Inflationary Cosmology

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BCVSPIN - MSPF - Mitchell, Dec.8 -13 2014
Manzanillo, Mexico

Hot Big Bang Cosmology

- Comes from combining Standard Model (SM) of high energy physics with Einstein' general relativity, and the assumption that on sufficiently large scales, the universe is isotropic and homogeneous.

isotropic: same in all directions

homogeneous: position independent

Three remarkable predictions (Consequences):

1. Expanding Universe
2. Cosmic Microwave Background Radiation (CMB)
3. Nucleosynthesis

STANDARD MODEL OF HE PHYSICS

Provides excellent description of strong, weak and electromagnetic interactions.

Based on local gauge symmetry

$$SU(3)_c \times SU(2)_L \times U(1)_Y$$

QCD - strong interactions involving ‘colored’ quarks & gluons

Electromagnetic interactions mediated by W and Z^0 bosons which have been found

Only ‘color neutral’ states exist in nature

Higgs Boson

- Spontaneously symmetry breaking

$$SU(2)_L \times U(1)_Y \longrightarrow U(1)_{\rm EM} \to \gamma \text{ (photon)}$$

$$\langle \phi \rangle \sim 10^2 \text{GeV} (t \sim 10^{-10} \text{sec})$$

$$m_h \approx 125 \text{GeV (Discovery on 4 July 2012)}$$

- compare to superconductor

- A homogeneous and isotropic universe is described by the Robertson-Walker metric

$$ds^2 = -dt^2 + a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta\, d\phi^2)\right],$$

where r, ϕ and θ are ‘comoving’ polar coordinates, which remain fixed for objects that follow the general cosmological expansion.

k is the scalar curvature of 3-space, with k = 0, +1, -1 describing a flat, closed and open universe respectively.

Geometry of the Universe

- Friedmann Equation

$$\Omega \equiv \frac{\rho}{\rho_c} = 1 + \frac{k}{(aH)^2}, \quad \text{where} \quad \rho_c = \frac{3H^2}{8\pi G} = \text{critical density}$$

- Closed ($\Omega > 1$ or $k = 1$)
- Open ($\Omega < 1$ or $k = -1$)
- Flat ($\Omega = 1$ or $k = 0$)

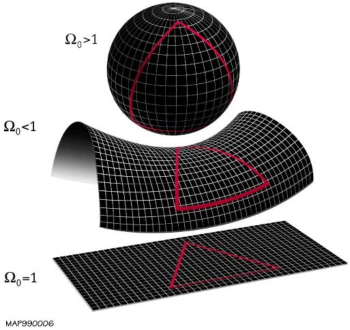


Image courtesy of NASA: http://map.gsfc.nasa.gov/universe/uni_shape.html

Solving Friedmann Equations:

- For flat universe

$$H^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 \propto \rho$$

- Matter $\left(\rho_m = \frac{NM}{V}\right)$

$$\rho_m \propto a^{-3} \Rightarrow a(t) \propto t^{2/3}$$

- Radiation $\left(\rho_\gamma = \frac{Nhc}{V\lambda}\right)$

$$\rho_\gamma \propto a^{-4} \Rightarrow a(t) \propto t^{1/2}$$

- Vacuum $(\rho_\Lambda = \text{const.})$

$$\rho_\Lambda \propto a^0 \Rightarrow a(t) \propto e^{Ht}$$

Cosmological Problems

- Flatness Problem

Present energy density of the universe is determined to be equal to its critical value corresponding to a flat universe. This means that in the early universe

$$\Omega - 1 = \frac{k}{(aH)^2} \propto t$$ (for a radiation dominated universe)

$$\Rightarrow \quad |\Omega_{BBN} - 1| \leq 10^{-16} \qquad \left(|\Omega_{GUT} - 1| \leq 10^{-55}\right)$$

How does this come about?

Horizon Problem

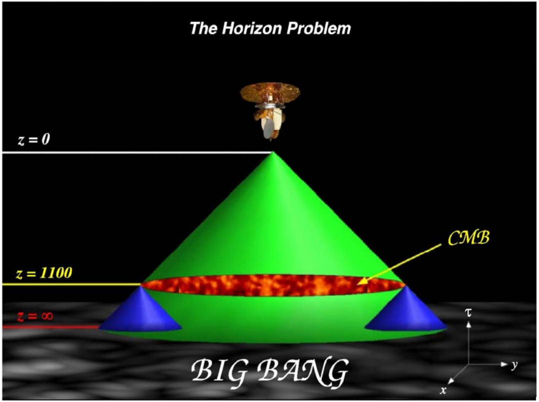


Image courtesy of W. Kinney

Why the CMB is so uniform on large scales?

- Origin of primordial density fluctuation which lead to Large Scale Structure and also explain

$$\delta T/T \sim 10^{-5}$$

observed by COBE/WMAP and other experiments.

- Origin of baryon asymmetry ($n_b/n_\gamma \sim 10^{-10}$)?

Inflationary Cosmology

[Guth, Linde, Albrecht & Steinhardt, Starobinsky, Mukhanov, Hawking, ...]

Successful Primordial Inflation should:

- Explain flatness, isotropy;
- Provide origin of $\frac{\delta T}{T}$;
- Offer testable predictions for n_s, r, $dn_s/d\ln k$;
- Recover Hot Big Bang Cosmology;
- Explain the observed baryon asymmetry;
- Offer plausible CDM candidate;

Physics Beyond the SM?

Cosmic Inflation

- Inflation can be defined as:

$$\frac{d}{dt}\left(\frac{1}{aH}\right) < 0,$$ a decreasing comoving horizon

$$\ddot{a} > 0,$$ an accelerated expansion

$$P < -\rho/3,$$ a negative pressure $\longrightarrow$ repulsive gravity $\downarrow$ drives inflation

- Consider a scalar field ϕ

$$\rho_\phi = \frac{1}{2}\dot{\phi}^2 + V(\phi) \approx V,$$

$$a(t) \approx e^{Ht}$$ $\longrightarrow$ inflation

Slow rolling scalar field acts as an inflaton

Cosmic Inflation

Tiny patch ~10^{-28} cm ⇨ > 1 cm after 60 e-foldings (time constant ~10^{-38} sec)

Inflation over ⇨ radiation dominated universe (hot big bang)

Quantum fluctuations of inflation field give rise to nearly scale invariant, adiabatic, Gaussian density perturbations

⇨ Seed for forming large scale structure

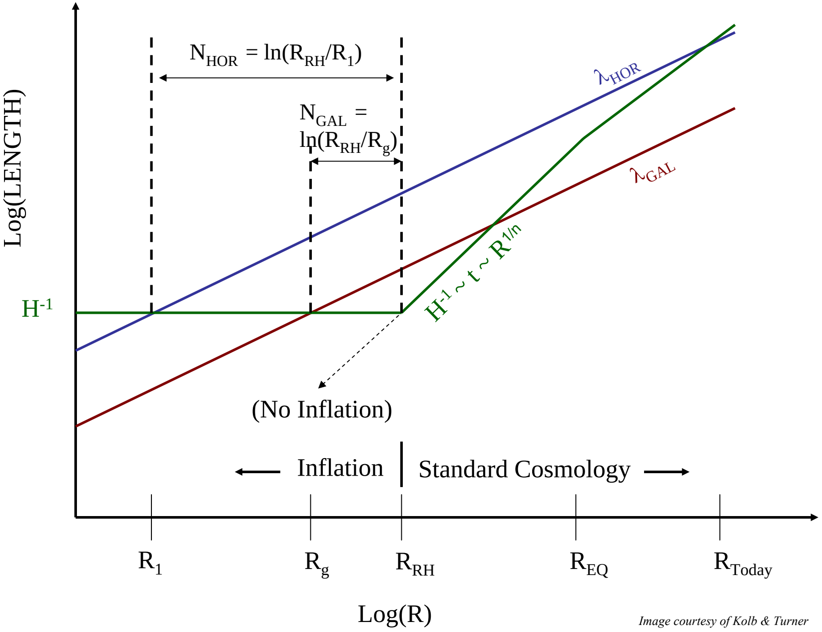


Image courtesy of Kolb & Turner

- Solution to the Flatness Problem $\left(\Omega - 1 = \frac{k}{(aH)^2}\right)$

$$\boxed{|\Omega_f - 1| = |\Omega_i - 1| e^{-2N} \to 0,}$$ where $N = \mathrm{H}\,\Delta \mathrm{t} \geq 50$

- Solution to the Horizon Problem

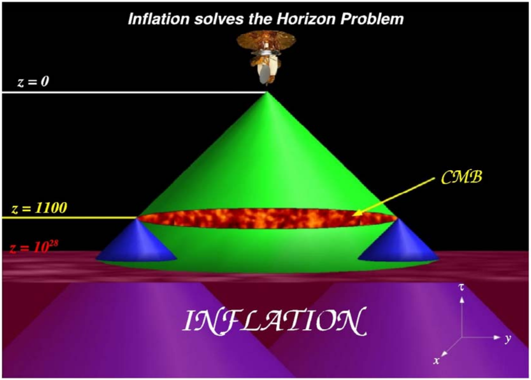
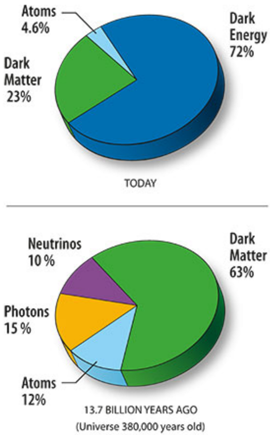


Image courtesy of W. Kinney

ΛCDM Model (current paradigm)

Λ stands for Dark Energy with Einstein's cosmological constant being the leading candidate

($P_\Lambda = w_\Lambda \, \rho_\Lambda$, with w_Λ = -1)



$$\rho_{Total} = \rho_\Lambda + \rho_{CDM} + \rho_M \approx \rho_c$$

$$\rho_\Lambda \approx 10^{-120} m_P^4$$ ⟵ Fine tuning?

CDM denotes 'cold dark matter' (particle have tiny velocities)

Image courtesy of NASA / WMAP Science Team

Where does ΛCDM come from?

Slow-roll Inflation

- Inflation is driven by some potential $V(\phi)$:
- Slow-roll parameters:

$$\epsilon = \frac{m_p^2}{2}\left(\frac{V'}{V}\right)^2, \ \eta = m_p^2\left(\frac{V''}{V}\right).$$

- The spectral index n_s and the tensor to scalar ratio r are given by

$$n_s - 1 \equiv \frac{d\ln\Delta_{\mathcal{R}}^2}{d\ln k}, \ r \equiv \frac{\Delta_h^2}{\Delta_{\mathcal{R}}^2},$$

where Δ_h^2 and $\Delta_{\mathcal{R}}^2$ are the spectra of primordial gravity waves and curvature perturbation respectively.

- Assuming slow-roll approximation (i.e. $(\epsilon, |\eta|) \ll 1$), the spectral index n_s and the tensor to scalar ratio r are given by

$$n_s \simeq 1 - 6\epsilon + 2\eta, \ r \simeq 16\epsilon.$$

- The tensor to scalar ratio r can be related to the energy scale of inflation via

$$V(\phi_0)^{1/4} = 3.3 \times 10^{16}\, r^{1/4} \text{ GeV}.$$

- The amplitude of the curvature perturbation is given by

$$\Delta^2_{\mathcal{R}} = \frac{1}{24\pi^2}\left(\frac{V/m_p^4}{\epsilon}\right)_{\phi=\phi_0} = 2.43 \times 10^{-9} \text{ (WMAP7 normalization)}.$$

- The spectrum of the tensor perturbation is given by

$$\Delta^2_h = \frac{2}{3\,\pi^2}\left(\frac{V}{m_P^4}\right)_{\phi=\phi_0}.$$

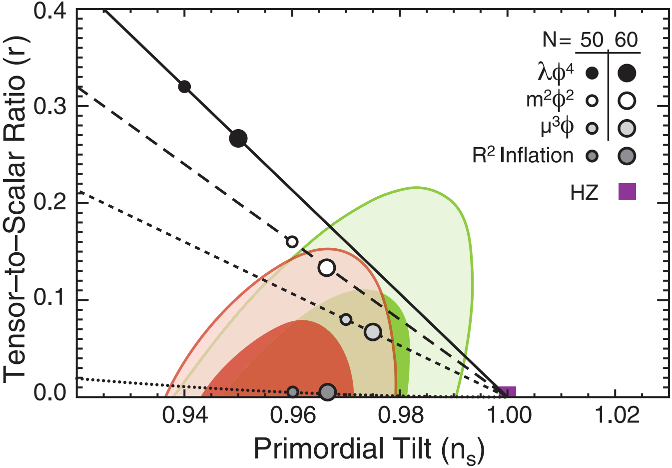
- The number of e-folds after the comoving scale $l_0 = 2\,\pi/k_0$ has crossed the horizon is given by

$$N_0 = \frac{1}{m_p^2}\int_{\phi_e}^{\phi_0}\left(\frac{V}{V'}\right) d\phi.$$

Inflation ends when $\max[\epsilon(\phi_e), |\eta(\phi_e)|] = 1$.

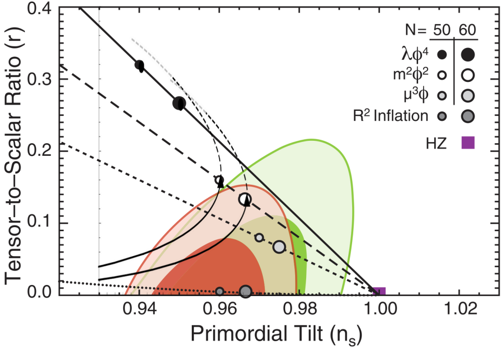
BICEP 2 Result

- BICEP 2 a few months ago surprised many people with their results that $r \sim 0.2$ (0.16).
- Some tension with the Planck upper bound $r < 0.11$.
- Somewhat earlier WMAP 9 stated that $r < 0.13$.



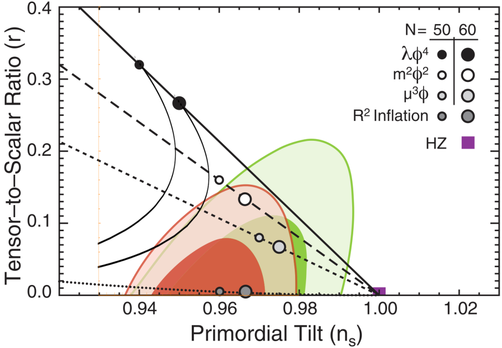
WMAP nine year data

Radiatively Corrected ϕ^2 Potential:



n_s vs. r for radiatively corrected ϕ^2 potential. The dashed portions are for $\kappa < 0$. The one loop radiative correction is larger than the tree level potential in the portions displayed in gray. N is taken as 50 (left curves) and 60 (right curves).

Radiatively Corrected ϕ^4 Potential:



n_s vs. r for radiatively corrected ϕ^4 potential. The dashed portions are for $\kappa < 0$. The one loop radiative correction is larger than the tree level potential in the portions displayed in gray. N is taken as 50 (left curves) and 60 (right curves).

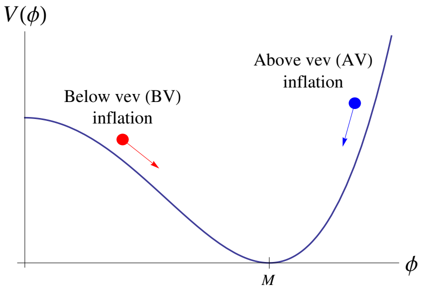
Tree Level Gauge Singlet Higgs Inflation

[Kallosh and Linde, 07; Rehman, Shafi and Wickman, 08]

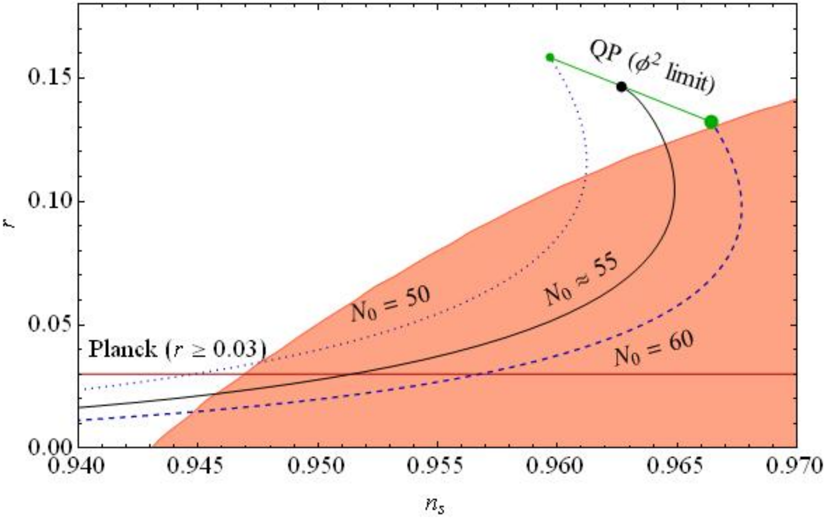
- Consider the following Higgs Potential:

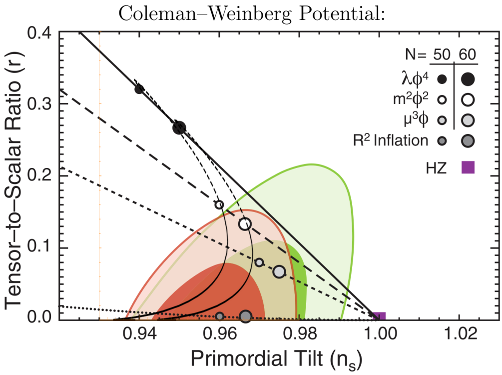
$$V(\phi) = V_0 \left[1 - \left(\frac{\phi}{M}\right)^2\right]^2 \quad \longleftarrow \text{(tree level)}$$

Here ϕ is a gauge singlet field.



- WMAP/Planck data favors BV inflation ($r \lesssim 0.1$).
- BUT now BICEP2 may have found $r \approx 0.2$.


QP (ϕ^2 limit)
0.15
0.10
0.05
0.00
r
Planck ($r \geq 0.03$)
$N_0 = 50$
$N_0 \approx 55$
$N_0 = 60$
0.940
0.945
0.950
0.955
0.960
0.965
0.970
n_s



n_s vs. r for Coleman–Weinberg potential. The dashed portions are for $\phi > v$. N is taken as 50 (left curves) and 60 (right curves).

Quartic Inflation with non-minimal coupling to gravity

- We consider a quartic inflaton potential with a non-minimal gravitational coupling.
- The basic action of non-minimal ϕ^4 inflation is given in the Jordan frame

$$S_J^{\rm tree} = \int d^4x\sqrt{-g}\left[-\left(\frac{1+\xi\phi^2}{2}\right)\mathcal{R} + \frac{1}{2}(\partial\phi)^2 - \frac{\lambda}{4!}\phi^4\right]$$

- The inflation potential in the Einstein frame is

$$V_E(\sigma_E(\phi)) = \frac{\frac{1}{4!}\lambda(t)\phi^4}{(1+\xi\,\phi^2)^2}.$$

Quartic Inflation with non-minimal coupling to gravity

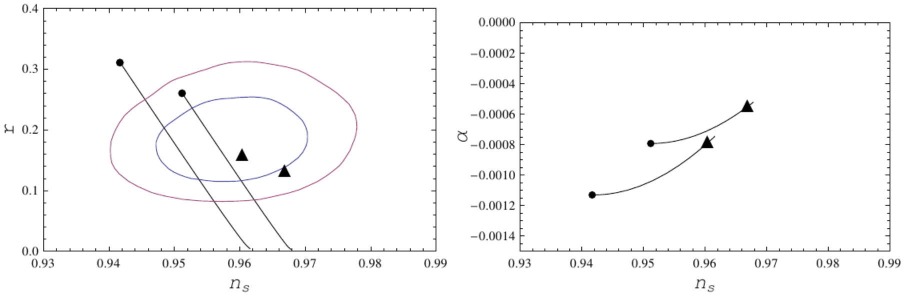
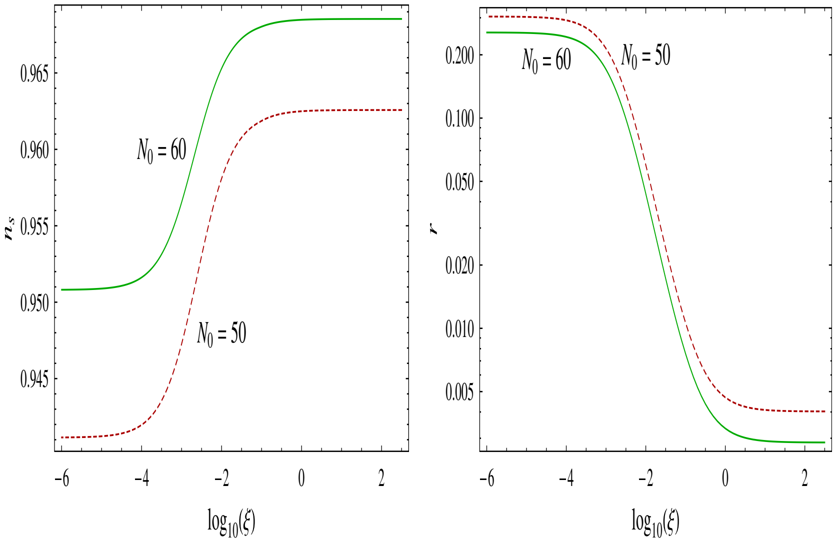


Figure 6. ϕ^4 potential with non-minimal gravitational coupling: n_s vs. r (left panel) and n_s vs. α (right panel) for various ξ values, along with n_s vs. r the contours (at the confidence levels of 68% and 95%) given by the BICEP2 collaboration (Planck+WP+highL+BICEP2). The black points and triangles are predictions in the textbook quartic and quadratic potential models, respectively. N is taken as 50 (left curves) and 60 (right curves).

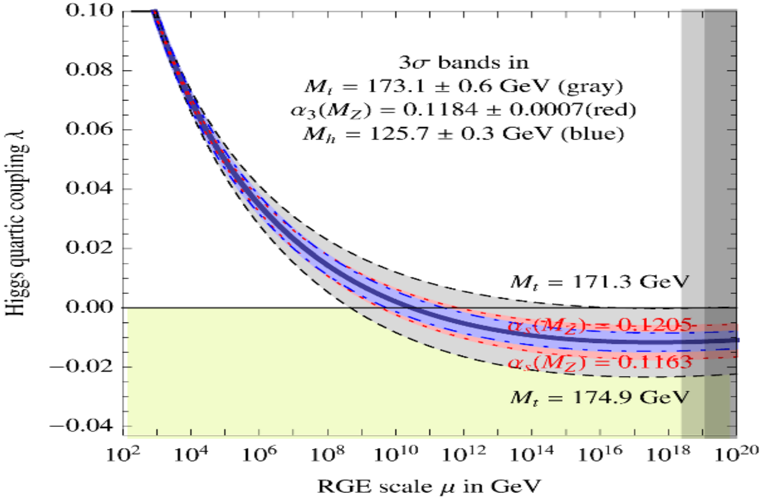
N. O.,Rehman & Shafi, Phys. Rev. D 82, 043502 (2010)



Standard Model Higgs Inflation?

Update of RGE analysis (@ 3-loop level)

Buttazzo et al.,
JHEP 12 (2013) 089



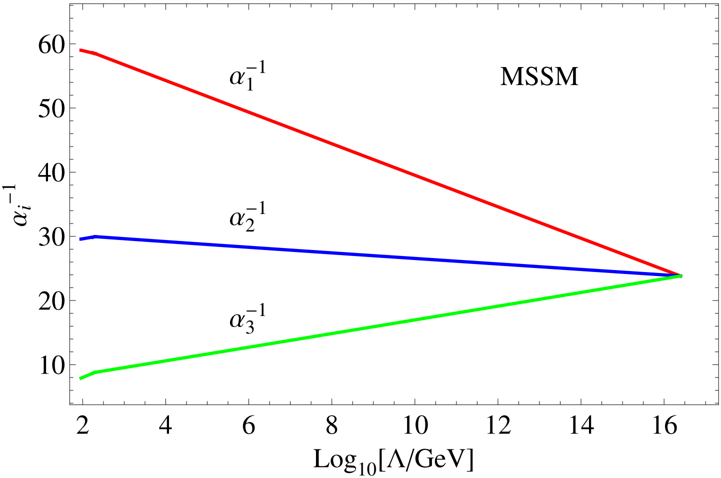
Supersymmetry

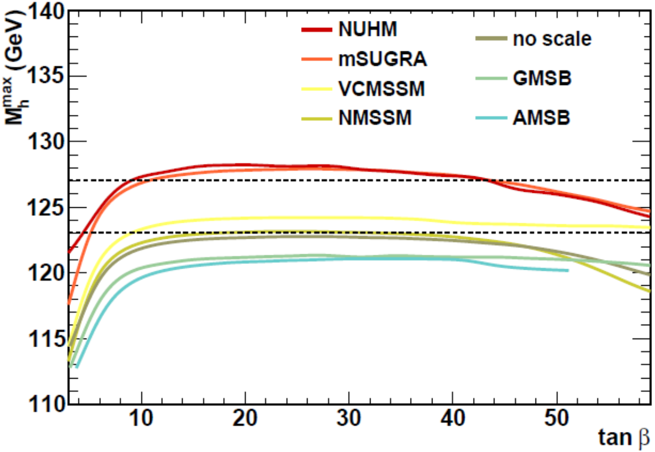
- Resolution of the gauge hierarchy problem
- Predicts plethora of new particles which LHC should find
- Unification of the SM gauge couplings at

  $M_{GUT} \sim 2 \times 10^{16}$ GeV
- Cold dark matter candidate (LSP)
- Radiative electroweak breaking
- String theory requires supersymmetry (SUSY)

Alas, SUSY not yet seen at LHC

Why Supersymmetry?





A. Arbey, M. Battaglia, A. Djouadi, F. Mahmoudi and J. Quevillon, Phys. Lett. B **708**, 162 (2012)

Venedig, Feb 23, 2005

1st Bet

Manfred Lindner & Jogesh Pati	Antonio Masiero, Qaisar Shafi, Mille	← say that Higgs will be found at ≤ 130 GeV.
NO!		

(Within 2σ)

2nd Bet

Jogesh Pati	Manfred Lindner, Antonio Masiero, Qaisar Shafi
says that neutron dipole moment will be discoverd with a factor 10 improvement (over today's ~~value~~ limit) Mille	SAY NO!

3rd Bet

Mille + Jogesh yes | Manfred, Antonio, Qaisar NO | $\mu \to e\gamma$ will be found at 10^{-12}

Winning Bet gets dinner at a fine restaurant

Qaisar Shafi (23 Feb, 05)

Manfred Lindner -u- [illegible]

Antonio Masiero 23 Feb. 05

Jogesh Pati "

Mille Cerk

Supersymmetric Higgs (Hybrid) Inflation

[Dvali, Shafi, Schaefer; Copeland, Liddle, Lyth, Stewart, Wands '94]

[Lazarides, Schaefer, Shafi '97][Senoguz, Shafi '04; Linde, Riotto '97]

- Attractive scenario in which inflation can be associated with symmetry breaking $G \longrightarrow H$
- Simplest inflation model is based on

$$W = \kappa\, S\, (\Phi\, \overline{\Phi} - M^2)$$

$S =$ gauge singlet superfield, $(\Phi\,, \overline{\Phi})$ belong to suitable representation of G

- Need $\Phi\,, \overline{\Phi}$ pair in order to preserve SUSY while breaking $G \longrightarrow H$ at scale $M \gg$ TeV, SUSY breaking scale.
- R-symmetry

$$\Phi\, \overline{\Phi} \rightarrow \Phi\, \overline{\Phi}, \;\; S \rightarrow e^{i\alpha}\, S, \;\; W \rightarrow e^{i\alpha}\, W$$

$\Rightarrow \quad W$ is a unique renormalizable superpotential

Supersymmetric Higgs (Hybrid) Inflation

- Attractive scenario in which inflation can be associated with symmetry breaking $G \longrightarrow H$
- Tree Level Potential

$$V_F = \kappa^2 \, (M^2 - |\Phi^2|)^2 + 2\kappa^2 |S|^2 |\Phi|^2$$

- Ground State

$$|\langle \Phi \rangle| = M, \;\; \langle S \rangle = 0$$

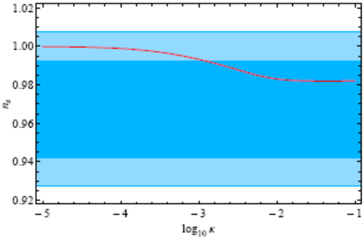
Cf: Superconductor, $\langle \Phi \rangle \rightarrow$ cooper pair, $\langle S \rangle \rightarrow$ temperature

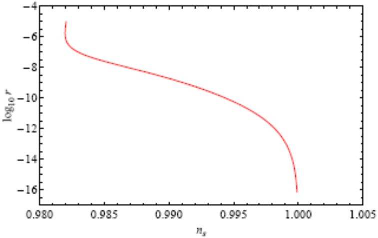
- To realize inflation

$S \gg M$ in early universe $(T \gg T_c)$

$\Rightarrow$ At tree level, $V \approx \kappa^2 M^4 \quad \Rightarrow \quad$ exponential expansion

Tree Level plus radiative corrections:





$$n_s \approx 1 - \frac{1}{N_0} \approx 0.98$$

$\delta T/T \propto (M/M_P)^2 \sim 10^{-5} \longrightarrow$ attractive scenario ($M \sim M_G$)

- Some examples of gauge groups:

$$G = U(1)_{B-L}, \text{ (Supersymmetric superconductor)}$$

$$G = SU(5) \times U(1), \quad (\Phi = 10), \quad \text{(Flipped } SU(5)\text{)}$$

$$G = 3_c \times 2_L \times 2_R \times 1_{B-L}, \; (\Phi = (1, 1, 2, +1))$$

$$G = 4_c \times 2_L \times 2_R, \; (\Phi = (\overline{4}, 1, 2)),$$

$$G = SO(10), \; (\Phi = 16)$$

- At renormalizable level the SM displays an ‘accidental’ global $U(1)_{B-L}$ symmetry.

- Next let us ‘gauge’ this symmetry, so that $U(1)_{B-L}$ is now promoted to a local symmetry. In order to cancel the gauge anomalies, one may introduce 3 SM singlet (right-handed) neutrinos.

  This has several advantages:

- See-saw mechanism is automatic and neutrino oscillations can be understood.

- RH neutrinos acquire masses only after $U(1)_{B-L}$ is spontaneously broken; Neutrino oscillations require that RH neutrino masses are $\lesssim 10^{14}\mathrm{GeV}$.

- RH neutrinos can trigger leptogenesis after inflation, which subsequently gives rise to the observed baryon asymmetry;

- Last but not least, the presence of local $U(1)_{B-L}$ symmetry enables one to explain the origin of Z_2 'matter' parity of MSSM. (It is contained in $U(1)_{B-L} \times U(1)_Y$, if $B-L$ is broken by a scalar vev, with the scalar carrying two units of $B-L$ charge.)

Take into account radiative corrections (because during inflation $V \neq 0$ and SUSY is broken by $F_S = -\kappa\, M^2$)

- Mass splitting in $\Phi - \overline{\Phi}$

$$m_{\pm}^2 = \kappa^2\, S^2 \pm \kappa^2\, M^2, \quad m_F^2 = \kappa^2\, S^2$$

- One-loop radiative corrections

$$\Delta V_{1\text{loop}} = \frac{1}{64\pi^2}\text{Str}[\mathcal{M}^4(S)(\ln \frac{\mathcal{M}^2(S)}{Q^2} - \frac{3}{2})]$$

- In the inflationary valley ($\Phi = 0$)

$$V \simeq \kappa^2\, M^4 \left(1 + \frac{\kappa^2 \mathcal{N}}{8\pi^2} F(x)\right)$$

where $x = |S|/M$ and

$$F(x) = \frac{1}{4}\left(\left(x^4+1\right)\ln \frac{\left(x^4-1\right)}{x^4} + 2x^2 \ln \frac{x^2+1}{x^2-1} + 2\ln \frac{\kappa^2 M^2 x^2}{Q^2} - 3\right)$$

Full Story

Also include supergravity corrections + soft SUSY breaking terms

- The minimal Kähler potential can be expanded as

$$K = |S|^2 + |\Phi|^2 + \left|\overline{\Phi}\right|^2$$

- The SUGRA scalar potential is given by

$$V_F = e^{K/m_p^2}\left(K_{ij}^{-1} D_{z_i}W D_{z_j^*}W^* - 3m_p^{-2}\,|W|^2\right)$$

where we have defined

$$D_{z_i}W \equiv \frac{\partial W}{\partial z_i} + m_p^{-2}\frac{\partial K}{\partial z_i}W;\; K_{ij} \equiv \frac{\partial^2 K}{\partial z_i \partial z_j^*}$$

and $z_i \in \{\Phi, \overline{\Phi}, S, ...\}$

[Senoguz, Shafi '04; Jeannerot, Postma '05]

- Take into account sugra corrections, radiative corrections and soft SUSY breaking terms:

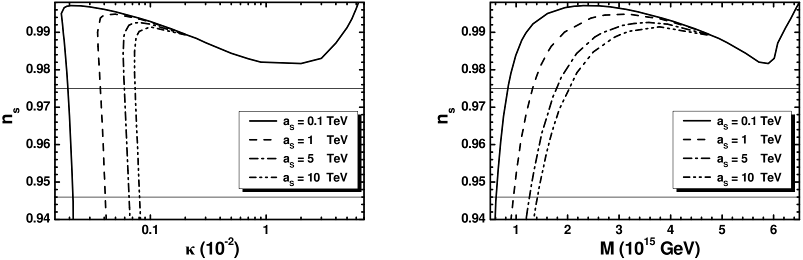
$$V \simeq \kappa^2 M^4 \left(1 + \left(\frac{M}{m_p}\right)^4 \frac{x^4}{2} + \frac{\kappa^2 \mathcal{N}}{8\pi^2} F(x) + a_s \left(\frac{m_{3/2} x}{\kappa M}\right) + \left(\frac{m_{3/2} x}{\kappa M}\right)^2\right)$$

where $a_s = 2\,|2 - A| \cos[\arg S + \arg(2 - A)]$, $x = |S|/M$ and $S \ll m_P$.

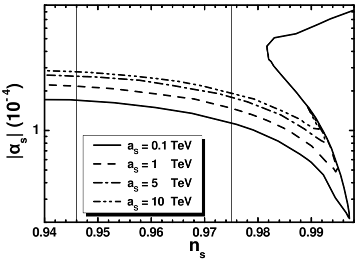
Note: No 'η problem' with minimal (canonical) Kähler potential !

Results

[Pallis, Shafi, 2013; Rehman, Shafi, Wickman, 2010]

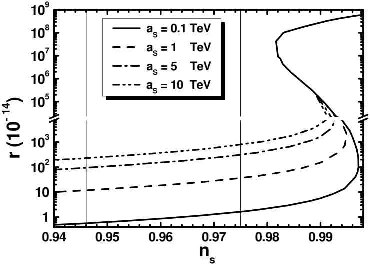


(a) (b)



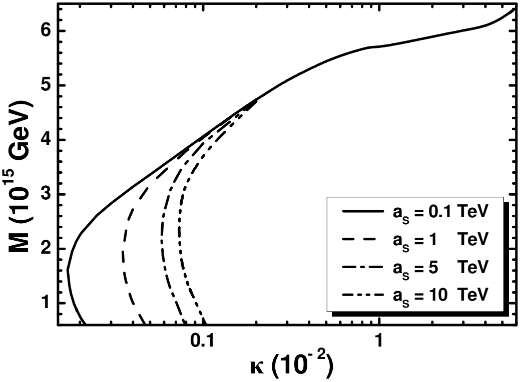
|α_s| (10^-4)
1
0.94 0.95 0.96 0.97 0.98 0.99
n_s
a_s = 0.1 TeV
a_s = 1 TeV
a_s = 5 TeV
a_s = 10 TeV

(a)

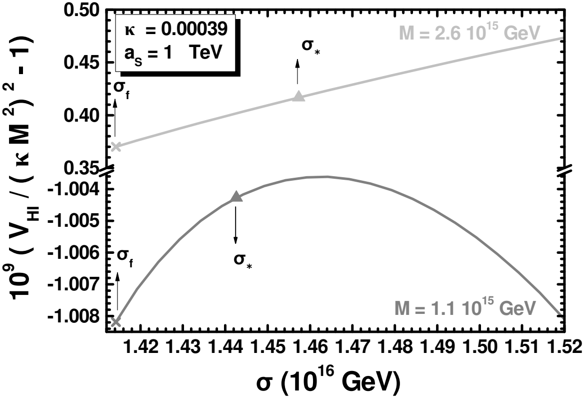


r (10^-14)
10^9 10^8 10^7 10^6 10^5 10^3 10^2 10 1
0.94 0.95 0.96 0.97 0.98 0.99
n_s
a_s = 0.1 TeV
a_s = 1 TeV
a_s = 5 TeV
a_s = 10 TeV

(b)



M (10^{15} GeV)
κ (10^{-2})
0.1
1
1
2
3
4
5
6
a_s = 0.1 TeV
a_s = 1 TeV
a_s = 5 TeV
a_s = 10 TeV



κ = 0.00039
a_S = 1 TeV
M = 2.6 10^15 GeV
M = 1.1 10^15 GeV
σ*
σf
σ*
σf
$10^9 \left(V_{HI} / (\kappa M^2)^2 - 1 \right)$
σ (10^16 GeV)
0.50
0.45
0.40
0.35
-1.004
-1.005
-1.006
-1.007
-1.008
1.42 1.43 1.44 1.45 1.46 1.47 1.48 1.49 1.50 1.51 1.52

Non-Minimal SUSY Hybrid Inflation and Tensor Modes

- Minimal SUSY hybrid inflation model yields tiny r values $\lesssim 10^{-10}$
- A more general analysis with a non-minimal Kähler potential can lead to larger r-values;
- The Kähler potential can be expanded as:

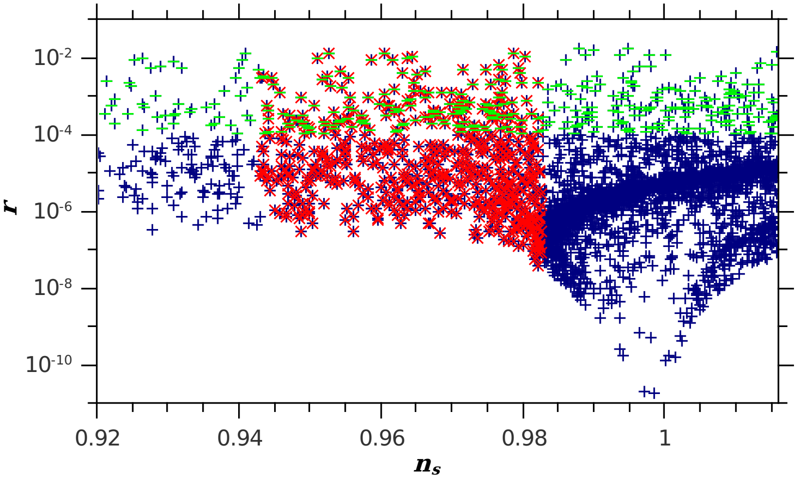
$$K = |S|^2 + |\Phi|^2 + |\overline{\Phi}|^2 + \frac{\kappa_S}{4}\frac{|S|^4}{m_P^2} + \frac{\kappa_\Phi}{4}\frac{|\Phi|^4}{m_P^2} + \frac{\kappa_{\overline{\Phi}}}{4}\frac{|\overline{\Phi}|^4}{m_P^2} + \kappa_{S\Phi}\frac{|S|^2|\Phi|^2}{m_P^2} + \kappa_{S\overline{\Phi}}\frac{|S|^2|\overline{\Phi}|^2}{m_P^2} + \kappa_{\Phi\overline{\Phi}}\frac{|\Phi|^2|\overline{\Phi}|^2}{m_P^2} + \frac{\kappa_{SS}}{6}\frac{|S|^6}{m_P^4} + \cdots \,,$$

The scalar potential becomes

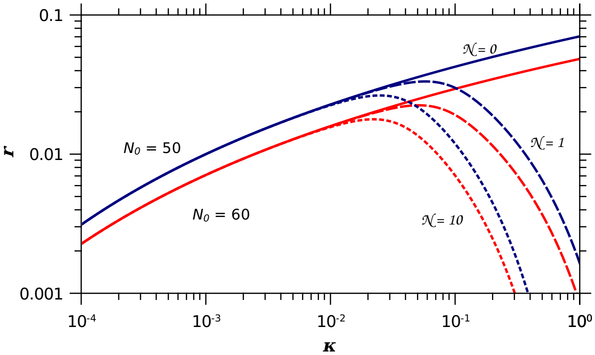
$$V \simeq \kappa^2 M^4 \left(1 - \kappa_S \left(\frac{M}{m_P}\right)^2 x^2 + \gamma_S \left(\frac{M}{m_P}\right)^4 \frac{x^4}{2} + \frac{\kappa^2 \mathcal{N}}{8\pi^2} F(x) + a \left(\frac{m_{3/2}\, x}{\kappa\, M}\right) + \left(\frac{M_S\, x}{\kappa\, M}\right)^2\right)$$

with (leading order) non-minimal Kähler, SUGRA, radiative, and soft SUSY-breaking corrections, and where

$$\gamma_S \equiv 1 - \frac{7}{2}\kappa_S + 2\kappa_S^2 - 3\kappa_{SS}$$



10-2
10-4
10-6
10-8
10-10
r
0.92
0.94
0.96
0.98
1
n_s



While radiative corrections are subdominant at large r, they play a crucial role in limiting the size of r. This limiting behavior comes in *indirectly* via the number of e-foldings N_0.

Summary

- If r lies close to 0.15, with n_s around 0.96, then chaotic inflation with ϕ^2 potential is an especially simple scenario. However, transplanckian field values remain a concern.
- If $r \sim 0.1 - 0.05$, then inflation models based on the Higgs / Coleman-Weinberg potentials can provide simple / realistic frameworks for inflation.
- If $r \leq 0.01$, then supersymmetric hybrid inflation models are especially interesting. These work with inflaton field values below M_{Planck}, and supergravity corrections are under control. The simplest versions employ TeV scale SUSY, and hopefully LHC 14 will find it.