BCVSPIN – MSPF – Mitchell 2014

LHC Implication Of Grand unification

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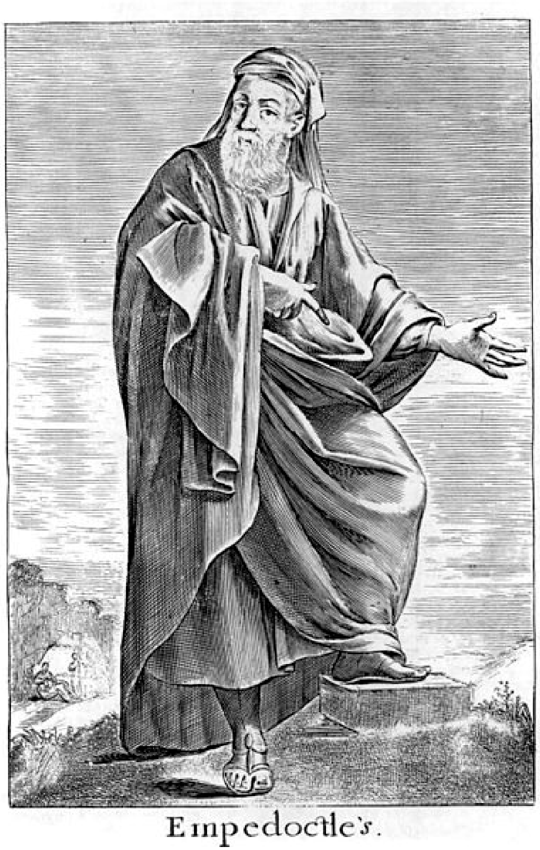
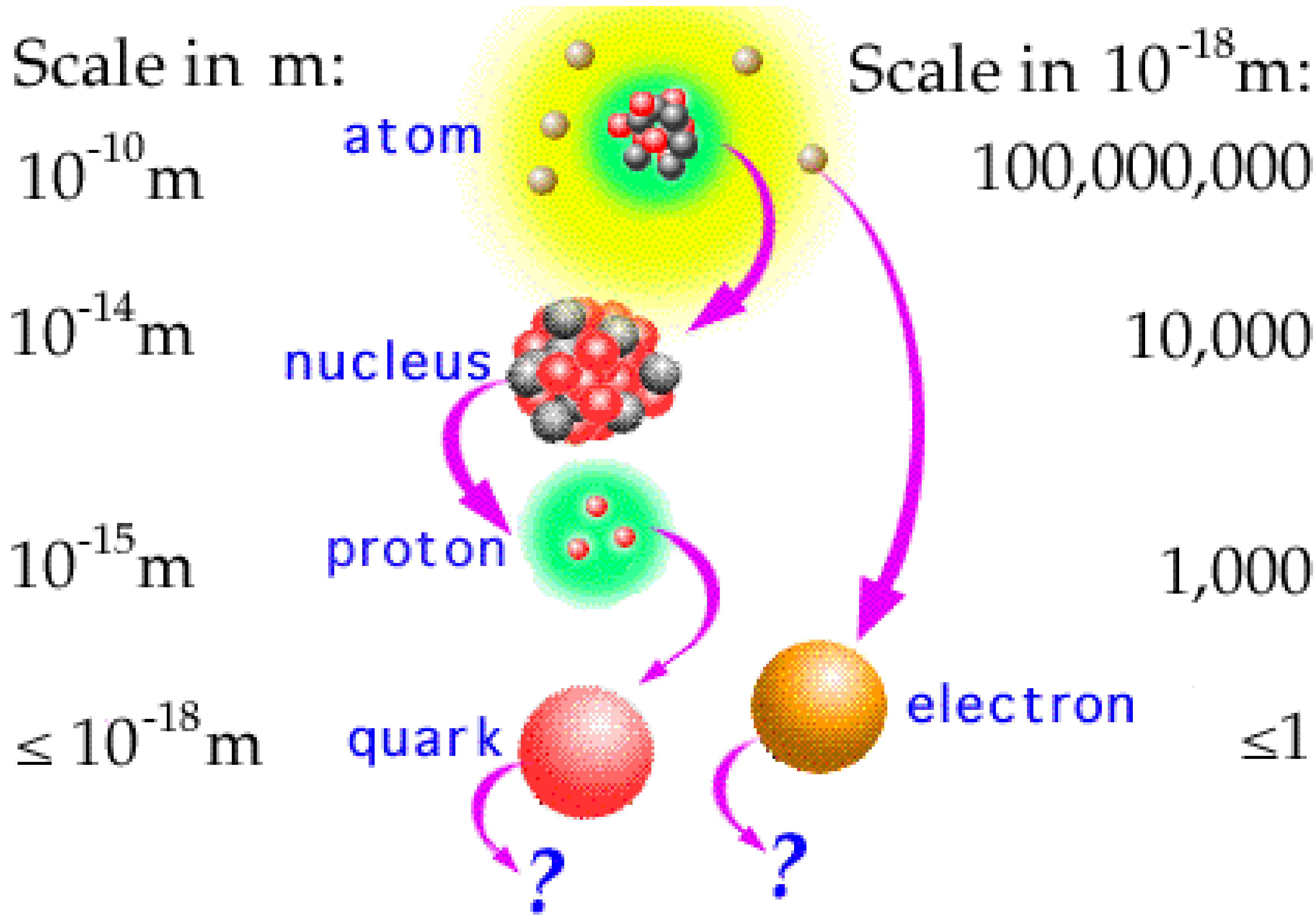
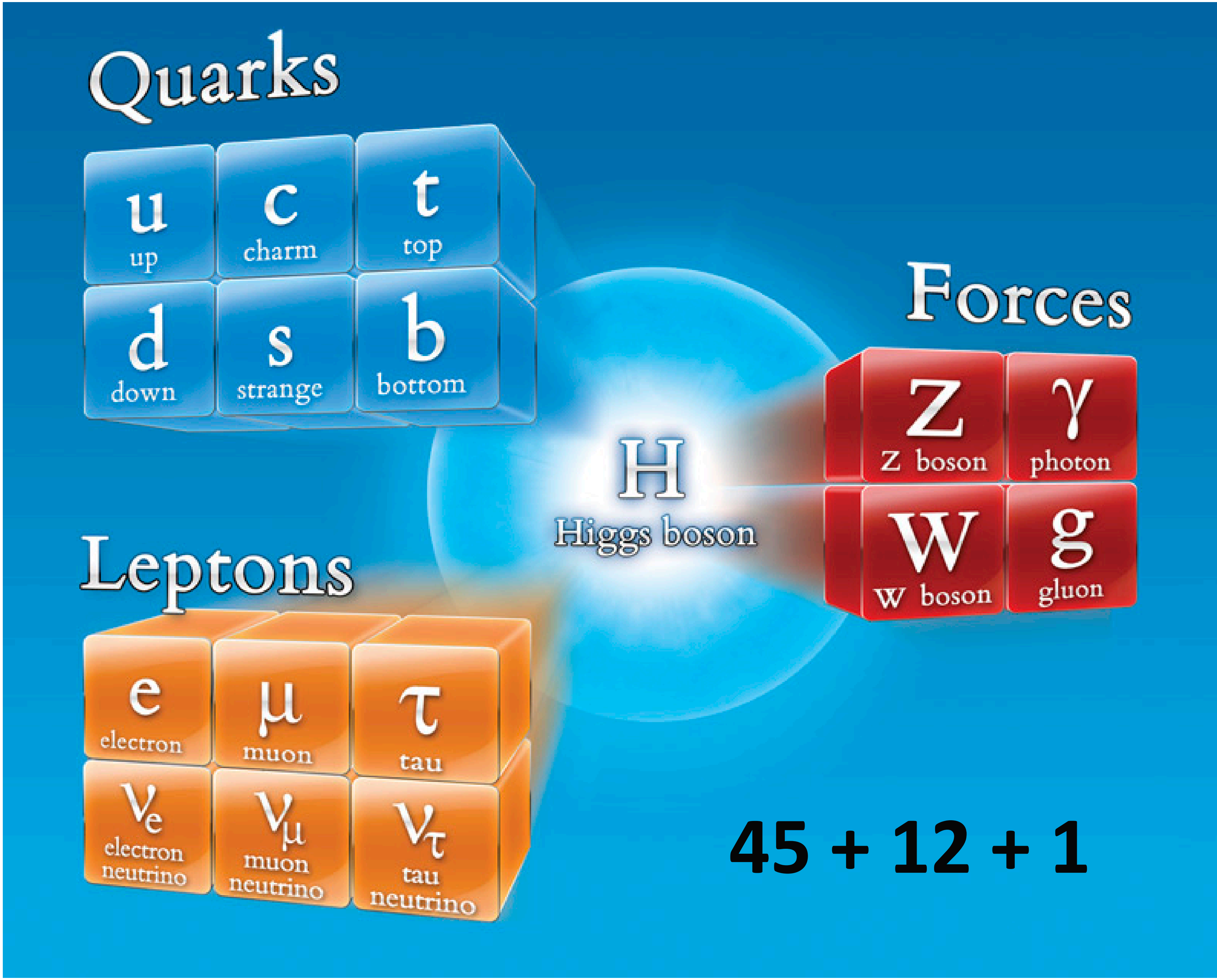
The concept that matter is composed of discrete units and cannot be divided into arbitrarily tiny quantities has been around for millennia



Empedocles, 490–430 BC



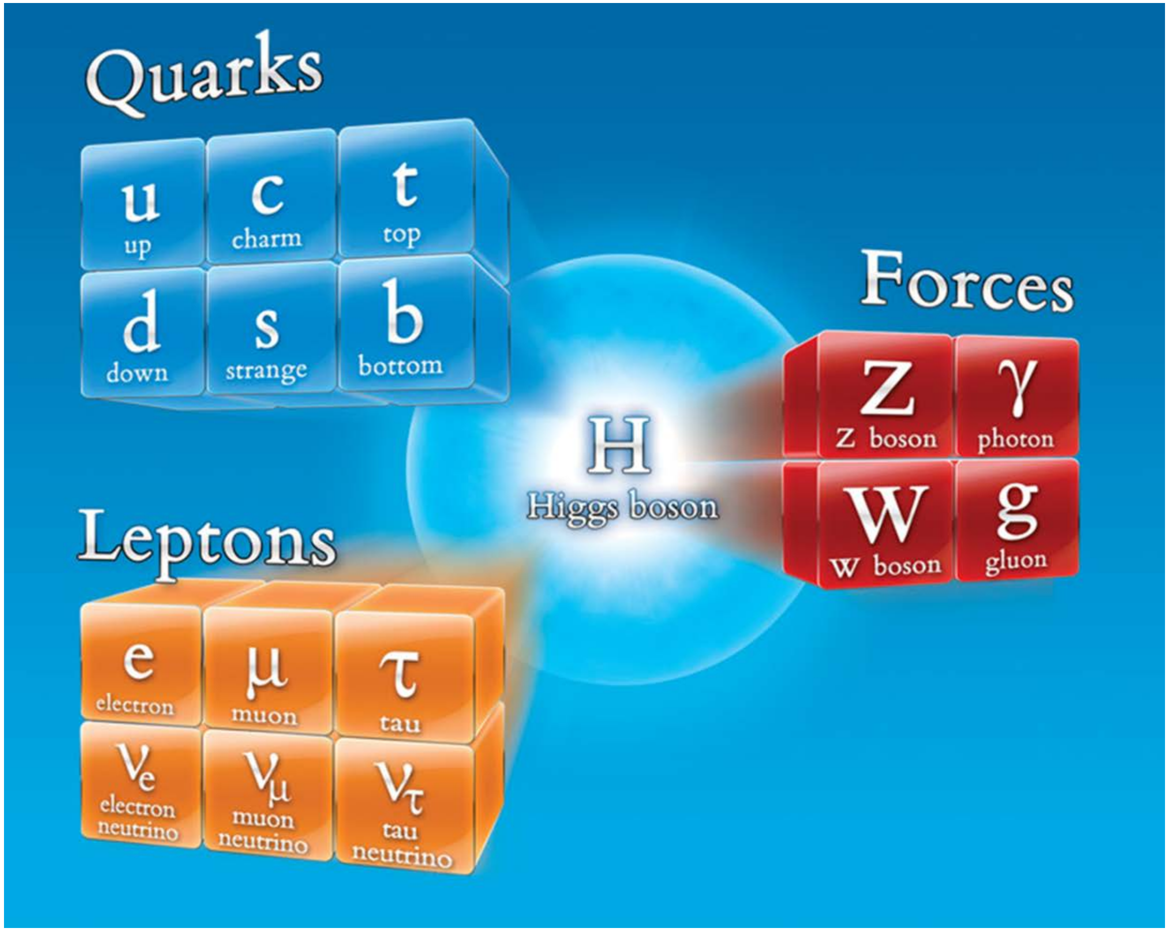
Scale in m:
Scale in 10^{-18}m:
atom
10^{-10}m
100,000,000
10^{-14}m
nucleus
10,000
proton
10^{-15}m
1,000
quark
$\leq 10^{-18}$m
electron
≤1
?
?


Quarks
u
up
c
charm
t
top
d
down
s
strange
b
bottom
Forces
Z
Z boson
γ
photon
W
W boson
g
gluon
H
Higgs boson
Leptons
e
electron
μ
muon
τ
tau
ν_e
electron neutrino
ν_μ
muon neutrino
ν_τ
tau neutrino

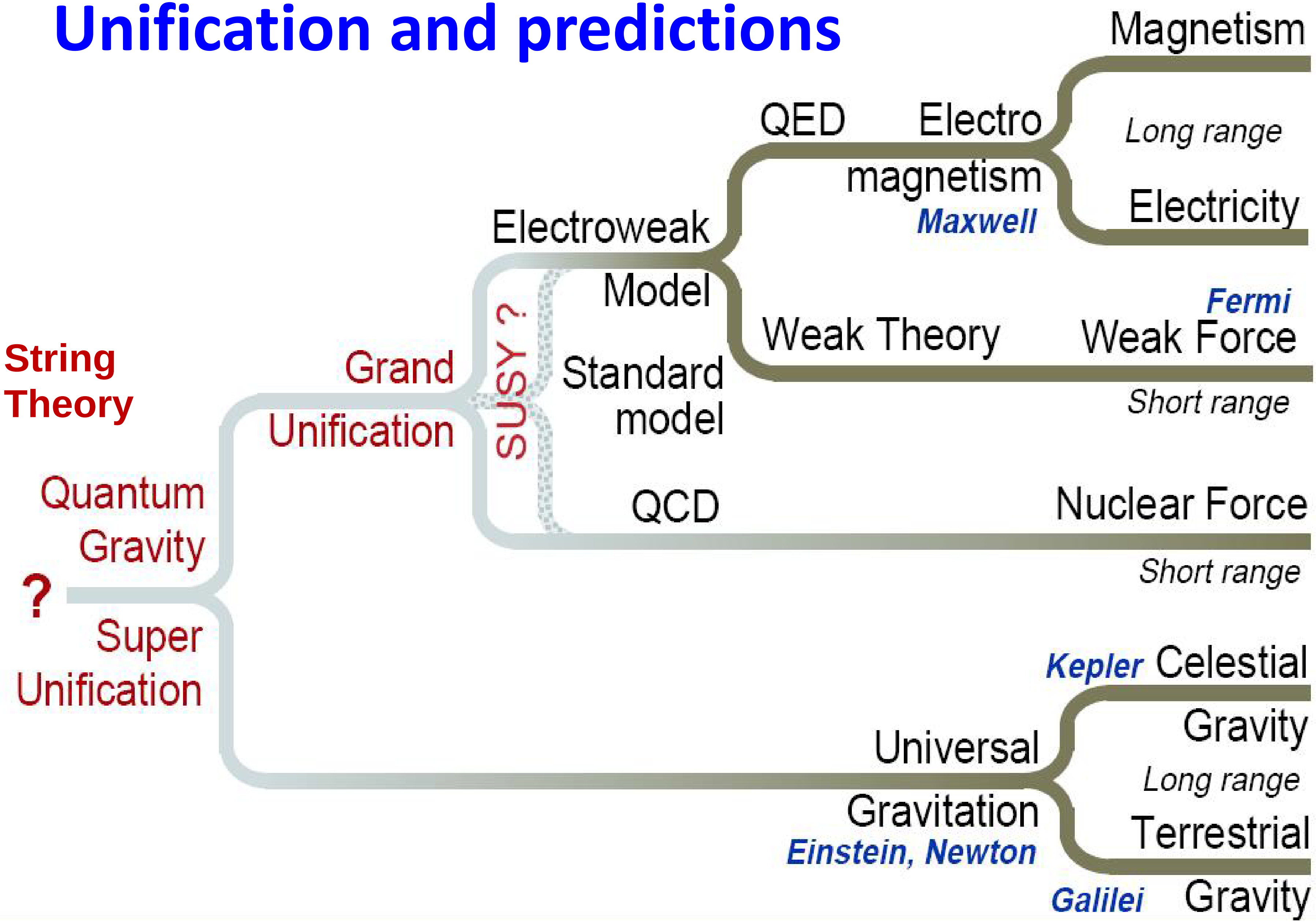
45 + 12 + 1

4 vs 58 ?


Air
Fire
Earth
Water


Quarks
u up
c charm
t top
d down
s strange
b bottom
Leptons
e electron
μ muon
τ tau
νe electron neutrino
νμ muon neutrino
ντ tau neutrino
H Higgs boson
Forces
Z Z boson
γ photon
W W boson
g gluon

Unification and predictions



The SM

The Standard Model is the theory governing fundamental particles and interaction (except Gravity)

For $L \geq 10^{-18}$ m ⇔ $E \leq 10^{2}$ GeV

SM is the Theory of Forces & the Particles

$$SU(3)_C \times SU(2)_L \times U(1)_Y$$

↓ ↓ ↓

8 Gluons | **$W^{\pm}$, W^{0}** | **B**

Spin 1 bosons

Particles are `` Chiral Fermions”

It is more convenient to work in left (or right) handed bases

We can just drop all “L” subscripts and write all field in terms of left-handed components

Q: $(3, 2, 1/6)$

L : $(1, 2, -1/2)$

u^c : $(\overline{3}, 1, -2/3)$

d^c : $(\overline{3}, 1, 1/3)$

e^c : $(1, 1, 1)$

$$Q_L = \left(3, 2, \frac{1}{6}\right), \quad but \quad NOT \quad \left(\overline{3}, 2, -\frac{1}{6}\right)$$

The conjugate of a **right-handed** component of a fermion is the **left-handed** component of the **conjugate** fermion!

Recall ``charge conjugate" operation (particle $\leftrightarrow$ antipartocle)

$$\Psi^c \equiv i\,\gamma^2\Psi^*$$

$$(\Psi_R)^c = i\gamma^2 \left(\frac{1}{2}(1+\gamma_5)\Psi\right)^*$$

Since $\gamma_5^* = \gamma_5$

$$= \frac{i}{2}\gamma^2(1+\gamma_5)\Psi^*$$

since $\{\gamma^\mu,\ \gamma^5\} = 0$

$$= \frac{1}{2}(1-\gamma_5)[i\gamma^2\Psi^*]$$

$$= (\Psi^c)_L$$

The SM Higgs sector

$$SU(3)_c \times SU(2)_L \times U(1)_Y \xrightarrow{\langle\varphi\rangle \text{ Higgs VEV}} U(1)_{EM}$$

φ – Higgs field is $SU(2)_L$ doublet, complex scalar field

$$\varphi = \begin{pmatrix} \varphi^+ \\ \varphi^0 \end{pmatrix}_{y=1/2} \Longleftarrow \text{Four degree of freedom}$$

Yukawa sector

$$L = Y_d \overline{Q}_L \varphi\, d_R + Y_u \overline{Q}_L (i\,\tau_2 \varphi^*) u_R + Y_e L_L \varphi\, e_R + h.c.$$

The fermions gain masses

$$m_i = Y_i < \varphi > = \frac{Y_i v}{\sqrt{2}}$$

We have three generation quarks and leptons.

We have mixing between generation.

Chiral Adler-Bell-Jackiw (ABJ) anomaly

The ABJ anomaly spoils the renormalizability of a gauge theory

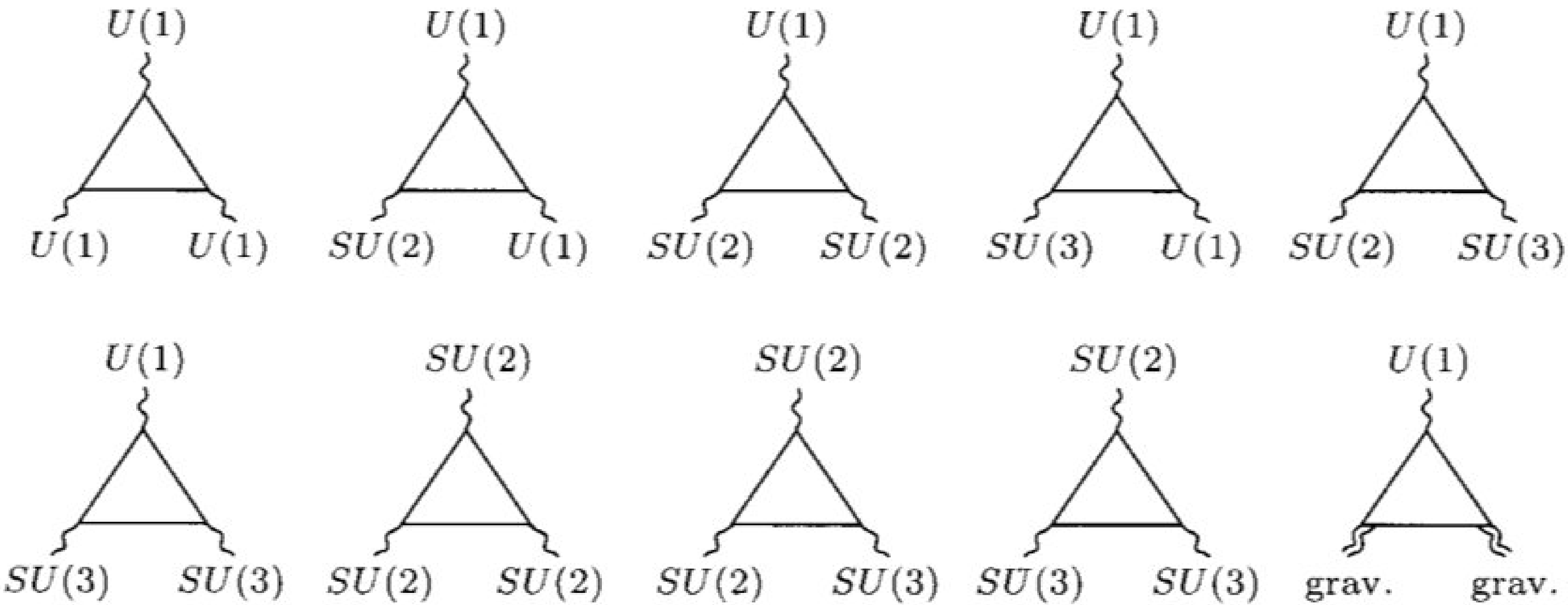


Figure 20.2. Possible gauge anomalies of weak interaction theory. All of these anomalies must vanish for the Glashow-Weinberg-Salam theory to be consistent.

From Peskin and Schroeder

Miraculous Cancellation of Anomalies

- $SU(3)_C^2 \times U(1)_Y$: $\frac{1}{2}\left[2\times(\frac{1}{6})+1\times(\frac{-2}{3})+1\times(\frac{1}{3})\right]=0$

- $SU(2)_L^2 \times U(1)_Y$: $\frac{1}{2}\left[3\times(\frac{1}{6})+1\times(\frac{-1}{2})\right]=0$

- $(\text{gravity})^2 \times U(1)_Y$:

  $\left[3\times 2\times(\frac{1}{6})+3\times(\frac{-2}{3})+3\times(\frac{1}{3})+2\times(\frac{-1}{2})+1\times 1\right]=0$

- $U(1)_Y^3$:

$$\left[3\times 2\times(\tfrac{1}{6})^3+3\times(\tfrac{-2}{3})^3+3\times(\tfrac{1}{3})^3+2\times(\tfrac{-1}{2})^3+1\times(1)^3\right]=0$$

Relative Y-values are fixed ⟹ charge quantization

But overall normalization still is not fixed

The SM - Things to remember

1) Lots of seemingly disconnected representations for gauge & particle content

2) 3 independent gauge couplings: $(g_1, \ g_2, \ g_Y)$

3) Yukawa sector is unconstrained.

4) Particle representations are chiral

$$Q_L = \left(3, 2, \frac{1}{6}\right), \quad but \quad NOT \quad \left(\overline{3}, 2, -\frac{1}{6}\right)$$

5) Overall normalization for hypercharge unfixed, (since $U(1)_Y$ Abelian), Even thought relative *Y*-values are fixed

6) Higgs mechanism breaks

$$SU(2)_L \times U(1)_Y \quad \rightarrow \quad U(1)_{EM}$$

In general, the subgroup which survives is the subgroup with respect the field getting the non zero VEV is neutral

7) In SM

Baryon # (B) conserved

Lepton # (L) conserved

Thus, proton is stable!

Note: B – is actually broken by instanton effects (very small)

L – can be broken by RH neutrino Majorama mass

The rank of The SM gauge symmetry

$$SU(3)_c \times SU(2)_L \times U(1)_Y$$

$$2 + 1 + 1 = 4$$

Our goal – to "unify" all of the forces and particles

↓

We need a bigger group

What groups G can we choose?

SM has rank = 4	1. group G must be rank ≥ 4 and contain SM as subgroup
SM has chiral reps (3, 2, 1/6) but not $(\overline{3}, 2, -1/6)$	Group G must also have chiral reps
SM is free of chiral anomaly	Group G must have reps for which chiral anomalies are canceled
If we wont to relate the gauge couplings to each other	G should be a simple group

Classification of Lie Groups					
Rank 1	U(1), SU(2)	SO(3)	Sp(2)		
Rank 2	SU(3)	SO(5)	Sp(4)	SO(4)	G_2
Rank 3	SU(4)	SO(7)	Sp(6)	SO(6)	
Rank 4	SU(5)	SO(9)	Sp(8)	SO(8)	F4
Rank 5	SU(6)	SO(11)	Sp(10)	SO(10)	
Rank 6	SU(7)	SO(13)	Sp(12)	SO(12)	E_6 ....

Does SU(5) symmetry have the potential for a successful unification ?

SU(5) symmetry has the following representations:

1, 5, 10, 15, 24, 45, 50, 78 etc.

Recall each SM generation contains 15 states and 3 generations. ($3 \times 15 = 45$)

$$SU(5) \rightarrow SU(3) \times SU(2) \times U(1)$$

$$15 = (3,1)_6 + (2,3)_1 + (1,6)_{-4}$$

$$45 = (2,1)_3 + (1,3)_1 + (3,3)_{-2} + (1,3)_8 + (2,\overline{3})_{-7} + (1,\overline{6})_{-2} + (2,8)_3$$

Here all U(1) charges are normalized to avoid fractions

But let's look at $\overline{5}$ and 10 dimensional representation

SU(5) $\longrightarrow$ SU(3) X SU(2) X U(1)

$$\overline{5} = (\overline{3},1)_{-2} + (1,2)_{-3}$$

$$10 = (\overline{3},1)_{-4} + (3,2)_{1} + (1,1)_{6}$$

we have to rescale U(1) quantum numbers by 1/6

$$10_{[\alpha\beta]} = \underbrace{(\overline{3},1)}_{u^c}{}_{-\frac{2}{3}} + \underbrace{(3,2)}_{Q}{}_{\frac{1}{6}} + \underbrace{(1,1)}_{e^c}{}_{1}$$

$$\overline{5} = \underbrace{(\overline{3},1)}_{d^c}{}_{1/3} + \underbrace{(1,2)}_{L}{}_{-1/2}$$

An Entire SM generation fits into: $\bar{5} + 10$

Nothing left over and no exotics!

Anomaly cancellation?

Since we have not added new exotic fermions, the anomaly cancelation still it is OK

$$\bar{\mathbf{5}} : (d_1^c, d_2^c, d_3^c, e, -\nu_e)$$

$$\mathbf{10} : \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & u_3^c & -u_2^c & u_1 & d_1 \\ -u_3^c & 0 & u_1^c & u_2 & d_2 \\ u_2^c & -u_1^c & 0 & u_3 & d_3 \\ -u_1 & -u_2 & -u_3 & 0 & e^c \\ -d_1 & -d_2 & -d_3 & -e^c & 0 \end{pmatrix}$$

Gauge bosons

$$24 \rightarrow (8,1)_0 + (1,3)_0 + (1,1)_0 + (3,2)_{-5/6} + (\bar{3},2)_{5/6}$$

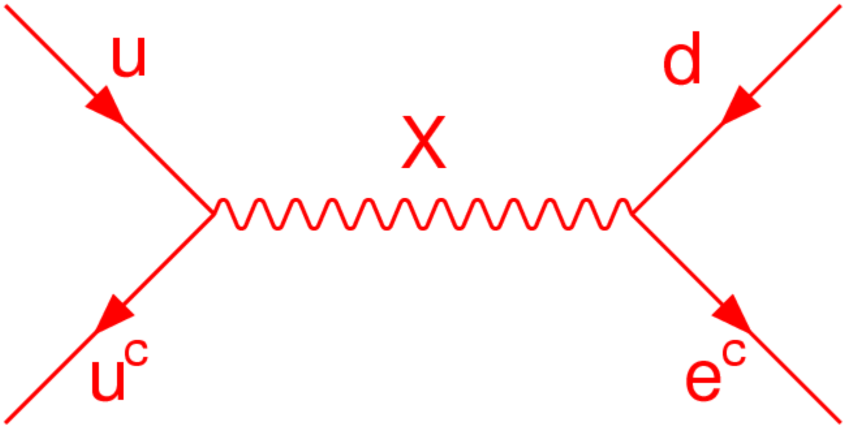
gluinos — $A^{\pm}, A^0$ — B — X, Y bosons

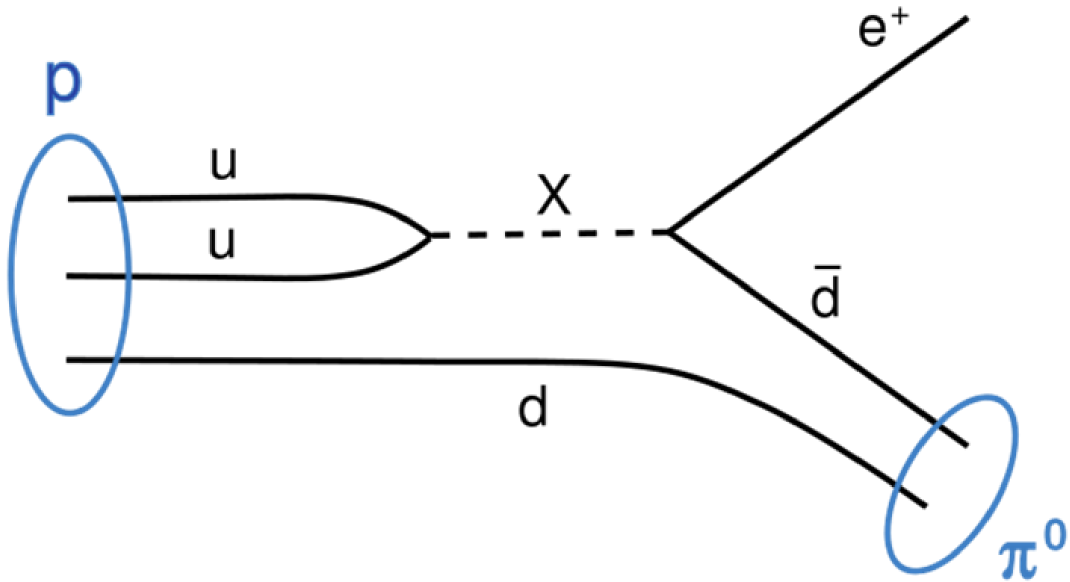
All SM gauge bosons are successfully embedded

X and Y gauge bosons carry both color and electroweak charges simultaneously !

They can connect quarks ⟺ leptons!

They can also turn quark directly to antiquark!





$$p \to e^+\pi^0,\ \tau_p^{-1} \approx \left[\frac{g^2}{M_X^2}\right]^2 m_p^5 \approx [10^{35\pm1} yr]^{-1}$$

The current experimental limit is:

$$\boldsymbol{\tau(p \to e^+\pi^0) > 1.4 \times 10^{34}\,\mathrm{yr}}$$

Overall hypercharge Y normalization finally fixed

$$SU(5) \Rightarrow SU(3) \times SU(2) \times U(1)$$

Hypercharge is one of the non-Abelian generator

$$Q_{EM} = T_3 + Y = T_3 + c\, T_0$$

$$\bar{5} : (d_1^c, d_2^c, d_3^c, e, -\nu_e)$$

$$Y(5) = \left(-\frac{1}{3}, -\frac{1}{3}, -\frac{1}{3}, \frac{1}{2}, \frac{1}{2}\right)$$

$$T_0 = \frac{1}{\sqrt{60}}((2,\ 2\ , 2, -3, -3)$$

$$\mathbf{c} = -\sqrt{\frac{3}{5}}$$

$$Y_{SU(5)} = \sqrt{\frac{3}{5}}\, Y_{SM}$$

$$[D_\mu = \partial_\mu + i\,\frac{g_y Y}{2}\, B_\mu]$$

The product $(g_y\, Y)$ must be preserved

$$g_Y{}^{SU5} = \sqrt{\frac{5}{3}}\, g_Y{}^{SM}$$

So, unification into a single GUT group such as SU(5) requires all generators to act with a common couplings

$$g_5 \equiv \left(g_3 = g_2 = g_1 = \sqrt{\frac{5}{3}}\, g_y \right)$$

$$\alpha_5 \equiv (\alpha_3 = \alpha_2 = \alpha_1 = \frac{5}{3}\, \alpha_Y)$$

$$\frac{g_3}{g_2} = 1$$

$$\sin^2\theta_W = \frac{g_Y^2}{g_2^2 + g_Y^2} = \frac{3}{8} = 0.375$$

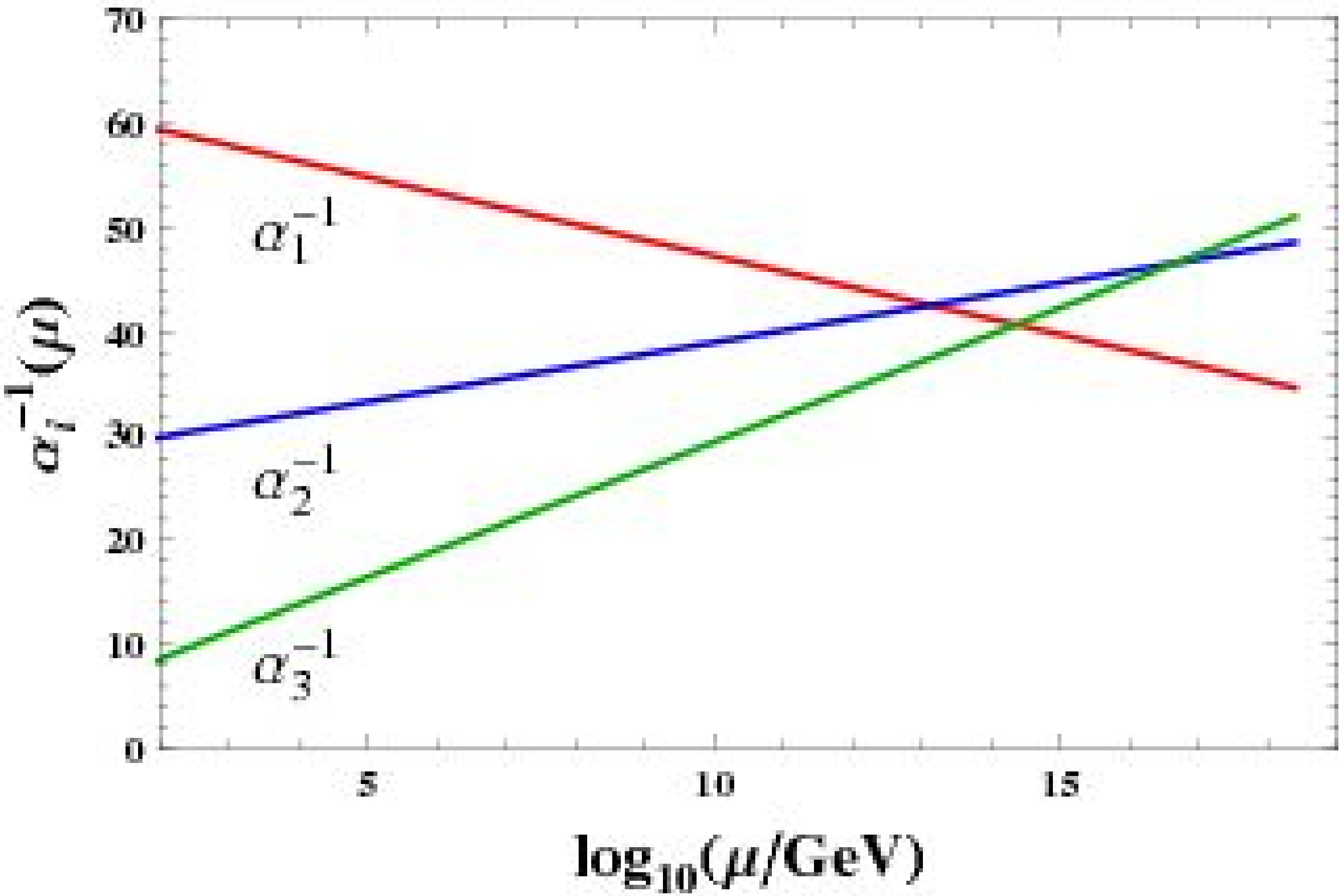
But at electroweak scale we have

$$\alpha_3^{-1} \approx 8.5$$

$$\alpha_2^{-1} \approx 29.6$$

$$\alpha_1^{-1} \approx 59.1$$

- Couplings are not equal
- $\sin^2\theta_W \approx 0.23$, $\mathit{NOT}\ 0.375$



70
60
50
40
30
20
10
0
5
10
15
$\alpha_i^{-1}(\mu)$
α_1^{-1}
α_2^{-1}
α_3^{-1}
$\log_{10}(\mu/\mathrm{GeV})$

Yukawa sector

$$Y_5\,\overline{5}_f 10_f\,5_H{}^* + Y_{10} 10_f\,10_f\,5_H$$

$$\Downarrow$$

$$Y_5\,(d^c\,Q\,H^* + L\,e^c\,H^*) \;\Rightarrow\; Y_d = Y_E^T$$

$$\Downarrow$$

$$m_e = m_d, \qquad m_\mu = m_s\,, \qquad m_\tau = m_b$$

The Yukawa (mass) matrix for down quarks is just the transpose of the Yukawa (mass) matrix for the charge leptons.

Introduce a 45 rep as another new Higgs

$$\left(\frac{m_e}{m_\mu} = \frac{m_d}{m_s}\right) \Rightarrow \left(\frac{m_e}{m_\mu} = \frac{1}{9}\frac{m_d}{m_s}\right)$$

Or consider effective non Non-renormalizable couplings

$$Y'_5\,\overline{5}_f 10_f \left(\frac{\Sigma}{\text{M}}\right)^n 5^*$$

$$10 = u^c\ (3,1)_{-2/3} + Q\ (3,2)_{1/6} + e^c\ (1,1)_1$$

$$10_f\ 10_f\ 5_H \Longrightarrow Y_U = Y_U^T$$

The Yukawa(mass) matrix for the top quark is symmetric

Open Questions for the Standard Model

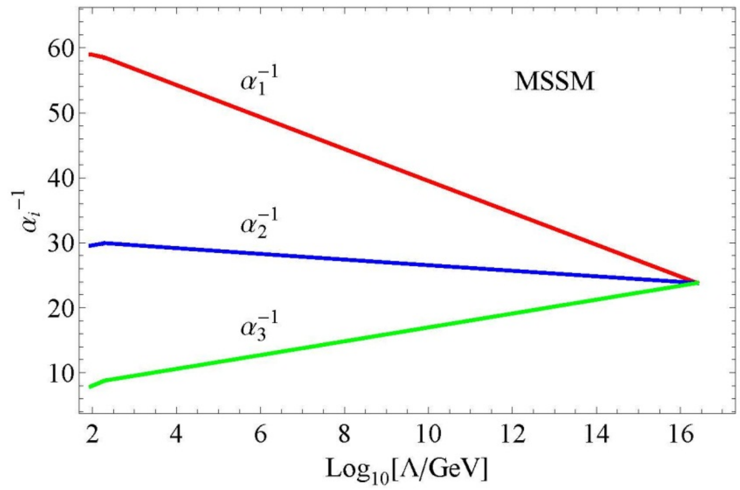
- Why local gauge $SU(3)_c \times SU(2)_L \times U(1)_Y$ interaction?
- Why $g_1 : g_2 : g_3 = 1 : 2 : 7$?
- Why electric charge is quantized

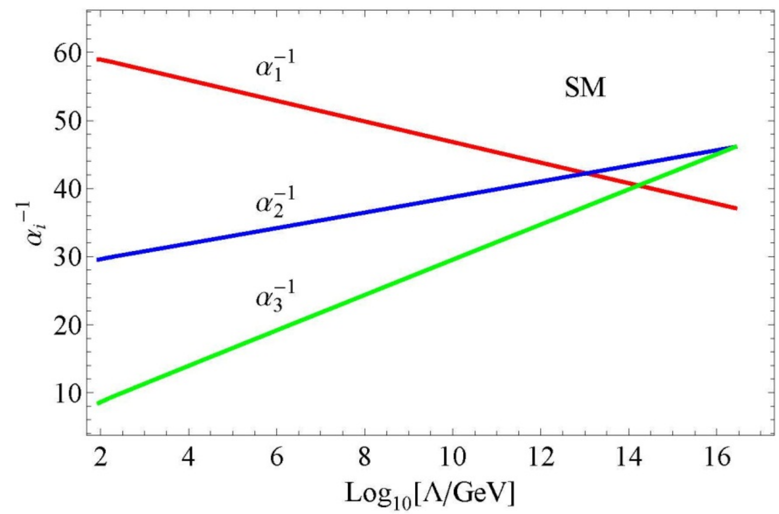
  $Q_p = Q_e$ to better than 1 part in 10^{21}
- Why 3 families of quarks and leptons?
- What is origin of quark and lepton masses and mixing?
- How neutrino masses and mixing angles are generated?
- What is nature of dark matter?
- Strong CP problem?

Low Energy Supersymmetry

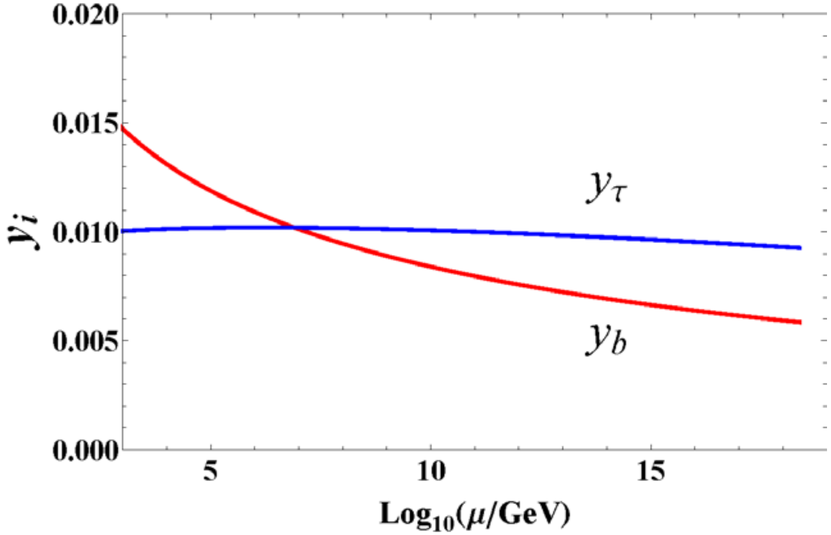
- Resolves the gauge hierarchy problem;
- Provides cold dark matter candidate (LSP);
- Implements radiative electroweak symmetry breaking;
- Predicts new particles accessible at the LHC;
- Improves unification of the SM gauge and Yukawa couplings.

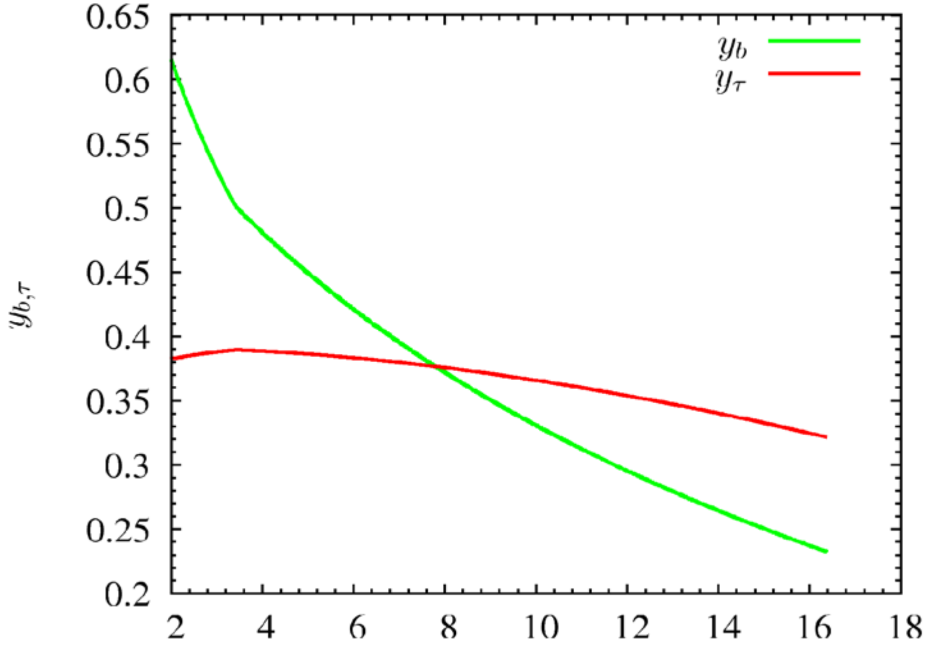
• **Improves unification of the SM gauge couplings.**



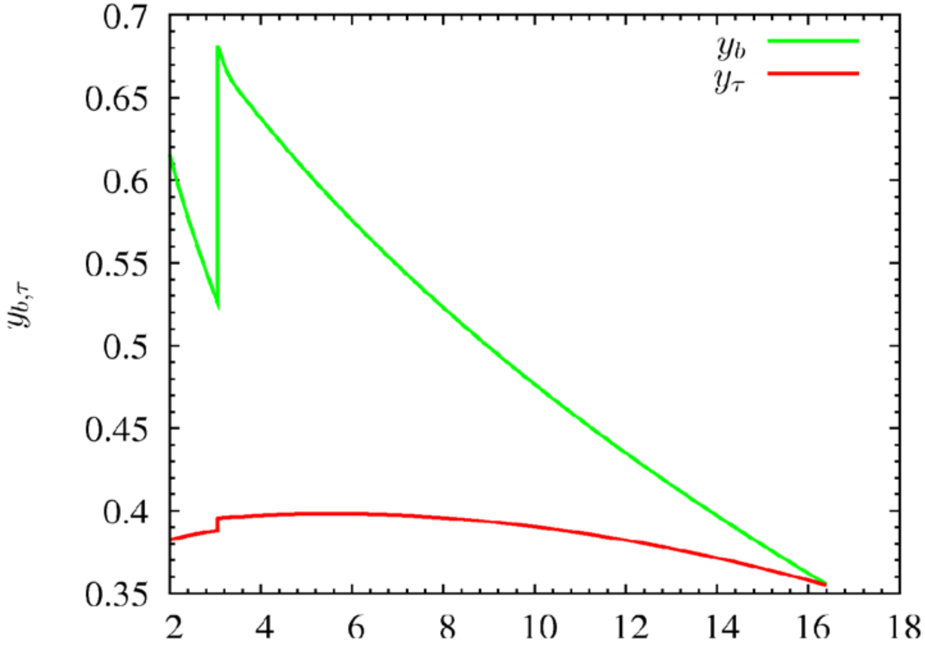


$b-\tau$ Yukawa coupling unification ♡ SUSY





(a) $\log_{10}(Q/\mathrm{GeV})$



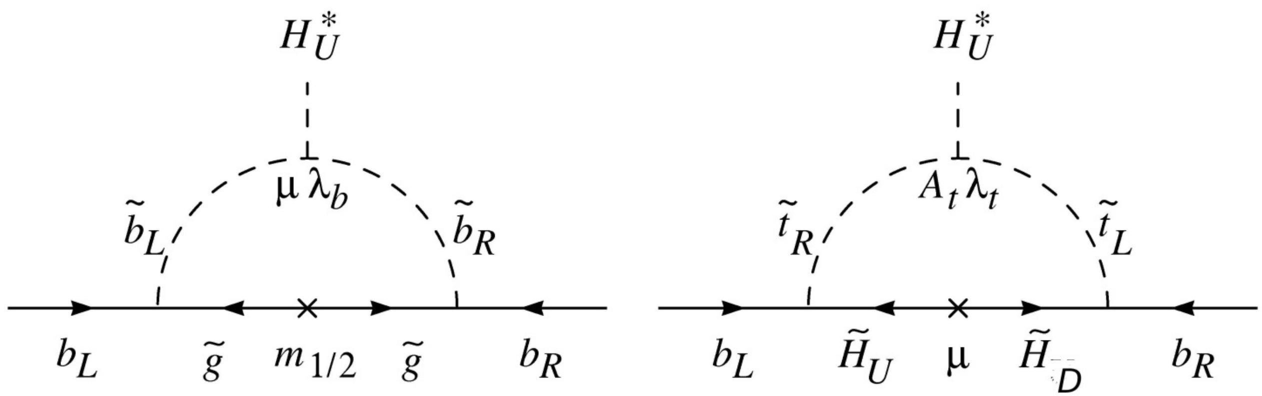
(b) $\log_{10}(Q/\mathrm{GeV})$

Finite SUSY threshold corrections

Dominant contributions to the bottom quark mass from the gluino and chargino loop

$$\delta y_b \approx \frac{g_3^2}{12\pi^2}\frac{\mu m_{\tilde{g}}\tan\beta}{m_{\tilde{b}}^2} + \frac{y_t^2}{32\pi^2}\frac{\mu A_t \tan\beta}{m_{\tilde{t}}^2} + \ldots$$

where $m_{\tilde{b}}$ and $m_{\tilde{t}}$ stands for sbottom and stop mass.



where $\lambda_b = y_b$ and $\lambda_t = y_t$

SU(5)	SO(10)
Fermion sector: $\mathbf{10}_i + \overline{\mathbf{5}_i} + \ (\mathbf{1}_i\,?\,)$	**Fermions:** $\mathbf{16}_i$ **existence of** ν_R **and thus neutrino mass via seesaw mechanism**
SM Higgs: $\overline{\mathbf{5}_H} + \mathbf{5}_H$	**SM Higgs:** **10**
Z_2 **R-parity** $\mathbf{10}_f\,\overline{\mathbf{5}}_f\,\overline{\mathbf{5}_f}$	**Automatic** Z_2 **matter parity** ~~$\mathbf{16}_f\mathbf{16}_f\mathbf{16}_f$~~
$\overline{\mathbf{5}_H} + \mathbf{5}_H \ + \ \mathbf{24}$	$\mathbf{144} + \overline{\mathbf{144}}$

Yukawa sector

$$\mathbf{16}_f \times \mathbf{16}_f \times \mathbf{10}_H$$

SUSY SO(10) GUT with non universal gauginos

- $m_{16},\ m_{10},\ M_i,\ A_0,\ \tan\beta,\ sign(\mu)$
- $m_{16} \equiv$ Universal soft SUSY breaking (SSB) sfermion mass
- $m_{10} \equiv$ Universal SSB MSSM Higgs mass.
- $M_i \equiv$ SSB gaugino masses. $M_1 : M_2 : M_3 = 1 : 3 : -2$ at M_{GUT} comes from $\frac{F_{\Phi_{ab}}}{M_P}\lambda^a\lambda^b$
- $A_0 \equiv$ Universal SSB trilinear interaction
- $\tan\beta = \frac{v_u}{v_d}$
- $\mu \equiv$ SUSY bilinear Higgs parameter $\mu > 0$

We have performed random scans for the following parameter range

$$0 \leq m_{16} \leq 10\, TeV$$

$$0 \leq m_{10} \leq 10\, TeV$$

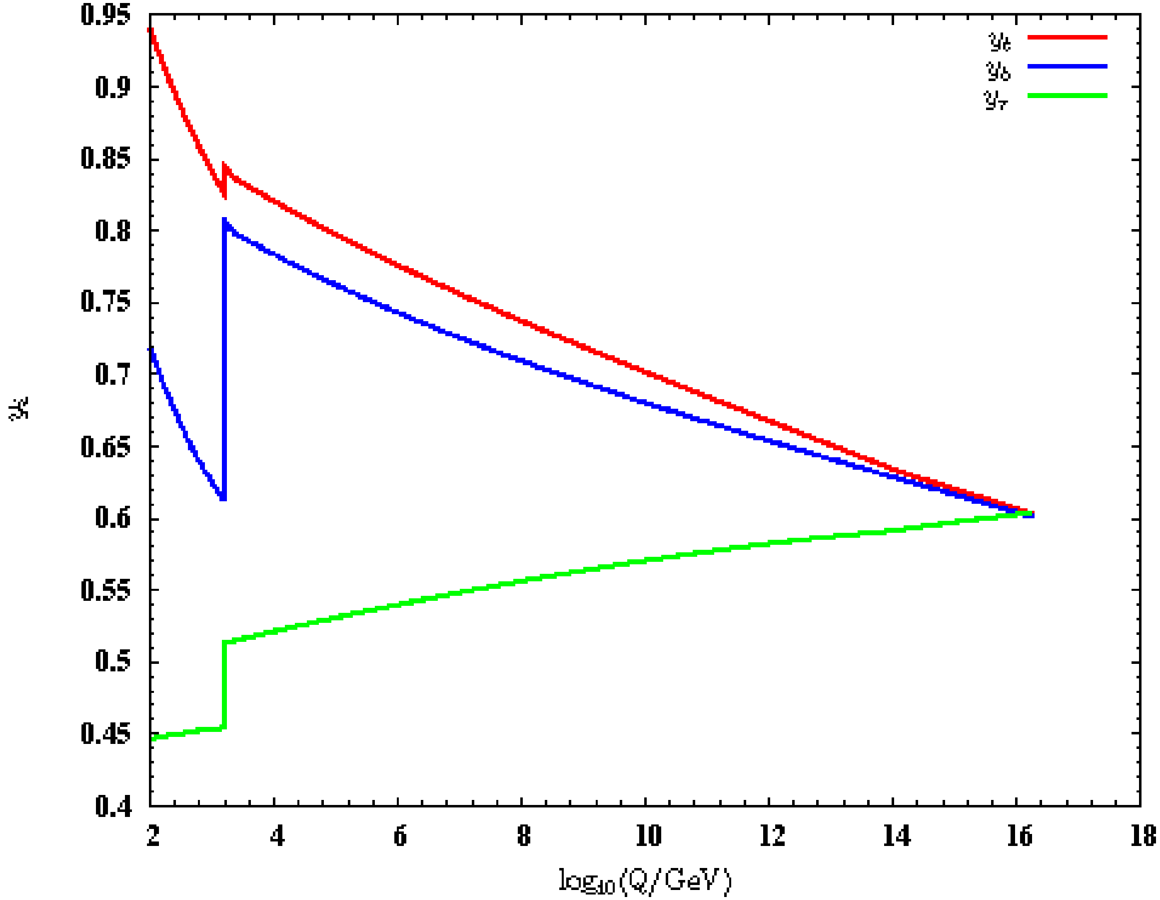
$$0 \leq m_{\frac{1}{2}} \leq 5\, TeV$$

$$35 \leq tan\beta \leq 55$$

$$-3 \leq \frac{A_0}{m_{16}} \leq 3$$

M. Adeel Ajaib, I. Gogoladze, Q. Shafi and C. S. Un, arXiv:1303.6964 [hep-ph].

- SUSY and $t - b - \tau$ Yukawa coupling unification



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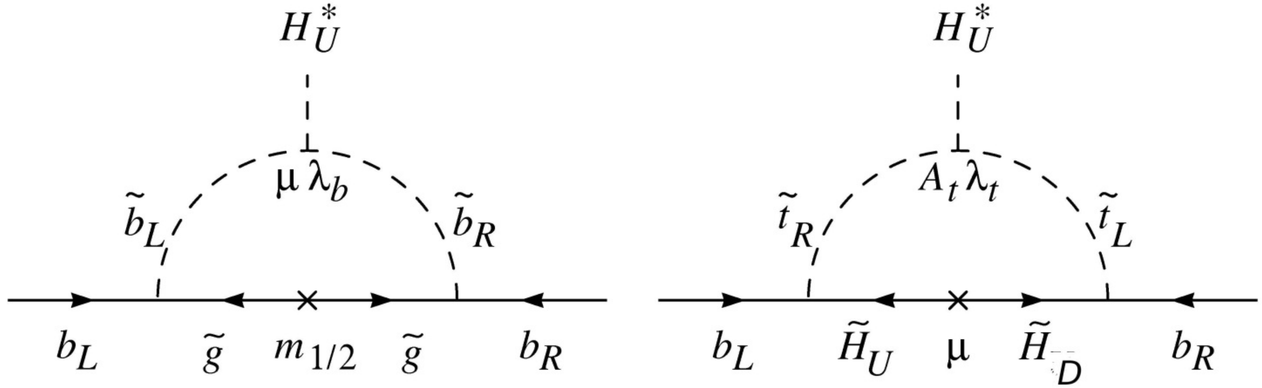
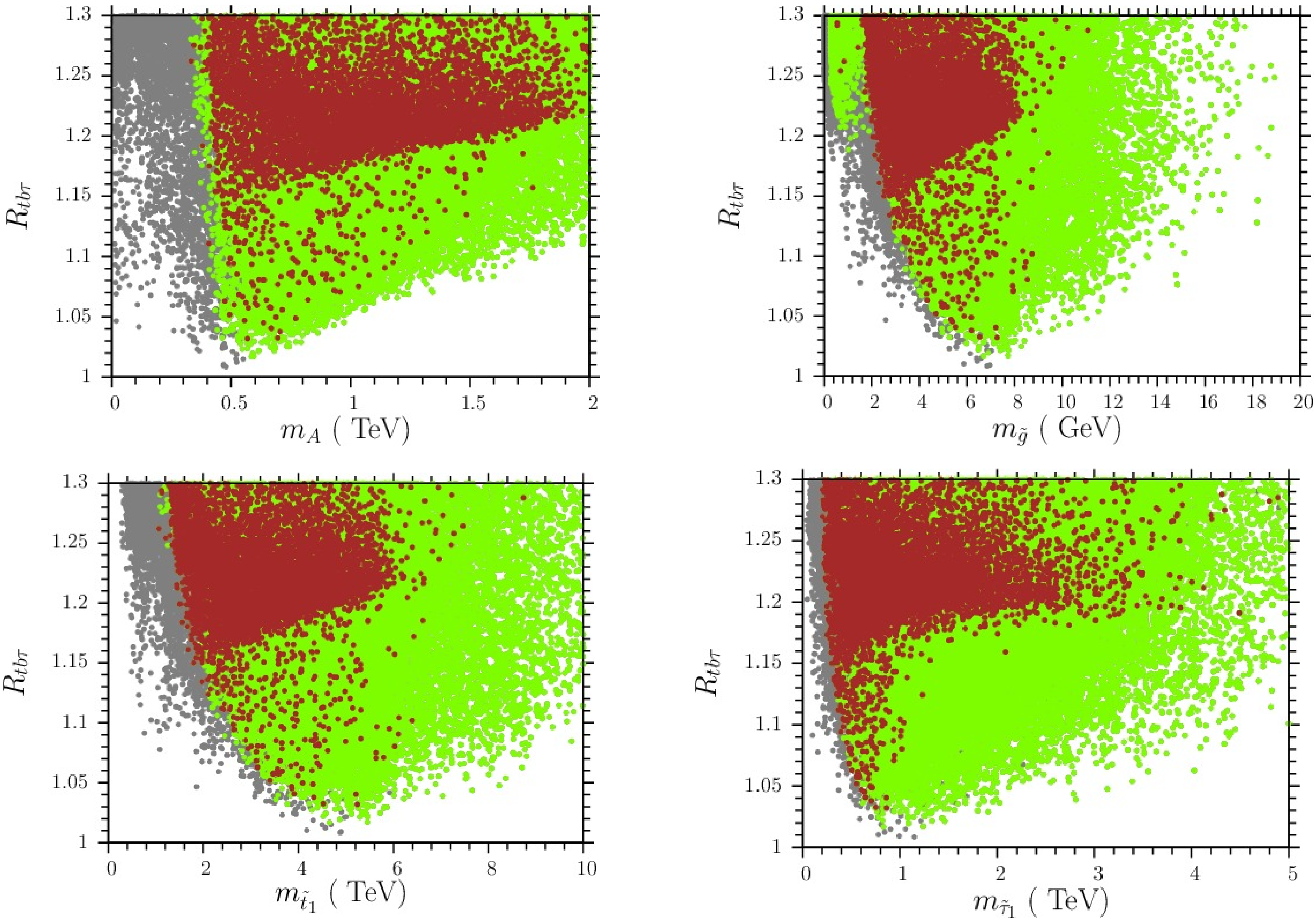
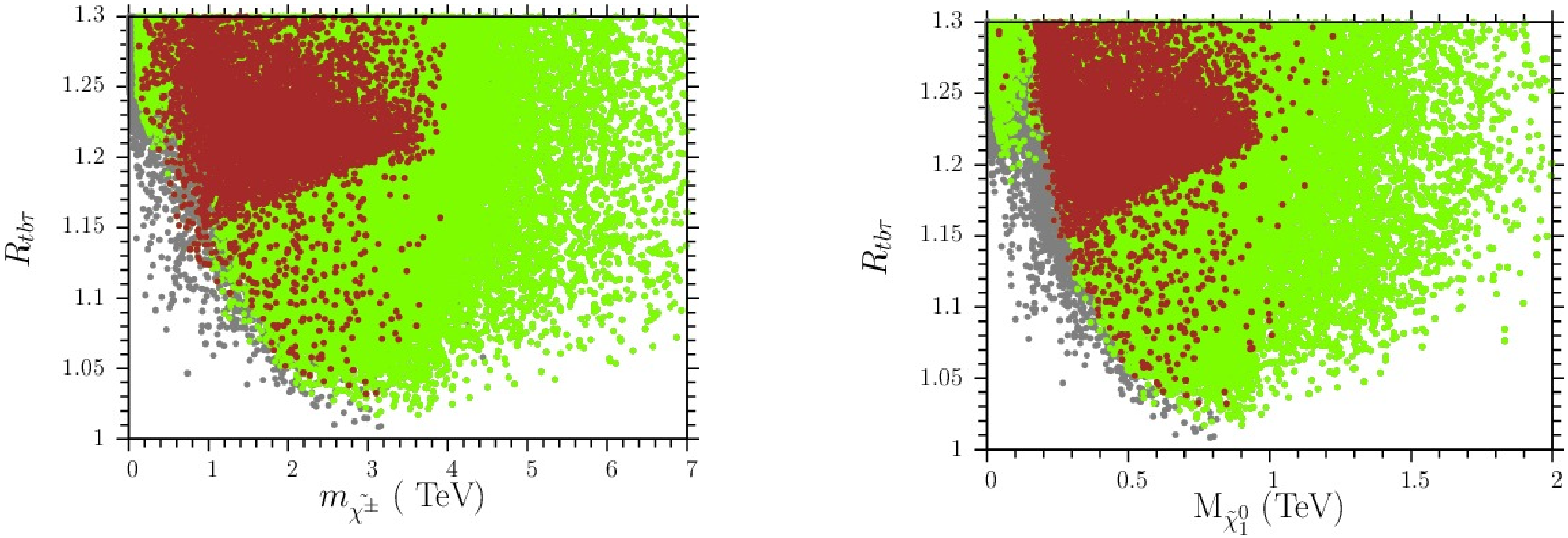
where $m_{\tilde{b}}$ and $m_{\tilde{t}}$ stands for sbottom and stop mass.



where $\lambda_b = y_b$ and $\lambda_t = y_t$



$R_{tb\tau}$
m_A (TeV)
$m_{\tilde{g}}$ (GeV)
$m_{\tilde{t}_1}$ (TeV)
$m_{\tilde{\tau}_1}$ (TeV)
1.3
1.25
1.2
1.15
1.1
1.05
1
0
0.5
1
1.5
2
4
6
8
10
12
14
16
18
20
3
5



$R_{tb\tau}$
$m_{\tilde{\chi}^{\pm}}$ (TeV)
$R_{tb\tau}$
$M_{\tilde{\chi}_1^0}$ (TeV)

The Lightest CP Even Higgs Boson Mass in the MSSM

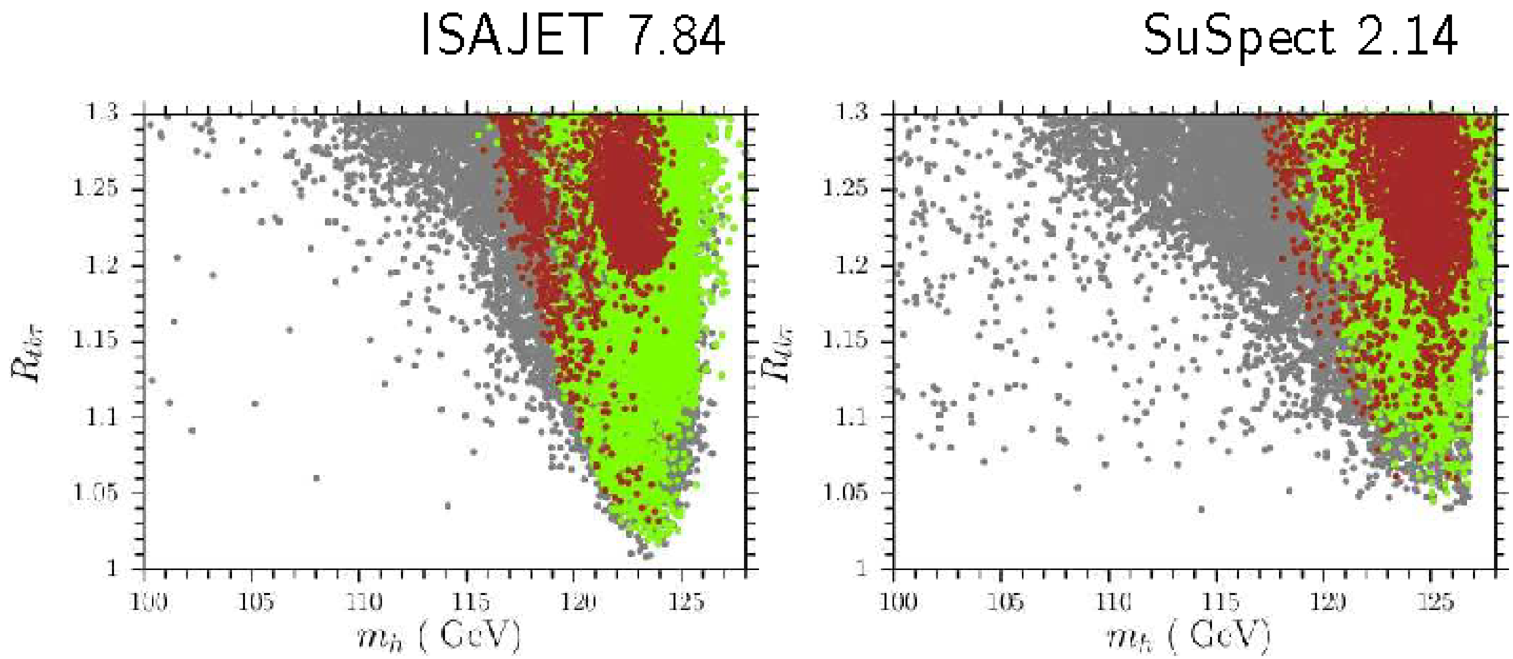
$$\mathrm{m_h^2} \approx \mathrm{M_Z^2} \cos^2 2\beta \left(1 - \frac{3}{8\pi^2} \frac{\mathrm{m_t^2}}{v^2} \, \mathrm{t} \right) + \frac{3}{4\pi^2} \frac{\mathrm{m_t^4}}{v^2} \left[\frac{1}{2} \widetilde{\mathrm{X}}_\mathrm{t} + \mathrm{t} \right.$$

$$\left. + \quad \frac{1}{16\pi^2} \left(\frac{3}{2} \frac{\mathrm{m_t^2}}{v^2} - 32\pi\alpha_3 \right) \left(\widetilde{\mathrm{X}}_\mathrm{t} \mathrm{t} + \mathrm{t}^2 \right) \right]$$

$$\mathrm{t} = \log \frac{\mathrm{M_{SUSY}^2}}{\mathrm{m_t^2}}, \qquad \widetilde{\mathrm{X}}_\mathrm{t} = \frac{2\mathrm{X_t^2}}{\mathrm{M_{SUSY}^2}} \left(1 - \frac{\mathrm{X_t^2}}{12\mathrm{M_{SUSY}^2}} \right)$$

$$\mathrm{X_t} = \mathrm{A_t} - \mu \cot\beta, \qquad M_{\mathrm{SUSY}} = \sqrt{m_{\tilde{t}_L} m_{\tilde{t}_R}}$$

Higgs mass from Yukawa coupling unification in SUSY

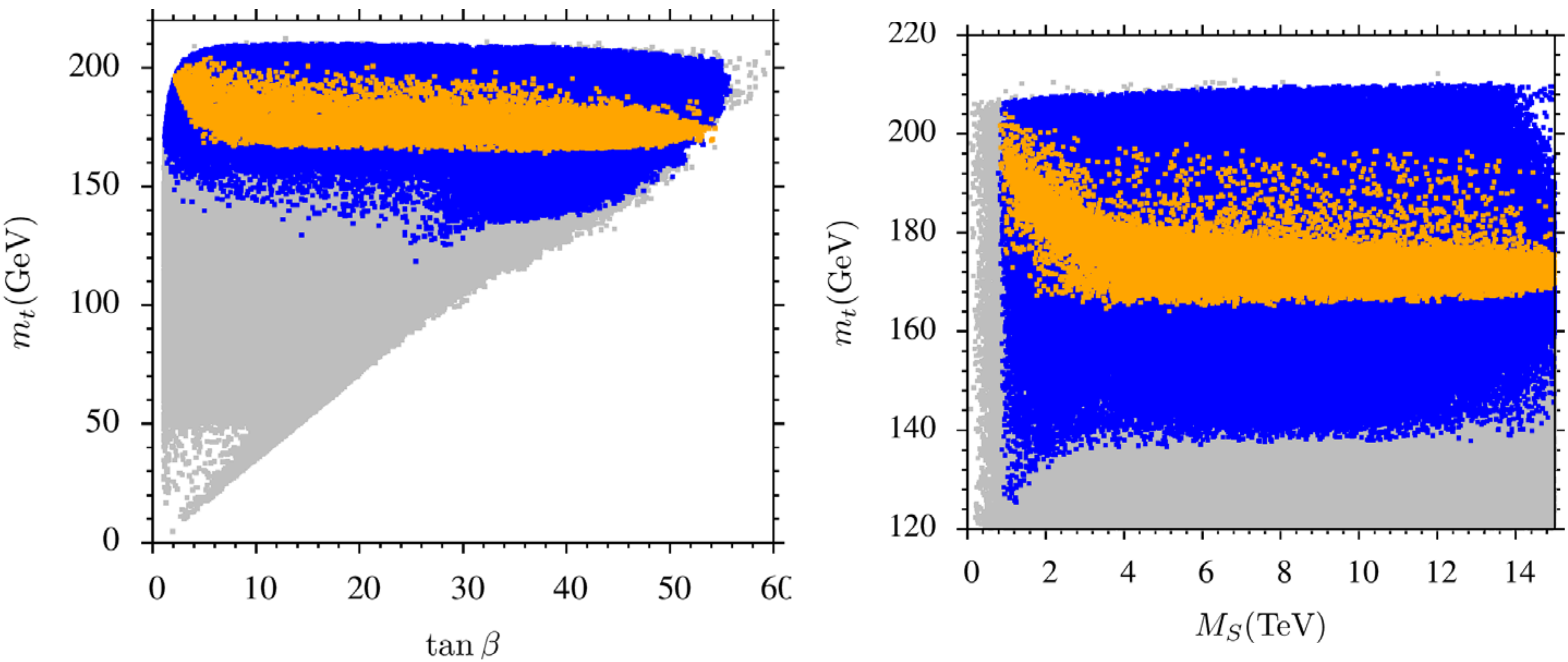


There is around 2 GeV theoretical error in the calculation of CP-even Higgs mass.

$m_h \approx 125.8 \pm 0.6$ GeV (CMS), $m_h \approx 126 \pm 0.6$ GeV (ATLAS)

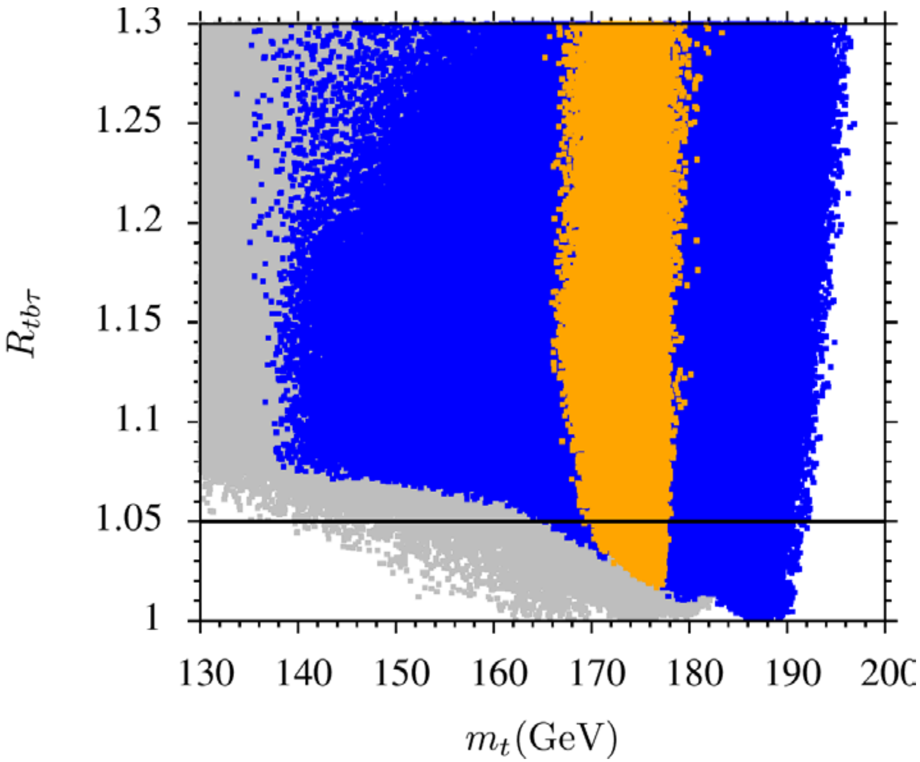
	Isajet	SuSpect	Isajet	Isajet
m_{10}	4.19×10^2	3.82×10^3	4.49×10^2	1.94×10^3
m_{16}	2.13×10^3	2.69×10^3	1.91×10^3	2.00×10^3
M_1	1.89×10^3	2.00×10^3	1.78×10^3	1.51×10^3
M_2	5.67×10^3	6.00×10^3	5.35×10^3	4.53×10^3
M_3	-3.78×10^3	-4.00×10^3	-3.57×10^3	-3.02×10^3
A_0/m_{16}	2.39	1.37	0.03	1.56
$\tan\beta$	47.18	48.05	47.93	47.46
m_t	174.2	173.1	174.2	173.1
μ	3729	1935	2913	2526
m_h	125	126	124	123
m_H	747	491	572	558
m_A	742	491	568	554
$m_{H^\pm}$	753	500	580	567
$m_{\tilde{\chi}^0_{1,2}}$	895, 3739	955, 1935	848, 2932	709, 2540
$m_{\tilde{\chi}^0_{3,4}}$	3742, 4822	1936, 5043	2935, 4562	2543, 3849
$m_{\tilde{\chi}^\pm_{1,2}}$	3789, 4774	1934, 5043	2978, 4516	2579, 3809
$m_{\tilde{g}}$	7694	7673	7266	6239
$m_{\tilde{u}_{L,R}}$	7667, 6824	8112, 7245	7219, 6415	6295, 5635
$m_{\tilde{t}_{1,2}}$	5331, 6560	5604, 6839	5239, 6367	4390, 5370
$m_{\tilde{d}_{L,R}}$	7668, 6814	8112, 7236	7220, 6406	6296, 5628
$m_{\tilde{b}_{1,2}}$	5553, 6526	5870, 6870	5434, 6333	4591, 5341
$m_{\tilde{\nu}_{1,2}}$	4148	4590	3870	3487
$m_{\tilde{\nu}_3}$	3898	4234	3641	3243
$m_{\tilde{e}_{L,R}}$	4153, 2234	4590, 2780	3875, 2009	3491, 2068
$m_{\tilde{\tau}_{1,2}}$	1094, 3875	1140, 4235	881, 3620	1061, 3225
$R_{tb\tau}$	1.02	1.05	1.03	1.04

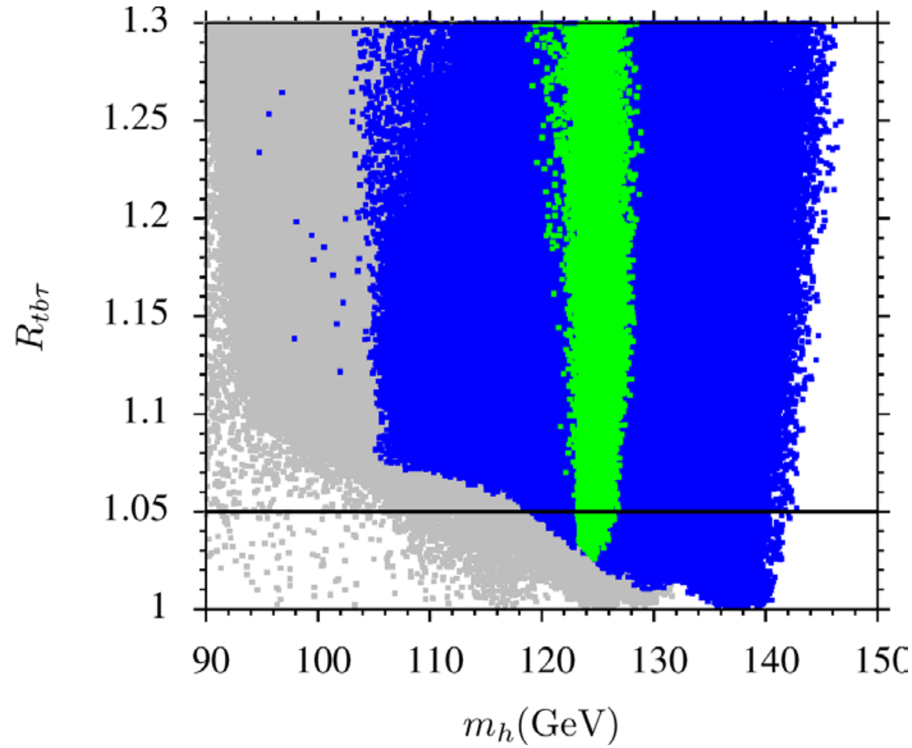
Top quark and Higgs boson masses



$0 < m_t < 220$ GeV

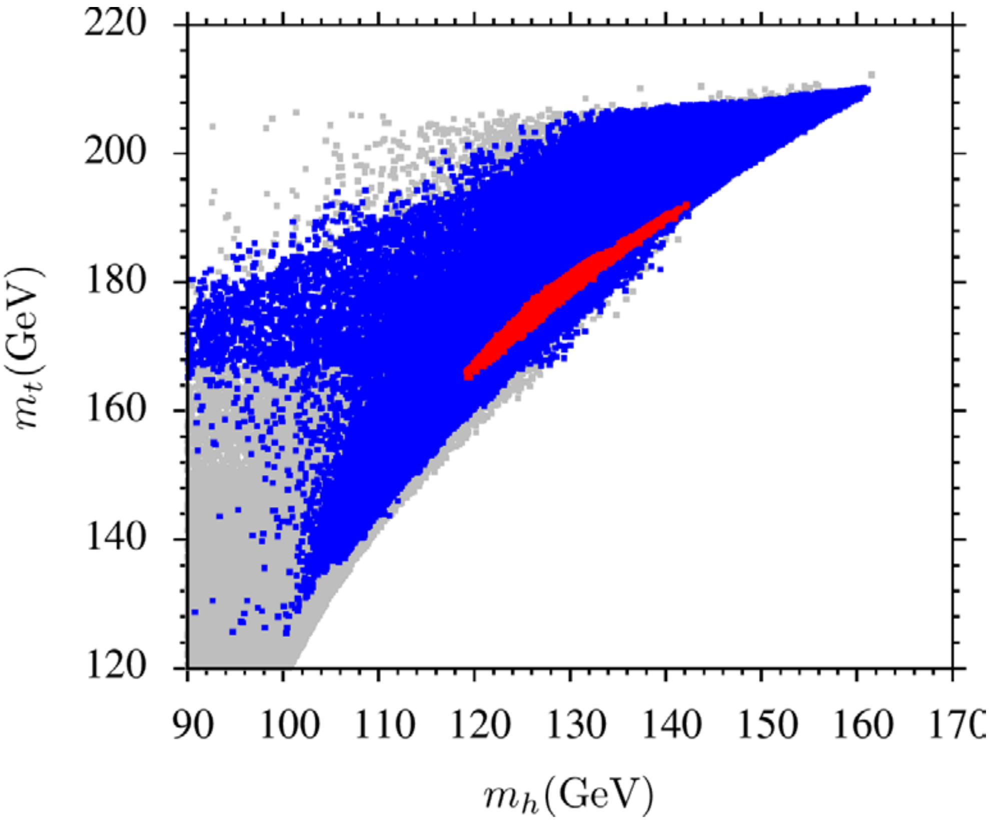
Top quark and Higgs boson masses

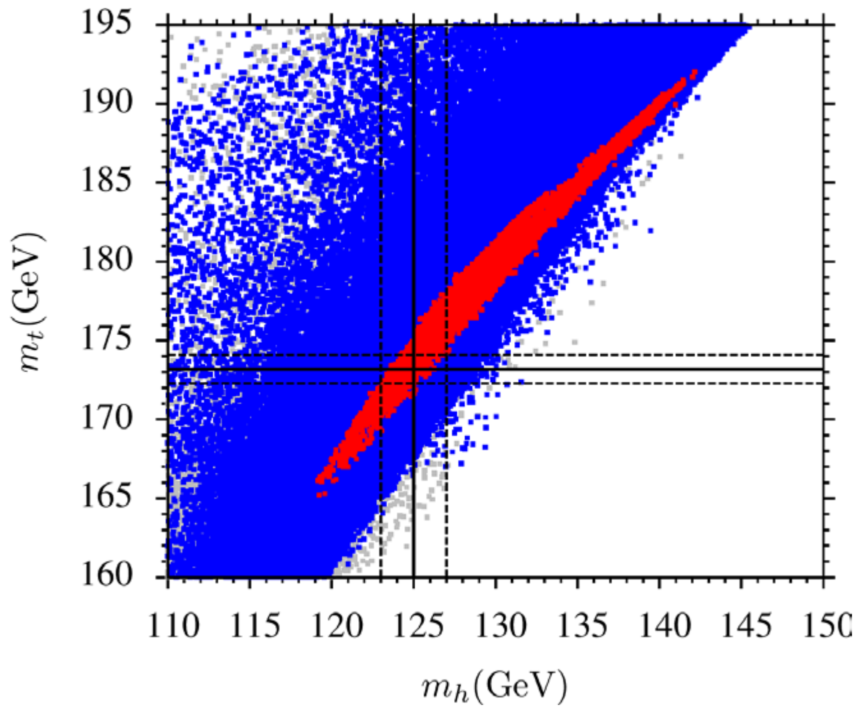




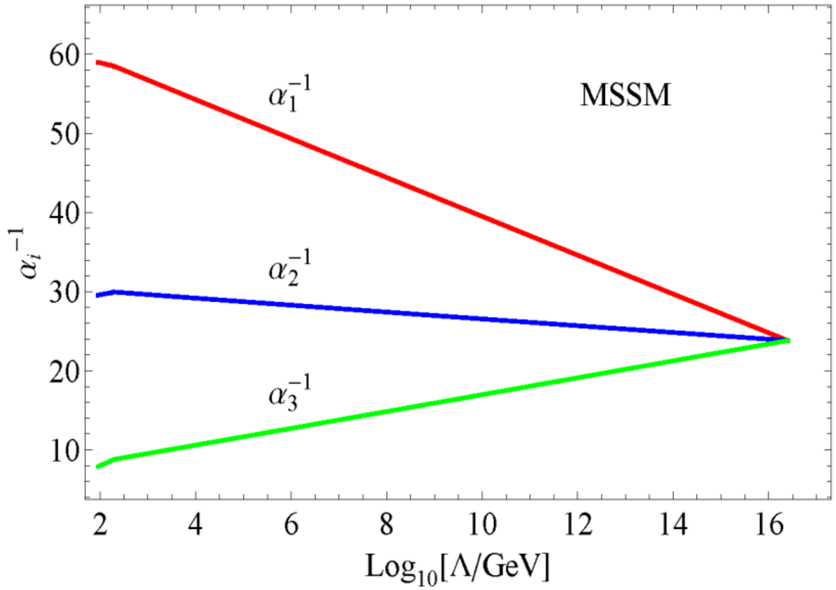
0 < m_t < 220 GeV

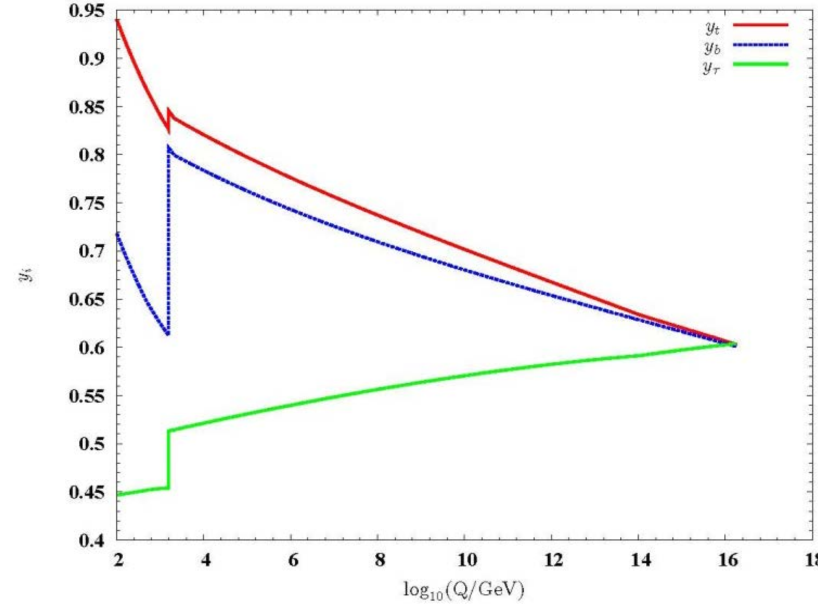
Top quark and Higgs boson masses

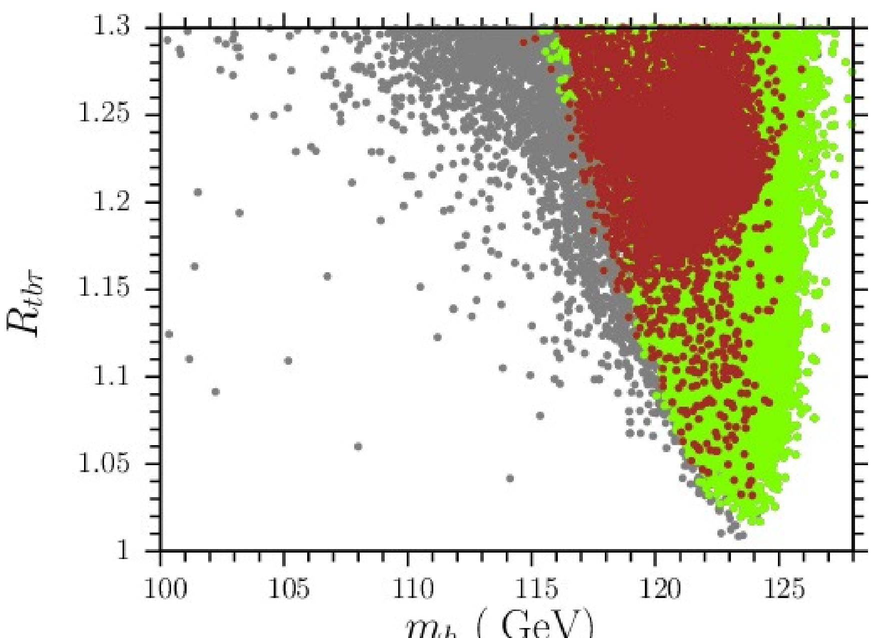


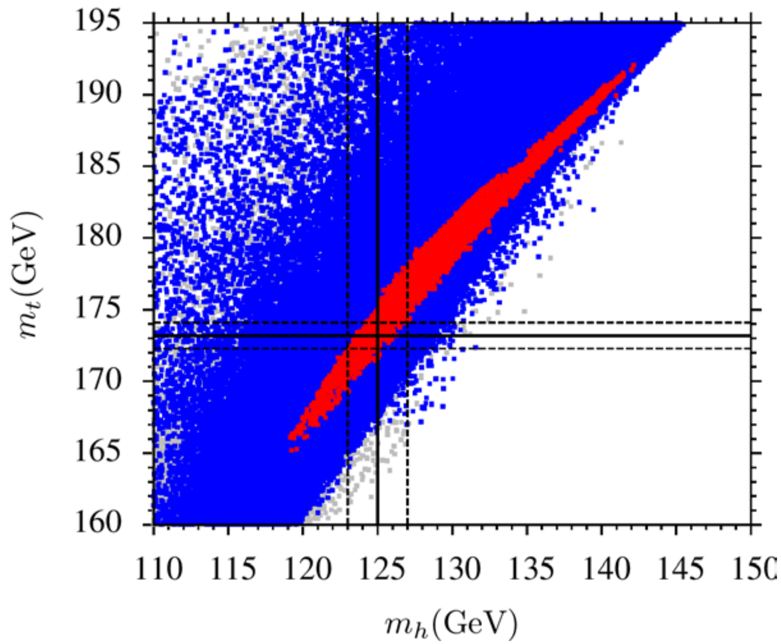


SUSY SO(10) GUT









Muchas Gracias !

Thank You !