor physics)

Example: Effective Lagrangian for b→s FCNC transitions
(see Buras lectures for a derivation)

$\lambda_p = V_{ps}^* V_{pb}$ (CKM matrix elements)

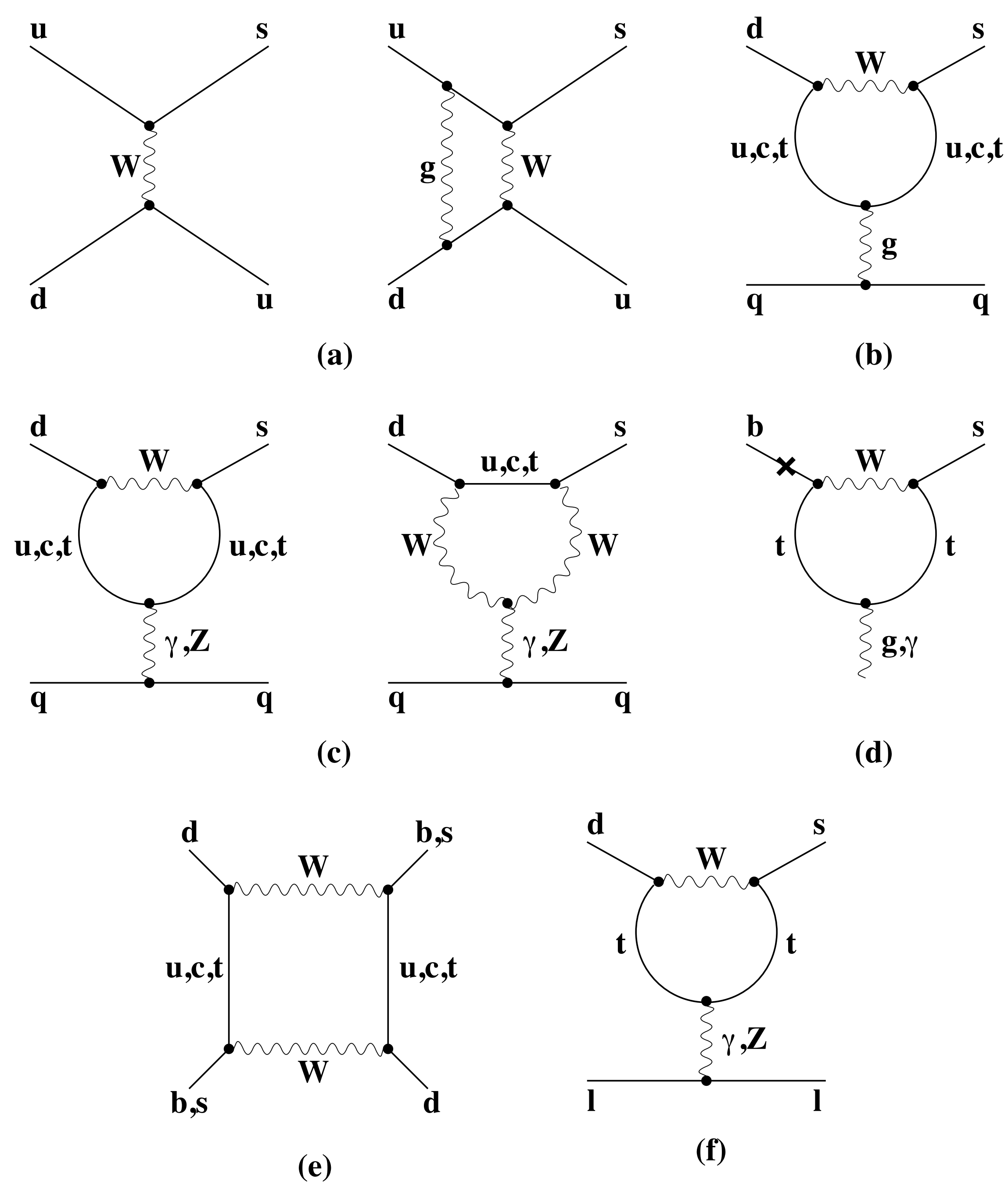
$$\boxed{\mathcal{H}_{\rm eff} = \frac{G_F}{\sqrt{2}} \sum_{p=u,c} \lambda_p \Big(C_1\, Q_1^p + C_2\, Q_2^p + \sum_{i=3,\dots,10} C_i\, Q_i + C_{7\gamma}\, Q_{7\gamma} + C_{8g}\, Q_{8g} \Big)}$$

$$\begin{aligned}
Q_1^p &= (\bar p b)_{V-A} (\bar s p)_{V-A}\,, & Q_2^p &= (\bar p_i b_j)_{V-A} (\bar s_j p_i)_{V-A}\,, \\
Q_3 &= (\bar s b)_{V-A} \textstyle\sum_q (\bar q q)_{V-A}\,, & Q_4 &= (\bar s_i b_j)_{V-A} \textstyle\sum_q (\bar q_j q_i)_{V-A}\,, \\
Q_5 &= (\bar s b)_{V-A} \textstyle\sum_q (\bar q q)_{V+A}\,, & Q_6 &= (\bar s_i b_j)_{V-A} \textstyle\sum_q (\bar q_j q_i)_{V+A}\,, \\
Q_7 &= (\bar s b)_{V-A} \textstyle\sum_q \tfrac{3}{2} e_q (\bar q q)_{V+A}\,, & Q_8 &= (\bar s_i b_j)_{V-A} \textstyle\sum_q \tfrac{3}{2} e_q (\bar q_j q_i)_{V+A}\,, \\
Q_9 &= (\bar s b)_{V-A} \textstyle\sum_q \tfrac{3}{2} e_q (\bar q q)_{V-A}\,, & Q_{10} &= (\bar s_i b_j)_{V-A} \textstyle\sum_q \tfrac{3}{2} e_q (\bar q_j q_i)_{V-A}\,, \\
Q_{7\gamma} &= \frac{-e}{8\pi^2}\, m_b\, \bar s \sigma_{\mu\nu} (1+\gamma_5) F^{\mu\nu} b\,, & Q_{8g} &= \frac{-g_s}{8\pi^2}\, m_b\, \bar s \sigma_{\mu\nu} (1+\gamma_5) G^{\mu\nu} b\,,
\end{aligned}$$

Weak interactions at low energies (flavor physics)

Example: Effective Lagrangian for b→s FCNC transitions
(see Buras lectures for a derivation)

SM diagrams involving virtual heavy-particle exchanges contributing to the low-energy effective weak Lagrangian



Weak interactions at low energies (flavor physics)

Example: Effective Lagrangian for b→s FCNC transitions
(see Buras lectures for a derivation)

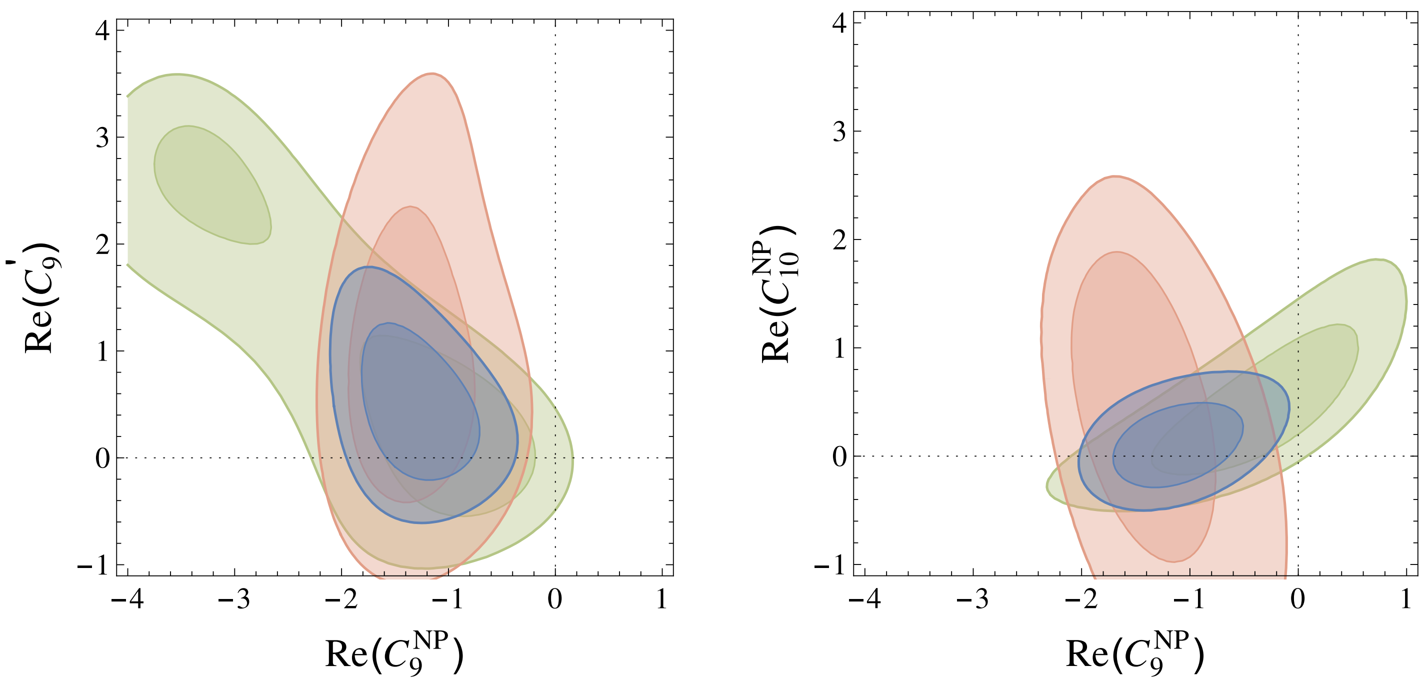
From the fact that the **leading operators** in the low-energy effective theory have **dimension 6,** it follows that the corresponding couplings are **irrelevant** and proportional to M_W^2, indeed:

$$\boxed{\frac{G_F}{\sqrt{2}} = \frac{g_2^2}{8M_W^2}}$$

The strong suppression of these contributions at low energies explains why we refer to these interactions as the **weak interactions,** even though the coupling constants of the $SU(2)_L \otimes U(1)_Y$ electroweak interactions is about as large as the electromagnetic coupling constant

Weak interactions at low energies (flavor physics)

A global analysis of experimental data on $B \to X_s\gamma$, $B \to K^*\gamma$, and $B \to K^{(*)}\mu^+\mu^-$ decay distributions provides information about various operator coefficients (defined to vanish in SM):



Allowed regions in the Re($C_9^{\rm NP}$)-Re(C_9') plane (left) and the Re($C_9^{\rm NP}$)-Re($C_{10}^{\rm NP}$) plane (right). The blue contours correspond to the 1 and 2σ best fit regions from the global fit. The green and red contours correspond to the 1 and 2σ regions if only branching ratio data or only data on $B \to K^*\mu^+\mu^-$ angular observables is taken into account.

Altmannshofer, Straub:1411.3161

$$\mathcal{H}_{\rm eff} = -\frac{4\,G_F}{\sqrt{2}} V_{tb}V_{ts}^* \frac{e^2}{16\pi^2} \sum_i (C_i O_i + C_i' O_i') + \text{h.c.}$$

SM operators:

$$O_7 = \frac{m_b}{e}(\bar{s}\sigma_{\mu\nu}P_R b)F^{\mu\nu}$$
$$O_9 = (\bar{s}\gamma_\mu P_L b)(\bar{\ell}\gamma^\mu \ell)$$
$$O_{10} = (\bar{s}\gamma_\mu P_L b)(\bar{\ell}\gamma^\mu \gamma_5 \ell)$$

Opposite chirality operators:

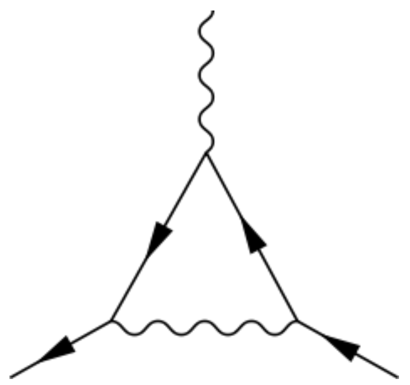
$$O_7' = \frac{m_b}{e}(\bar{s}\sigma_{\mu\nu}P_L b)F^{\mu\nu}$$
$$O_9' = (\bar{s}\gamma_\mu P_R b)(\bar{\ell}\gamma^\mu \ell)$$
$$O_{10}' = (\bar{s}\gamma_\mu P_R b)(\bar{\ell}\gamma^\mu \gamma_5 \ell)$$

A first hint of New Physics?

Anomalous magnetic moment of the muon

In a celebrated calculation that was the birth of modern QFT, Schwinger computed the anomalous magnetic moment of the electron in 1948 and found:

$$\boxed{\mu_e = \frac{g_e}{2m_e} \quad \text{, with} \quad a_e = \frac{g_e - 2}{2} = \frac{\alpha}{2\pi} + \ldots}$$



How will this result be affected if the SM is considered as an effective field theory?

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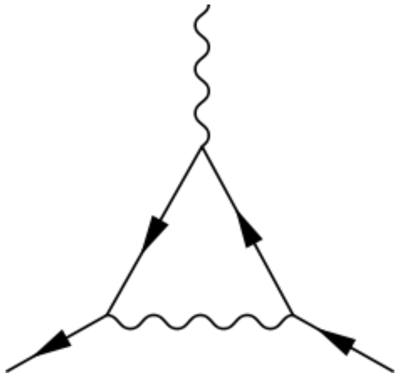

How will this result be affected if the SM is considered as an effective field theory?

Add **unique** dimension-5 operator ($\delta = 5\,,\ \gamma = -1$):

$$\boxed{\frac{gv}{M^2}\,\bar{\psi}\,\sigma_{\mu\nu}F^{\mu\nu}\psi}$$

factor *v* required by EWSB

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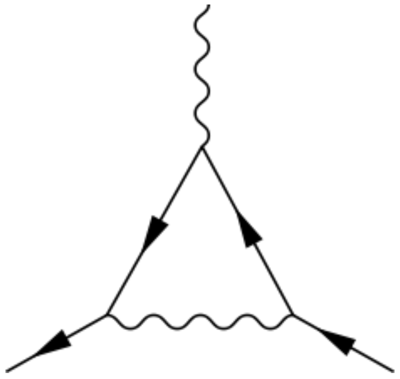

This adds g/M to μ_e and hence:

$$\boxed{a_e = \frac{\alpha}{2\pi} + \frac{g m_e v}{M^2} + \ldots}$$

As long as $M \gg m_e$ the additional term will be very small, and by comparing a measurement of μ_e with theory we can constrain M

Anomalous magnetic moment of the muon

Analogous discussion (with m_e replaced by m_μ) holds for the muon

In this case, there is presently a **3.6 σ discrepancy** between theory and experiment:

$$\boxed{a_\mu^{\mathrm{SM}} - a_\mu^{\mathrm{exp}} \approx -2.8 \cdot 10^{-9}}$$

Interpreting this effect in terms of our irrelevant operator implies that:

contains loop factor (small)

$$M \sim \sqrt{g} \times 100\,\mathrm{TeV}$$

One of the best hints for BSM physics!

Proton decay

Suppose you know the gauge symmetry $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ of the SM but nothing else (no GUTs). What could you say about proton decay?

The effective Lagrangian must contain at least **three quark fields** (change baryon number by 1 unit) and **one lepton field** (change lepton number by 1 unit)

Hence:

$$\mathcal{L}_{\text{proton decay}} \sim \frac{g}{M^2}\, qqq\ell$$

Since the lowest-dimension operators have dimension 6 (corresponding to $\gamma_i = -2$), the proton can be made sufficiently long-lived by raising the fundamental scale M into the 10^{16} GeV range

Proton decay

Now imagine that you do not know about the existence of quarks (no one has seen any) but you do know about protons and pions

Then an effective Lagrangian giving proton decay could be:

$$\mathcal{L}_{\text{proton decay}} \sim g\,\pi\,\bar{\psi}_e\,\psi_p$$

This is a marginal operator, and hence proton decay would not be suppressed by any large mass scale!

In some sense, we see that the **longevity of the proton** provides a hint for a substructure of the proton: **replacing a fundamental field by a composite of several fields** raises the dimension of the operators and hence gives rise to additional suppression

Proton decay

The same trick can be applied to other fine-tuning problems

For example, the hierarchy problem can be solved by supposing that the **Higgs boson is not an elementary scalar** particle but instead a **composite of a pair of elementary fermions**

If this is the case, then the Higgs mass term corresponds to a 4-fermion operator, which is irrelevant

This is the main idea of **composite Higgs** and **technicolor** theories

Baryon and lepton number conservation

In the construction of the SM, the conservation of baryon and lepton number is **not imposed** as a condition

There are no corresponding U(1) symmetries of the Lagrangian

How can we understand that in nature we have not seen any hints of baryon- or lepton-number violating processes?

Baryon and lepton number conservation

In the construction of the SM, the conservation of baryon and lepton number is **not imposed** as a condition

There are no corresponding U(1) symmetries of the Lagrangian

How can we understand that in nature we have not seen any hints of baryon- or lepton-number violating processes?

The answer is that **it is impossible to construct any relevant or marginal operator** that would respect the gauge symmetries of the SM and **violate baryon or lepton number!**

Hence, at the level of renormalizable interactions, baryon- and lepton-number conservation are **accidental symmetries** of the SM

Higgs production at the LHC

The protons collided at the LHC contain only light quarks (u,d, and a little bit of s), which in the SM have negligible couplings to the Higgs boson, and gluons, which do not couple to the Higgs boson at all

How, then, is the Higgs boson produced in pp collisions at the LHC?

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How, then, is the Higgs boson produced in pp collisions at the LHC?

We can gain insight by assuming (as seems to be the case) that the Higgs boson is lighter than the top quark

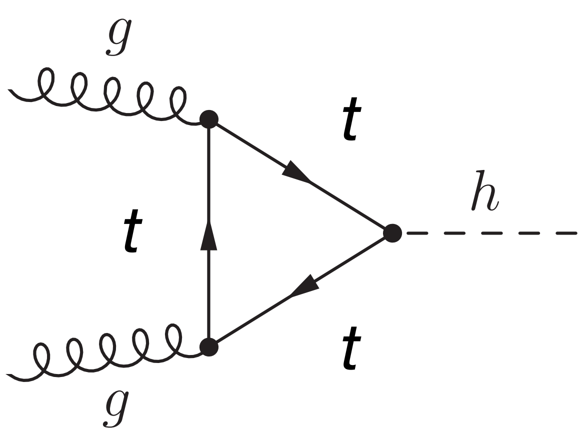
We can then construct an effective low-energy theory for Higgs physics, in which the top quark is integrated out

Higgs production at the LHC

In this effective low-energy theory, direct couplings of the Higgs boson to pairs of gluons and photons arise at the level of **irrelevant dimension-5 operators,** with coefficients that scale like $1/m_t$, e.g.:

$$\boxed{\mathcal{L}_{hgg} = \frac{y_t}{\sqrt{2}m_t}\,\frac{\alpha_s}{12\pi}\,h\,G^a_{\mu\nu}G^{\mu\nu,a}}$$

These operators appear first at one-loop order, via the exchange of a virtual top-quark



The **effective hgg interaction** provides the dominant production mechanism for the Higgs boson in **gluon-gluon fusion** at the LHC

Summary

Effective field theories are a very powerful tool in quantum field theory

The are of great **practical use,** but also provide the **conceptual tools** to understand scale separation (factorization) and renormalization in a physical and systematic way

Effective field theories are abundant, since any QFT can be considered as an effective low-energy theory of some more fundamental theory, which is often not yet known

Because of this fact, effective field theories provide the tools to perform **indirect searches for new physics** beyond the Standard Model

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Amplitudes, Motives and Beyond
Francis Brown **IHES Paris**, Marcus Spradlin **Brown Univ.**, Don Zagier **MPI-M Bonn**, Stefan Weinzierl **JGU Mainz**, Stefan Müller-Stach **JGU Mainz**
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Higher Orders and Jets at the LHC
Matteo Cacciari **LPTHE Paris**, Paolo Nason **INFN Milan**, Giulia Zanderighi **CERN & Univ. Oxford**
June 29-July 17, 2015

Crossroads of Neutrino Physics
Steen Hannestad **Aarhus Univ.**, Patrick Huber **Virginia Tech**, Alexei Smirnov **MPI-K Heidelberg**, Joachim Kopp **JGU Mainz**
July 20-August 14, 2015

Fundamental Parameters from Lattice QCD
Gilberto Colangelo **Bern Univ.**, Heiko Lacker **HU Berlin**, Georg von Hippel **JGU Mainz**, Hartmut Wittig **JGU Mainz**
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September 14-25, 2015

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