

Quantum chromodynamics in the LHC era

Pavel Nadolsky

Department of Physics
Southern Methodist University (Dallas, TX)

December 8, 2014
Lecture 1

Objectives of these lectures

- Introduce basic theoretical methods in quantum chromodynamics (QCD)
 1. perturbation theory for hard scattering
 2. determination of nonperturbative QCD functions
- convey the richness of ideas encountered in modern QCD – contributed by diverse branches of theory, experiment, and mathematics

Selection of topics and publications is far from complete – my apologies!
Complementary material can be found in lectures at 2014 CTEQ school in Beijing (www.cteq.org) and 2014 CTEQ-DESY workshop “Proton structure in the LHC era” (<http://bit.ly/1vuvpGK>).

QCD is fascinating and important

QCD is the only **non-Abelian quantum field theory** that can be experimentally tested in several phases. It is also the base theory for the majority of measurements at the LHC and other facilities.

Since its inception in 1973 by Gross, Wilczek, and Politzer, perturbative QCD has developed into a precise theory that will soon predict the key LHC cross sections with about **1% accuracy**.

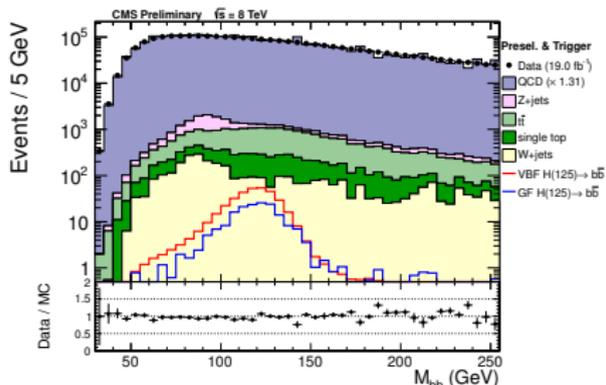
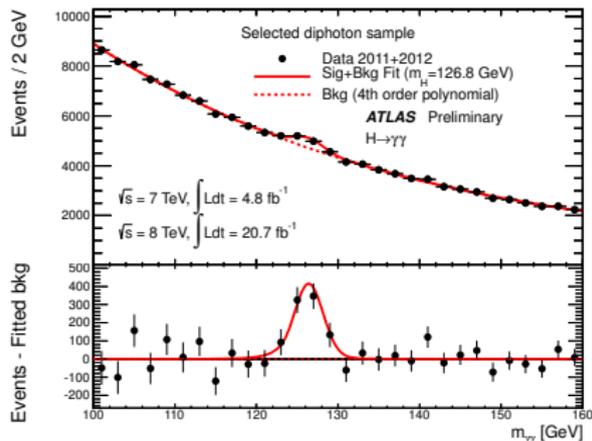
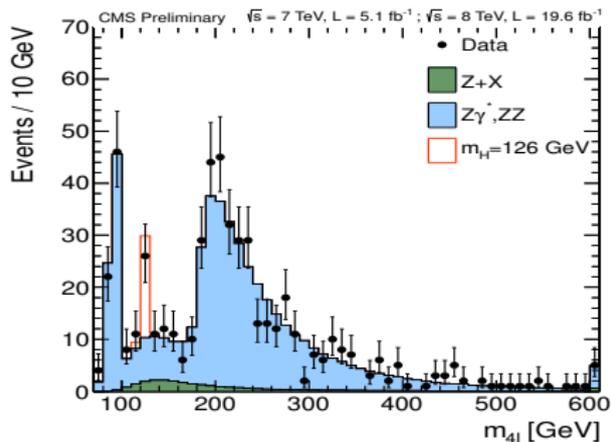
We will have difficulty understanding new physics if we don't understand QCD.

Higgs boson discovery in world news



The quick discovery of the Higgs boson resulted from precise understanding of hadronic interactions, which is also essential for future LHC measurements.

Higgs searches require accurate QCD calculations



Continuous backgrounds are large in most Higgs searches; must be predicted with accuracy of $< 5 - 10\%$ in order to identify the nature of Higgs boson

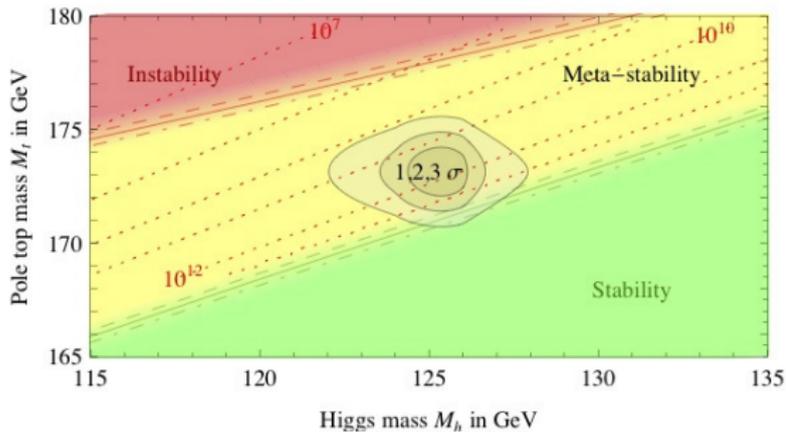
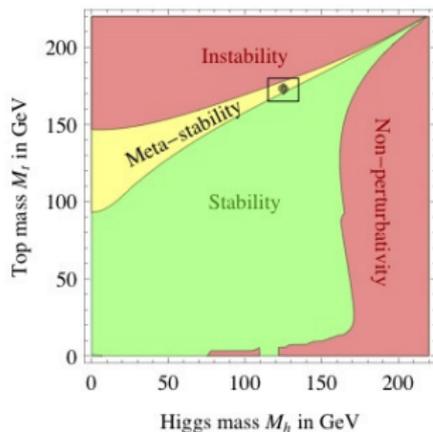
Which Higgs mechanism is it?

Now that the candidate Higgs particle is discovered, an ambitious program is underway to pin down the mechanism of electroweak breaking. It will require a combination of precision measurements of...

- ... Higgs mass and couplings of the Higgs excitation to SM particles;
- ... masses M_W and M_t of W boson and t quarks, sensitive to the interactions with Higgs via loop effects;

... and searches for sequential Higgs resonances predicted by supersymmetry and other new physics models.

Meta-stability of vacuum in the Standard Model

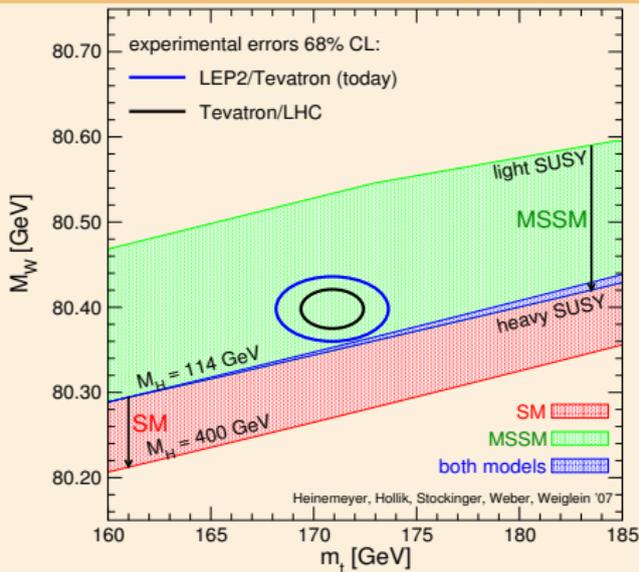


In the absence of non-SM particles, stability of EM vacuum at high scales can be estimated from the measured values of $M_H = 125.7 \pm 0.4$ GeV, $M_t^{pole} = 176 \pm 4$ GeV, and $M_W = 80.385 \pm 0.015$ GeV. With the current M_H , M_t , and M_W , the vacuum is predicted to be meta-stable at about 10^{12} GeV. M_t must be measured to about 1 GeV to confidently conclude on the vacuum stability.

Constraints on the supersymmetric parameter space

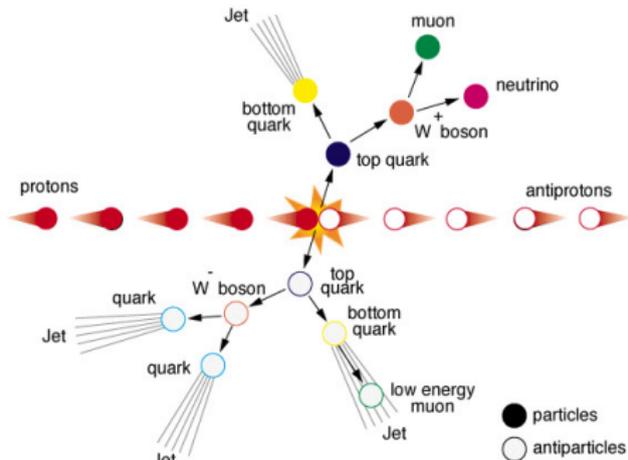
Precision measurements on M_H , M_W , M_t can distinguish between SM and its popular extensions, such as supersymmetry

EW fits + direct Higgs searches



SM band: $114 \leq M_H \leq 400$ GeV
 SUSY band: random scan

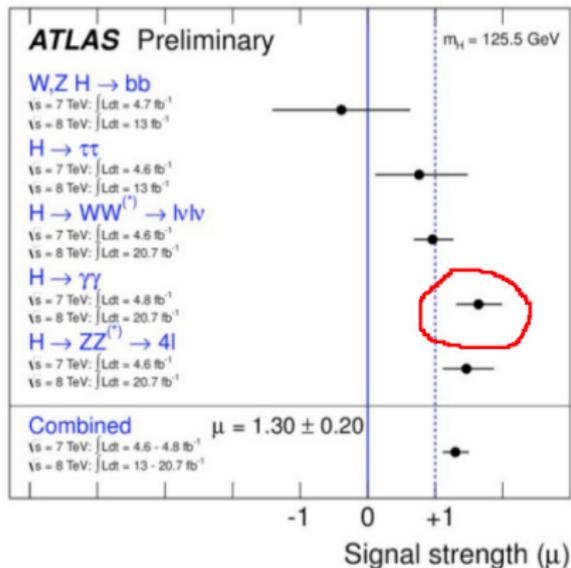
Determination of M_t, M_W is highly non-trivial: even simplest processes involve multiple particle production.



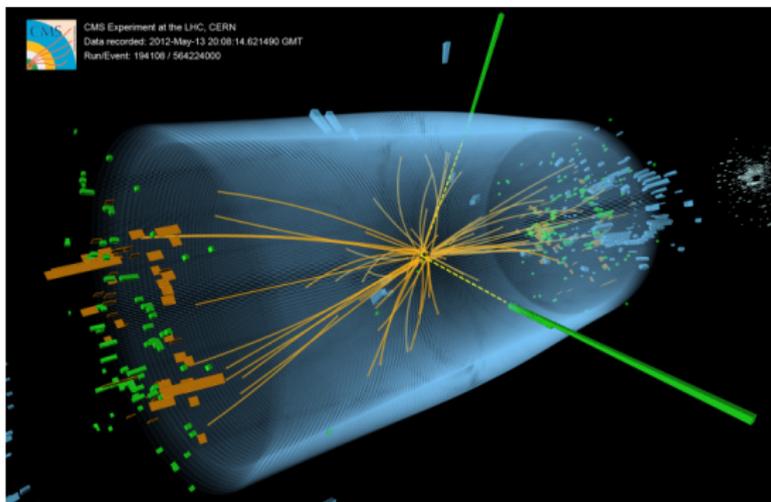
A Top Antitop Quark Event from the D-Zero Detector at Fermilab

Does the total rate in $H \rightarrow \gamma\gamma$ exceed the SM prediction?

NNLO

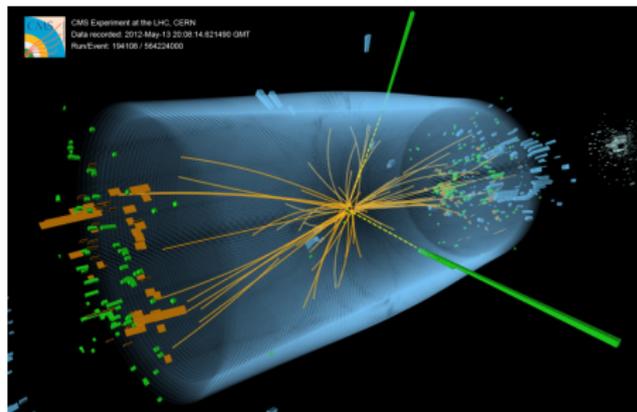


Production of a Higgs boson and decay into two high- p_T photons

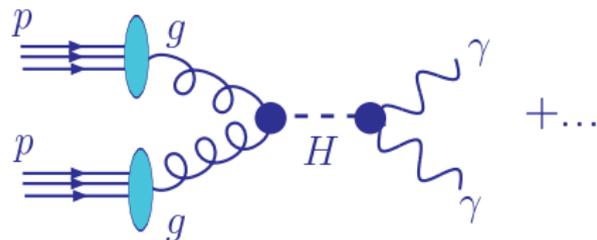


Long green lines indicate isolated γ 's, selected to be away from prominent hadronic activity. But there are still some soft particles around each $\gamma \Rightarrow$ enhanced radiative contributions from all α_s orders \Rightarrow must be evaluated using all-order resummation or a showering program

QCD calculations for LHC processes



A $H \rightarrow \gamma\gamma$ event at CMS



The lowest-order Feynman diagram

The simplest calculation you can set up

Question to the audience (1 minute)

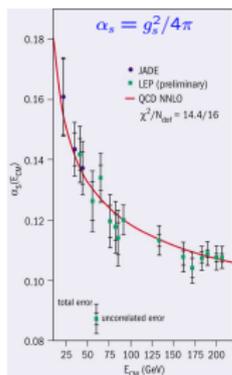
Which features of QCD make perturbative calculations possible? Suggest 2-3 features.

Essential concepts of QCD

1. **Asymptotic freedom** of quarks and gluons at **large energy (short distance)**
2. **Confinement** of quarks and gluons at **small energy (large distance)**
3. **Infrared safety** of some QCD observables
4. **Factorization** of high-energy and low-energy contributions

1. Asymptotic freedom of strong interactions

- Strong interactions are extremely intensive at small energies; weaken at large energies



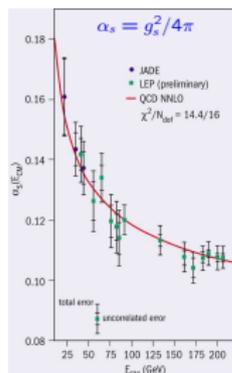
- At $E > 1 \text{ GeV}$, the proton or another **hadron** (bound state) is a loosely bound system of **partons** (quarks and gluons)



- hard scatterings of partons are independent from one another
- probability of emissions quickly reduces with the number of emitted particles \Rightarrow is described by **perturbation theory**

2. Confinement

- Strong interactions are extremely intensive at small energies; weaken at large energies



- At $E < 1 \text{ GeV}$, **partons** clump together because of increasing strength of interaction and phase transitions



- Probability of partonic emissions grows with the number of emitted particles \Rightarrow requires **non-perturbative computations**

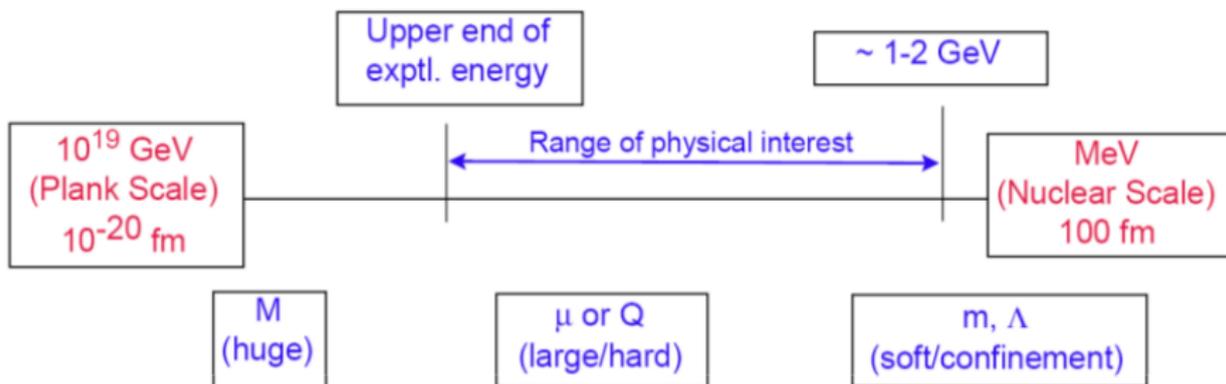
Simple visualization: colored quarks and gluons

Atom \Rightarrow Nucleus \Rightarrow Nucleon \Rightarrow Partons



As the resolution of the microscope (energy of the probing field) increases, **colored** quarks and gluons are observed inside **colorless** systems

The importance of *Scales* -- Renormalization and Factorization



Bare QCD amplitudes are singular **both** in UV and IR limits. “Effective field theory” and renormalization group analysis quantify the UV and IR contributions by introducing the scale-dependent coupling $\alpha_s(\mu)$ and nonperturbative functions $f_a(x, \mu)$.

What to do with the long-distance physics associated with colinear/soft singularities in PQCD?

1st strategy:

Identify physical observables which are insensitive to the singularities! (Infra-red-safe (IRS) quantities)

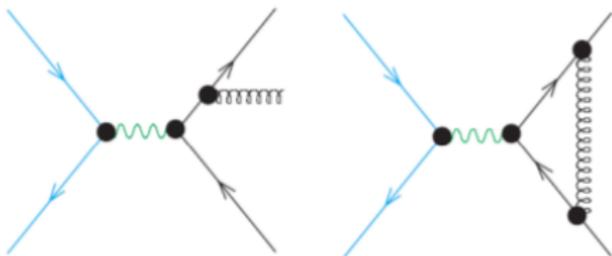
Total Hadronic Cross-section (*inclusive*):

$$\sigma_{tot}(s) = \sigma_0(s) [1 + \alpha_s(s) c_1 + \dots]$$

Kinoshita-Lee-Nauenberg theorem: c_i are finite, i.e. IRS (unitarity)

Order α_s :

Cancellation of the colinear/soft singularities between real and virtual diagrams



Infra-Red-Safe observables:

Total hadronic Cross-section $\sigma_{\text{tot}}/\sigma_{\mu+\mu-}$

Sterman-Weinberg jet cross-sections and their modern variations (*Jade*-, *Durham*-, ... algorithms);

Jet shape observables: Thrust, ... ;

energy-energy correlation ;

Essential feature of a general IRS physical quantity:

the observable must be such that it is insensitive to whether n or $n+1$ particles contributed -- if the $n+1$ particles has n -particle kinematics



e.g. a IRS "jet algorithm"

σ and R in e^+e^- Collisions

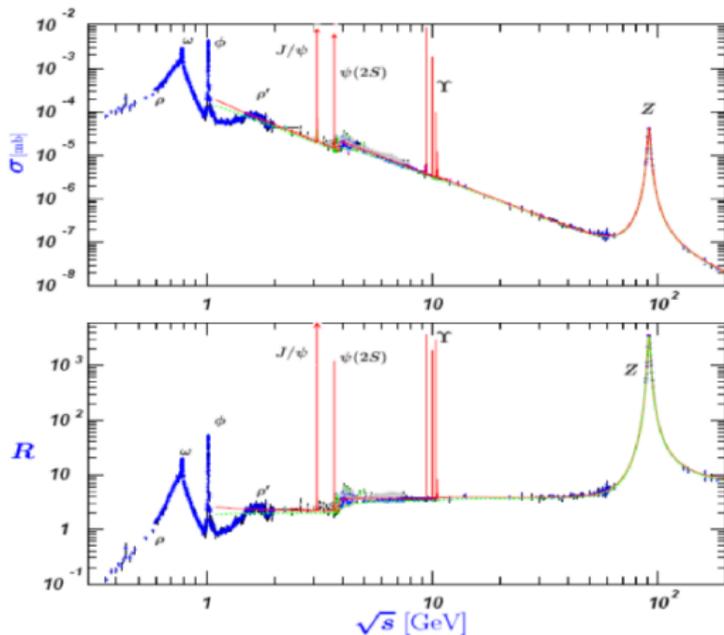
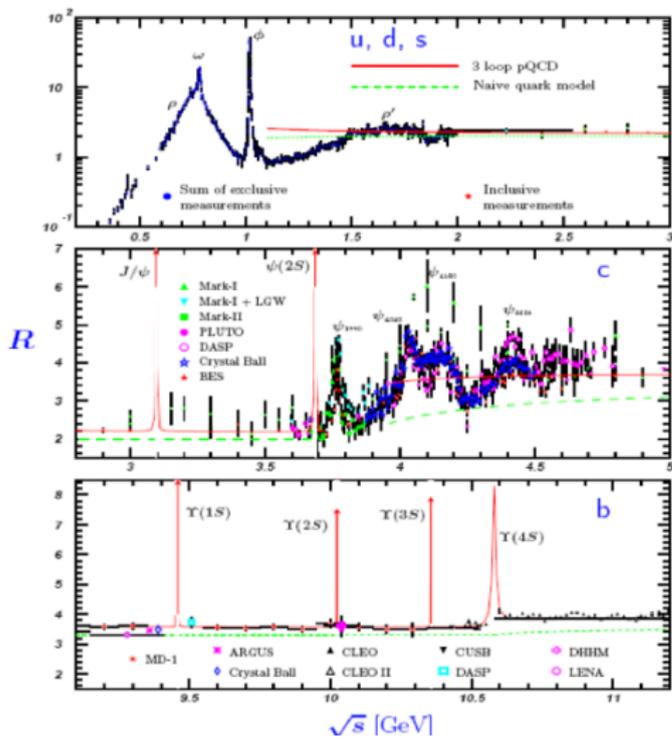


Figure 40.9: World data on the total cross section of $e^+e^- \rightarrow \text{hadrons}$ and the ratio $R(s) = \sigma(e^+e^- \rightarrow \text{hadrons}, s) / \sigma(e^+e^- \rightarrow \mu^+\mu^-, s)$. $\sigma(e^+e^- \rightarrow \text{hadrons}, s)$ is the experimental cross section corrected for initial state radiation and electron-positron vertex loops, $\sigma(e^+e^- \rightarrow \mu^+\mu^-, s) = 4\pi\alpha^2(s)/3s$. Data errors are total below 2 GeV and statistical above 2 GeV. The curves are an educated guess: the broken one (green) is a naive quark-parton model prediction and the solid one (red) is 3-loop pQCD production (see “Quantum Chromodynamics” section of this Review, Eq. (9.12) or, for more details, K. C. Chetyrkin et al., Nucl. Phys. B596, 58 (2000) [Erratum ibid. B634, 413 (2002)], Breit-Wigner parameterizations of J/ψ , $\psi(2S)$, and $\Upsilon(nS)$, $n = 1, 2, 3, 4$ see also shown. The full list of references to the original data and the details of the R ratio extraction from them can be found in [arXiv:hep-ph/0912114]. Corresponding computer-readable data files are available at <http://pdg.lbl.gov/2008/contents.html>. (Courtesy of the COMPAS(Protvino) and HEPDATA(Durham) Groups, August 2009. Corrections by P. Janot (CERN) and M. Schmitt (Northwestern U.) See full-color version on color page at end of book.

R in Light-Flavor, Charm, and Beauty Threshold Regions



The 2nd strategy:

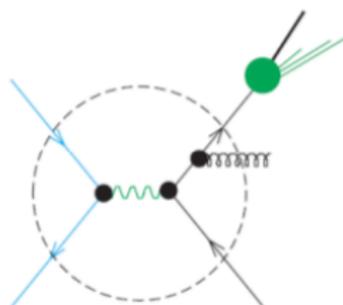
Factorization



QCD Parton model

Factorize the physical observable into a calculable *IRS* part and a non-calculable but *universal* piece.

Example: One particle inclusive cross-section



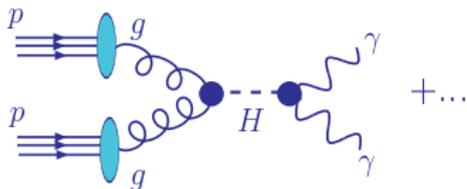
Fragmentation function:
Long-distance physics;
Universal.

Hard scattering:
Short distance physics;
IRS, perturbatively cal.

$$\sigma(s, z) = \int_z^1 \frac{d\zeta}{\zeta} \hat{\sigma}^a\left(\frac{s}{\mu}, \frac{z}{\zeta}, \alpha_s(\mu)\right) \cdot D_a(\zeta, \mu)$$

Example 1: QCD factorization for $H \rightarrow \gamma\gamma$ process

A. Cross section $\sigma_{pp \rightarrow H \rightarrow \gamma\gamma}$ for production and decay of H , e.g. via $g + g \rightarrow H$; at lowest order in g_s

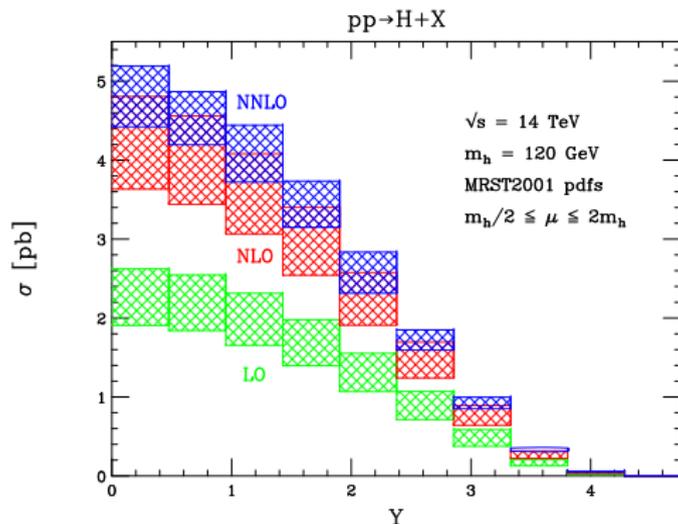


$$\sigma_{pp \rightarrow H \rightarrow \gamma\gamma} = \sigma_{gg \rightarrow H \rightarrow \gamma\gamma} f_{g/p}(x_1, M_H) f_{g/p}(x_2, M_H) + \dots$$

- $\sigma_{gg \rightarrow H \rightarrow \gamma\gamma}$ is the cross section for scattering of two gluons; can be computed as a perturbation series in g_s , at least formally
- $f_{g/p}(x, \mu)$ is the probability to find a gluon g with momentum $x\vec{P}$ in a proton with momentum \vec{P} ($|\vec{P}| \approx E \approx \mu > 1 \text{ GeV}$); $f_{g/p}(x, \mu)$ is nonperturbative (no full calculation yet)

NNLO predictions for $gg \rightarrow H$

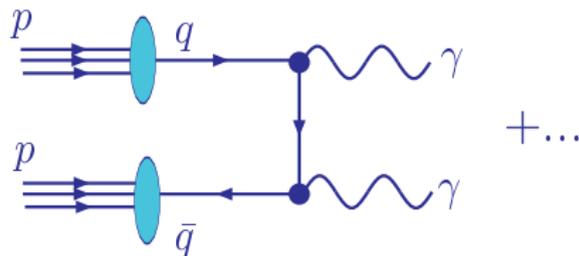
Anastasiou, Melnikov, Petriello, 2002-05



In $gg \rightarrow \text{Higgs}$, convergence of the series in α_s is relatively slow. NNLO computations and/or NNLL resummations are mandatory.

Example 2: Factorization for the $\gamma\gamma$ background

B. Cross section (probability) $\sigma_{pp \rightarrow \gamma\gamma}$ for $pp \rightarrow \gamma\gamma$ via conventional channels, at the lowest order in g_s

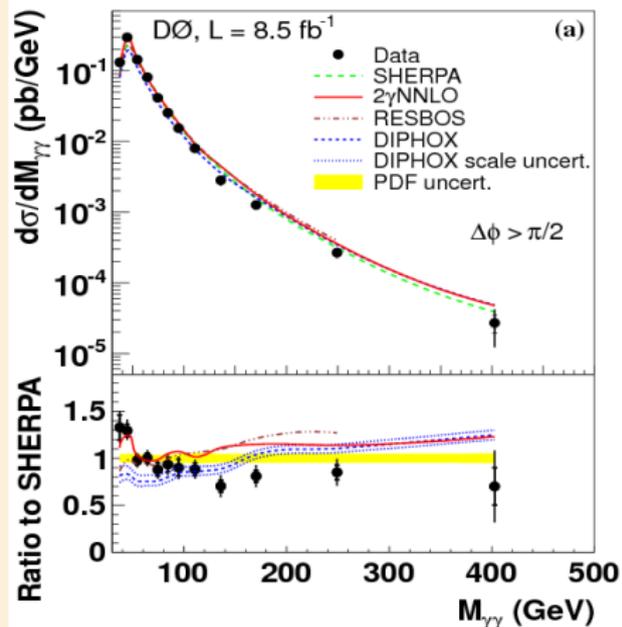


$$\sigma_{pp \rightarrow \gamma\gamma} = \sum_{q=u,d,s,\dots} [\sigma_{q\bar{q} \rightarrow \gamma\gamma} f_{q/p}(x_1) f_{\bar{q}/p}(x_2) + (q \leftrightarrow \bar{q})] \dots$$

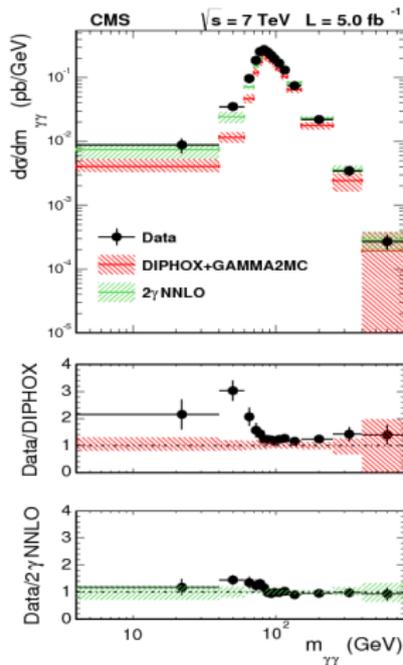
- $\sigma_{q\bar{q} \rightarrow \gamma\gamma}$ ($\sigma_{gg \rightarrow H \rightarrow \gamma\gamma}$) is the cross section for $q\bar{q}$ scattering; **perturbative!**
- $f_{q/p}(x, \mu)$ is the probability to find a quark q in the proton; **nonperturbative!**
- Other scattering channels (“...”) are **formally** suppressed by g_s

Invariant mass distributions of $\gamma\gamma$ pairs

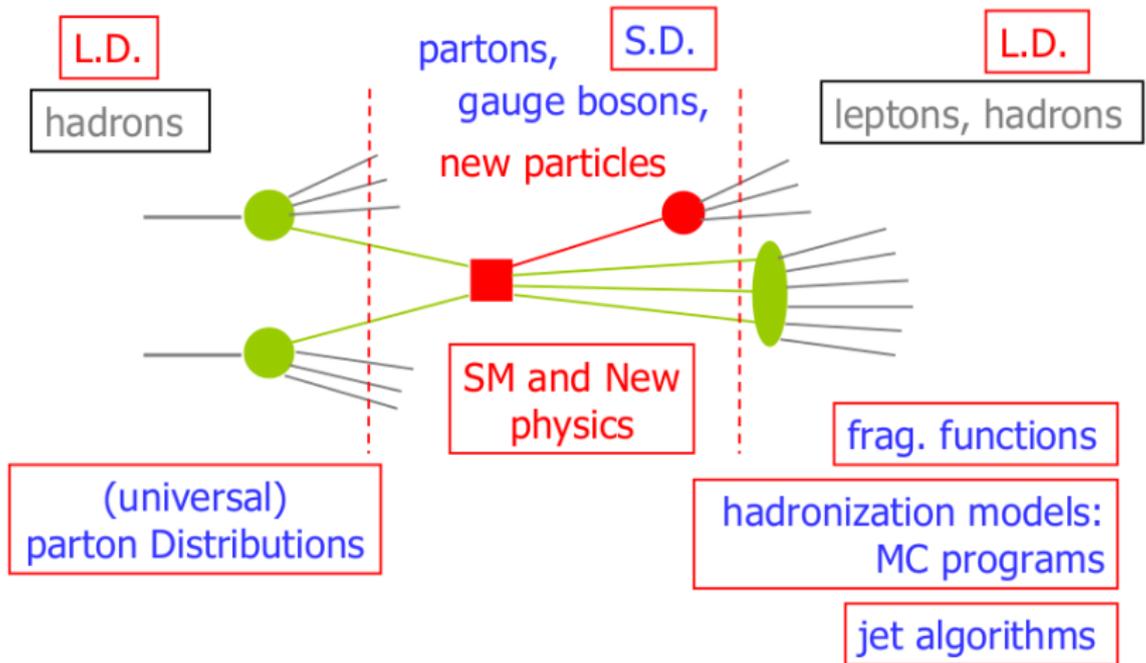
D0-Run2, PLB725 (2013) 6



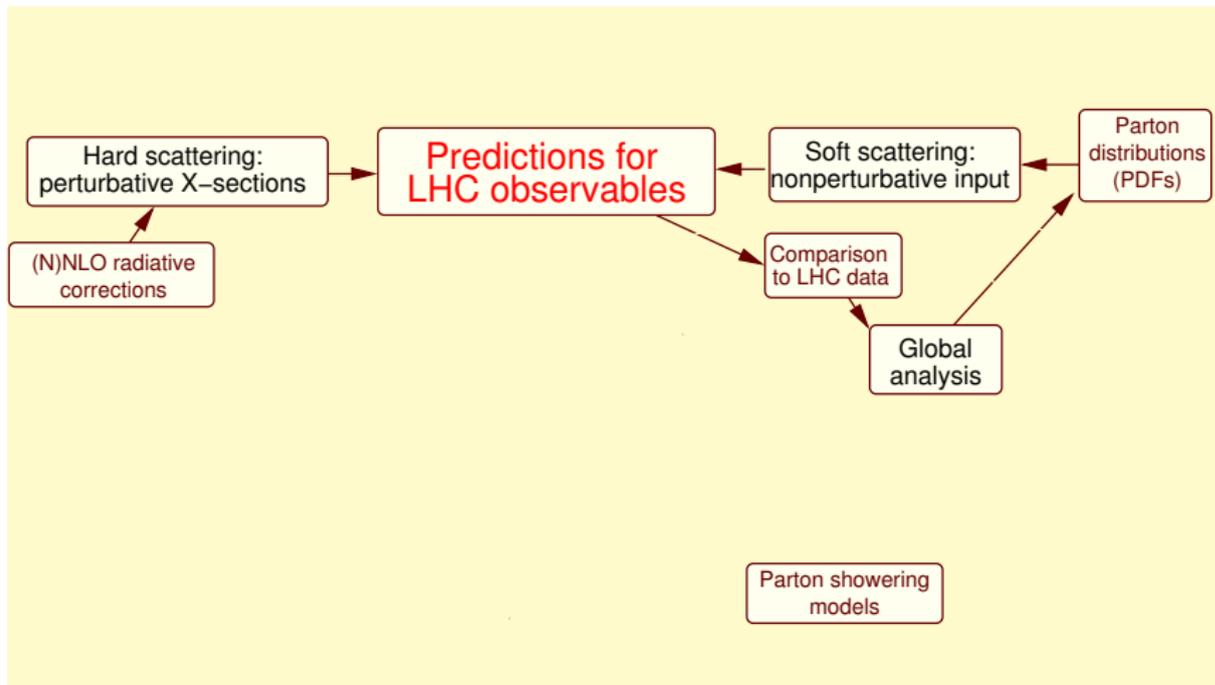
CMS, Eur.Phys.J. C74 (2014) 11



Data vs. theory up to NNLO (2γ NNLO). D0 cuts out poorly controlled QCD contributions from $\Delta\phi < \pi/2$. No such cut is applied by CMS.

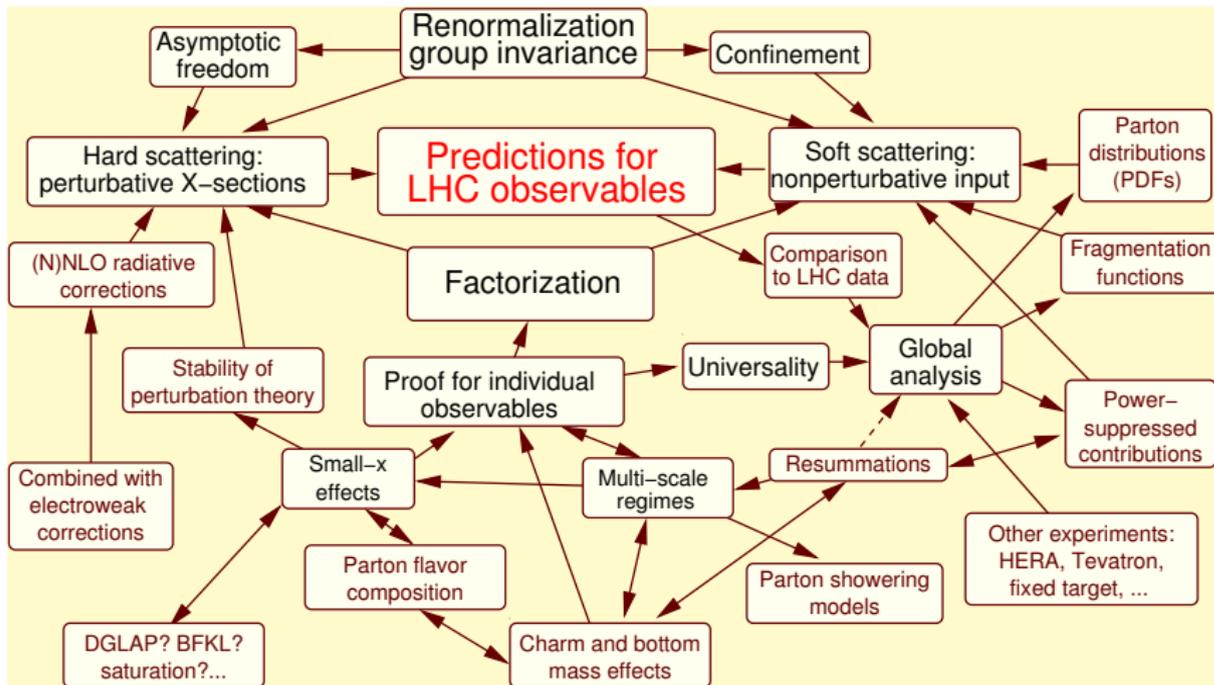


Factorization of QCD cross sections



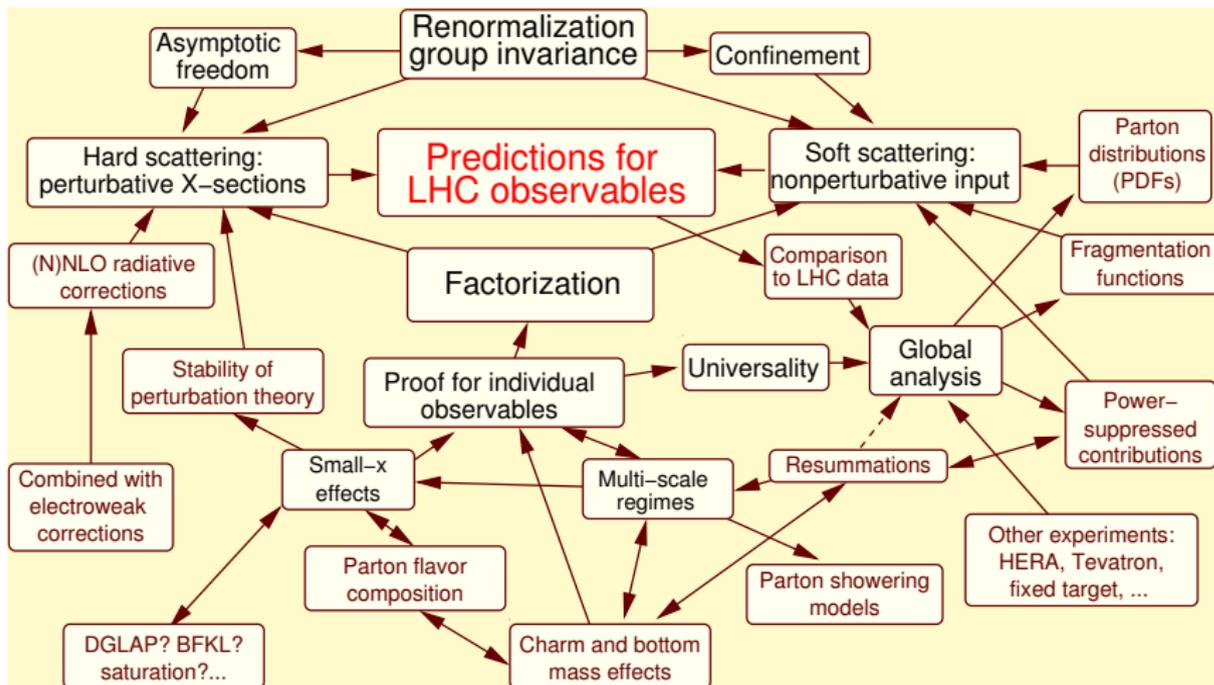
The very basic picture

Factorization of QCD cross sections



The full underlying theory is very rich

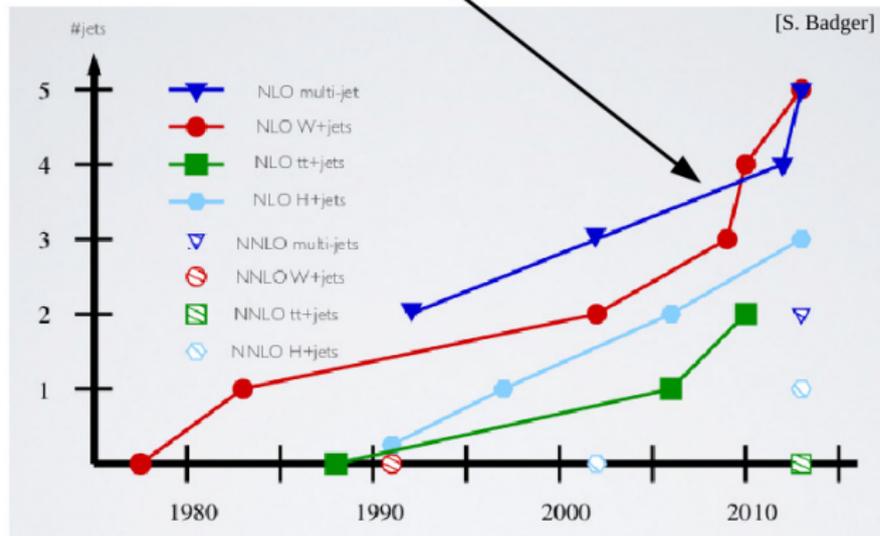
Factorization of QCD cross sections



Accuracy of hard QCD cross sections must be **matched** by the accuracy of PDFs

Perturbative QCD loop revolution

The NLO Revolution



# of jets	# 1-loop Feynman diagram
1	11
2	110
3	1,253
4	16,648
5	256,265

[L.Dixon]

Since 2005, “generalized unitarity” and related methods dramatically advanced the computations of **perturbative** NLO/NNLO/N3LO hard cross sections.

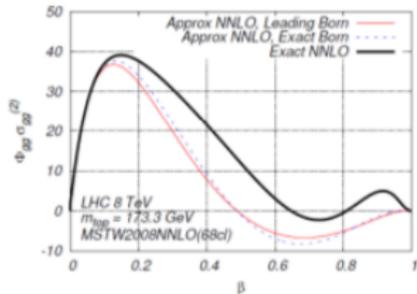
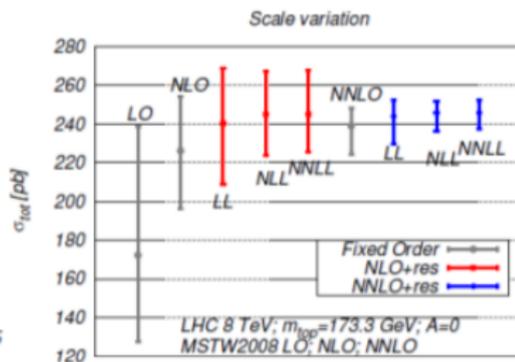
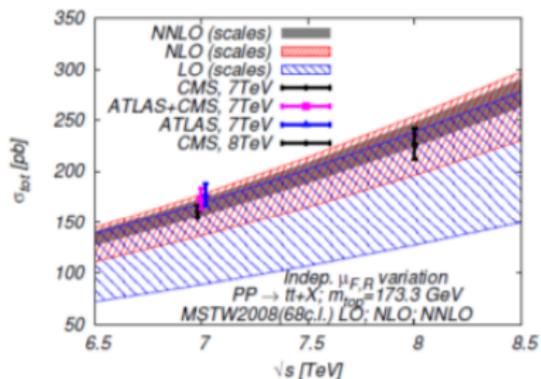
[S.Badger]

recent NNLO progress



$pp \rightarrow \gamma\gamma$	[Catani, Cieri, de Florian, Ferrera, Grazzini (2011)]
$pp \rightarrow WH$	[Ferrera, Grazzini, Tramontano (2011)]
$gg \rightarrow gg$	[Currie, Gehrmann de Ridder, Gehrmann, Glover, Pires (2013)]
$pp \rightarrow t\bar{t}$	[Czakon, Fiedler, Mitov (2013)]
$gg \rightarrow Hg$	[Boughezal, Caola, Melnikov, Petriello, Schulze (2013)]
$pp \rightarrow Z\gamma$	[Grazzini, Kallweit, Rathlev, Torre (2013)]
$pp \rightarrow tj$	[Bruchseifer, Caola, Melnikov (2014)]
$pp \rightarrow ZZ$	[Cascioli, Gehrmann, Grazzini, Kallweit, von Manteuffel, Pozzorini, Rathlev, Tancredi, Weihs (2014)]
$pp \rightarrow HH$	[de Florian, Mazzitelli (2014)]
$pp \rightarrow ZH$	[Ferrera, Grazzini, Tramontano (2014)]

“Impossible” $t\bar{t}$ total cross sections at NNLO have been computed



Bärnreuther, Czakon, Fiedler, Mitov

Theoretical methods for modern PQCD

Traditional analytic derivation of squared matrix elements $|M|^2$ fails \Rightarrow
Too complex expressions

Modern approaches derive scattering amplitudes M using recursive and numeric techniques, massively parallel computations

- Recursive evaluation of Feynman integrals (*La Porta algorithm*)

- Reduction of tensor structures in Feynman integrals

(*Denner, Dittmaier, Binoth, Ciccolini, Heinrich; ...*)

- Construction of Feynman amplitudes based on **generalized unitarity**

(*Bern, Dixon, Dunbar, Forde, Kosower; Britto, Cachazo, Feng; Badger; Ellis, Giele, Kunzst, Melnikov, Zanderighi; Ossola, Papadopoulos, Pittau; ...*)

Theoretical methods for modern PQCD

Traditional analytic derivation of squared matrix elements $|M|^2$ fails \Rightarrow
Too complex expressions

Identification and removal of IR singularities

1. Soft and collinear subtractions

NLO: Catani-Seymour dipole formalism

NNLO: antenna subtraction

(Boughezal, Daleo, Gehrmann-De Ridder, Gehrmann, Glover, Luisoni, Maitre, Monni, Pires, Ritzman)

2. Phase space slicing

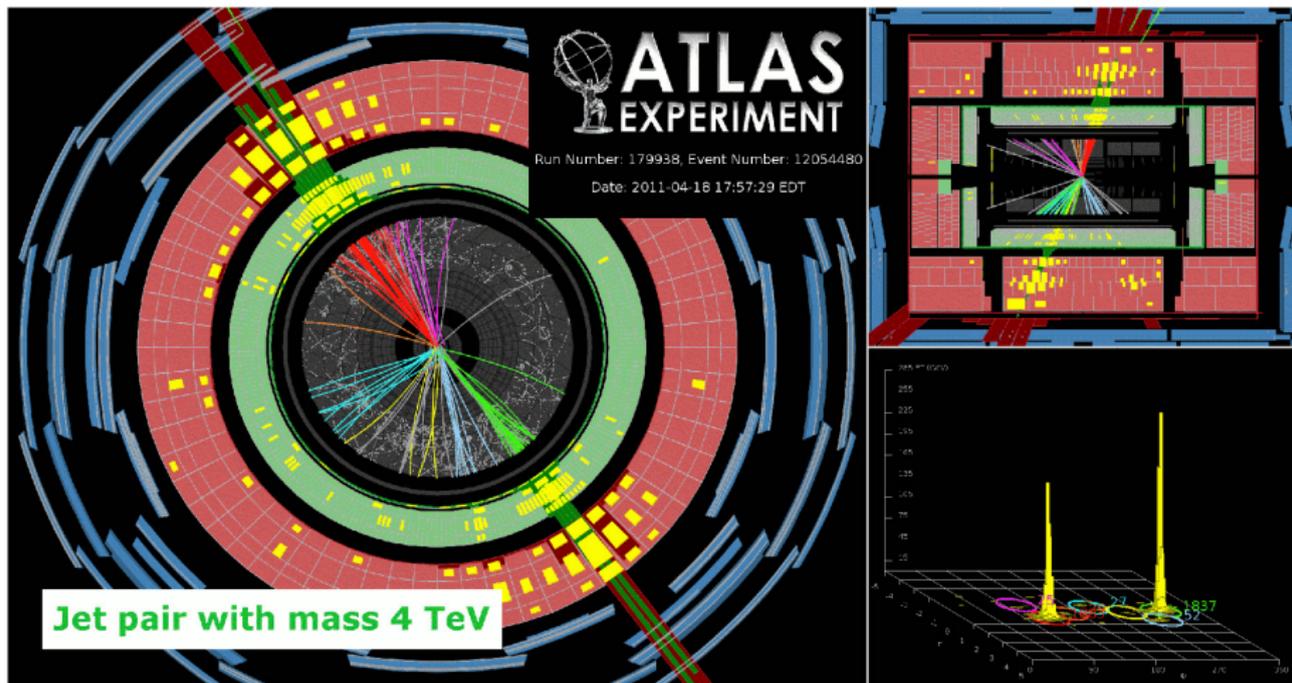
NLO: many implementations

NNLO: FKS-improved sector decomposition

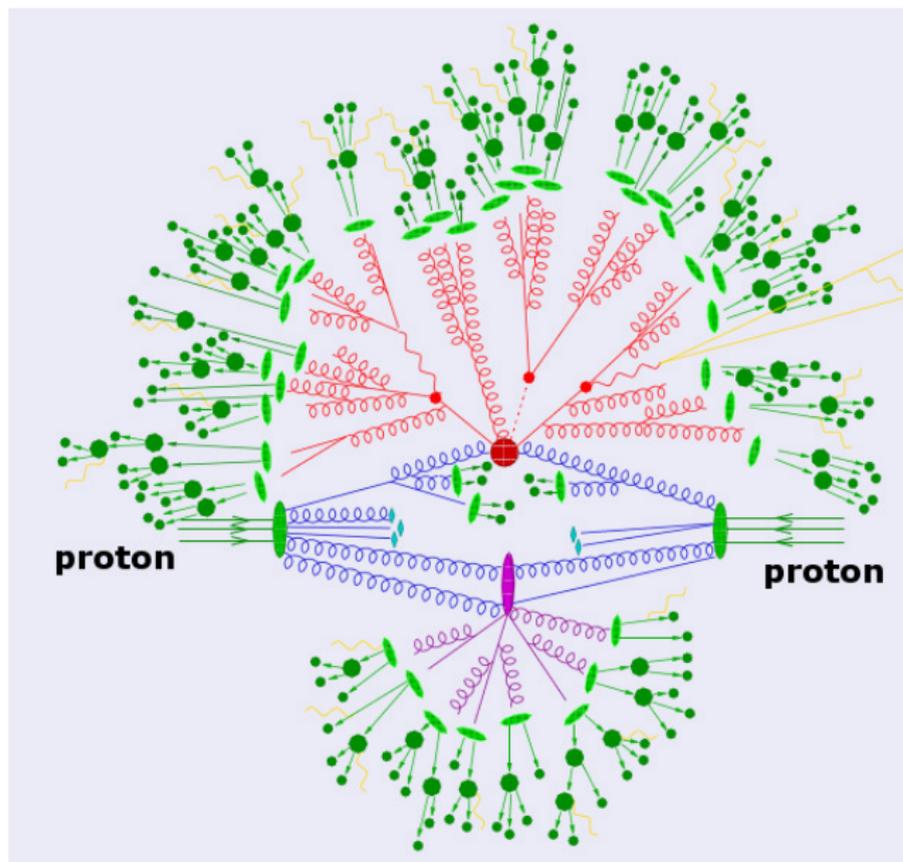
(Czakon; Boughezal, Melnikov, Petriello);

Q_T -dependent slicing *(Czakon) [Boughezal, Melnikov, Petriello] Catani, Grazzini, ...)*

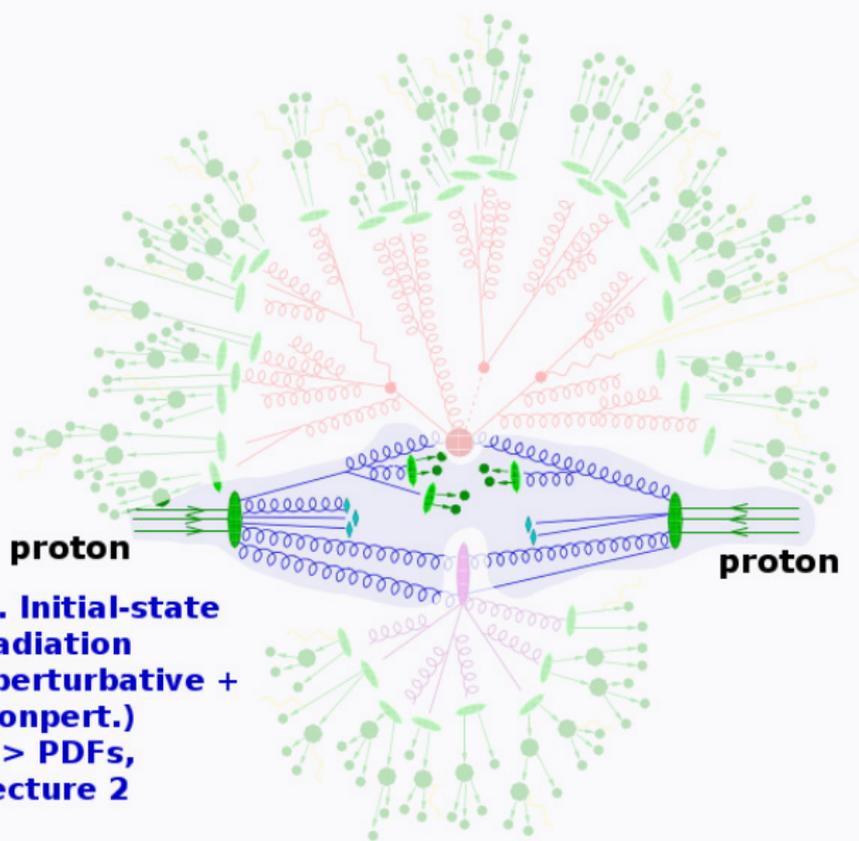
Example 3: Hadronic jet production at ATLAS



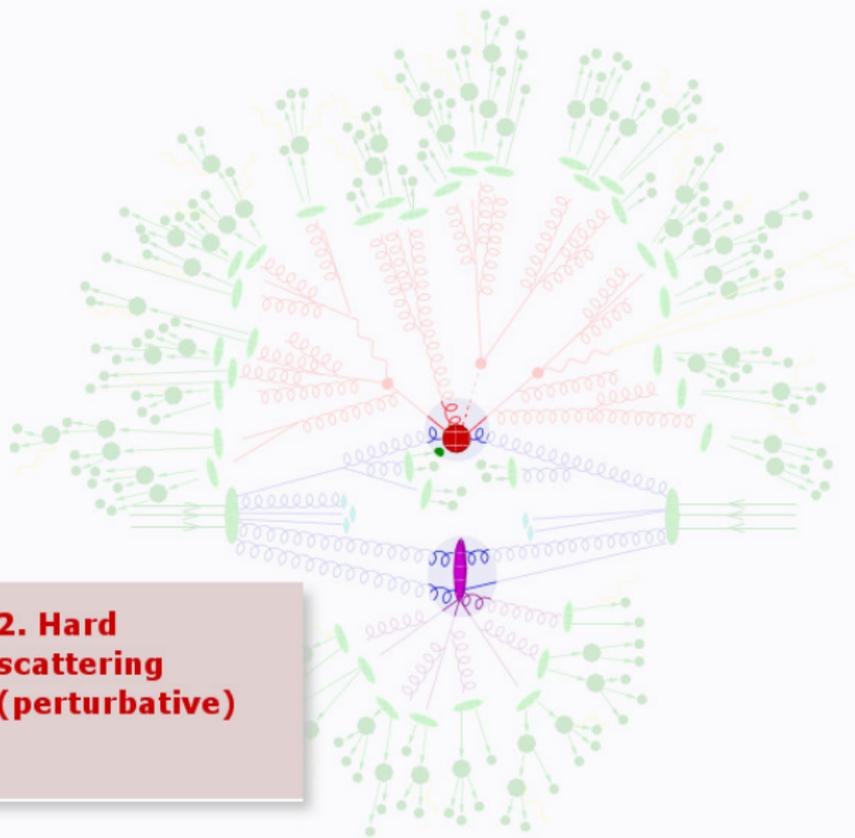
Structure of the collision event



Structure of the collision event



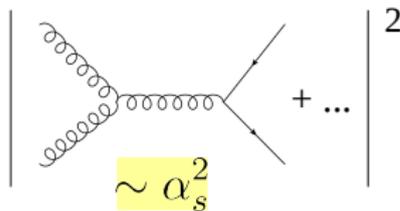
Structure of the collision event



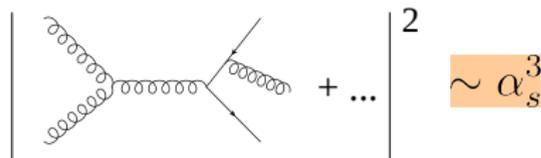
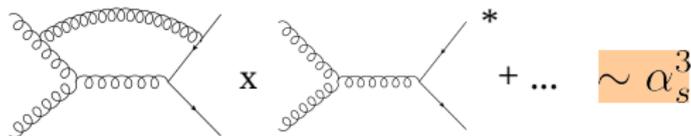
Jet hard cross sections are known at NLO. An NNLO calculation is in progress and requires completely new techniques that have not been available even at NLO.

Fixed-order corrections at NNLO

$$\sigma = \alpha_s^{n_0} \times [\sigma_{(0)} + \alpha_s \sigma_{(1)} + \alpha_s^2 \sigma_{(2)} + \dots]$$

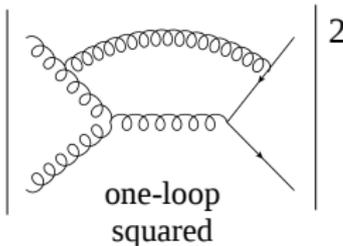
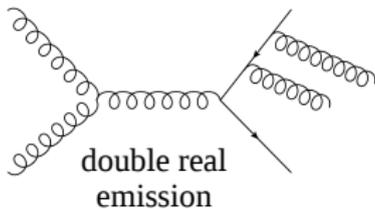
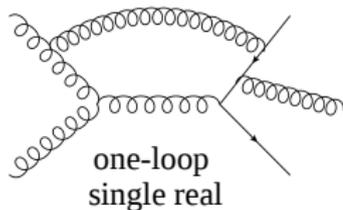
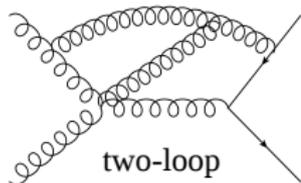


2



Fixed-order corrections at NNLO

$$\sigma = \alpha_s^{n_0} \times [\sigma_{(0)} + \alpha_s \sigma_{(1)} + \alpha_s^2 \sigma_{(2)} + \dots]$$



Fixed-order corrections at NNLO

$$\sigma_{(2)}^X = \frac{\#}{\varepsilon^4} + \frac{\#}{\varepsilon^3} + \frac{\#}{\varepsilon^2} + \frac{\#}{\varepsilon^1} + \# \text{ finite}$$

$X = \{ \text{two-loop, double real, one-loop single real, one-loop squared} \}$

One major difficulty at NNLO:
Extraction of (infrared) $1/\varepsilon$ poles

$$\begin{aligned} \widehat{\sigma}_{ij \rightarrow n}^{\delta NNLO} &= \int_n \left(d\sigma^{VV} + \int_1 d\sigma^{S_1(RV)} + \int_2 d\sigma^{S_2(RR)} \right) \\ &+ \int_{n+1} \left(d\sigma^{RV} - d\sigma^{S_1(RV)} + \int_1 d\sigma^{S_1(RR)} \right) \\ &+ \int_{n+2} \left(d\sigma^{RR} - d\sigma^{S_1(RR)} - d\sigma^{S_2(RR)} \right) \end{aligned}$$

[J.Currie]

	analytic	FS colour	IS colour	local
antenna subtraction	✓	✓	✓	✗
STRIPPER	✗	✓	✓	✓
q_T subtraction	✓	✗	✓	✓
reverse unitarity	✓	✗	✓	-
Trócsányi et al	✗	✓	✗	✓

Fixed-order corrections at NNLO

- Most Recent progress for $2 \rightarrow 2$ processes at NNLO with colored initial and final state was only possible thanks to the appearance of *new* techniques.
- None of the methods used for $pp \rightarrow V,H$ and $pp \rightarrow VV, VH, HH$ would work for any of the $pp \rightarrow jj, Hj, ttbar, tj$ processes. Bottleneck: extraction of poles
- New techniques:

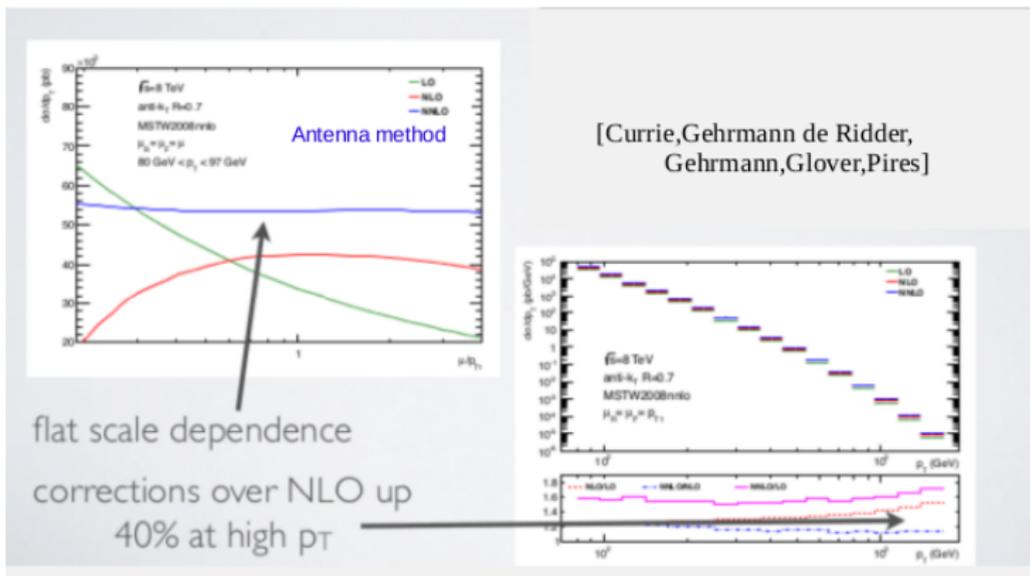
- “***FKS-improved sector decomposition***”
partitioning of the phase space + clever parameterization

[Czakon]
[Boughezal, Melnikov, Petriello]

- “***Antenna subtraction method***”
sophisticated version of dipole subtraction at NNLO

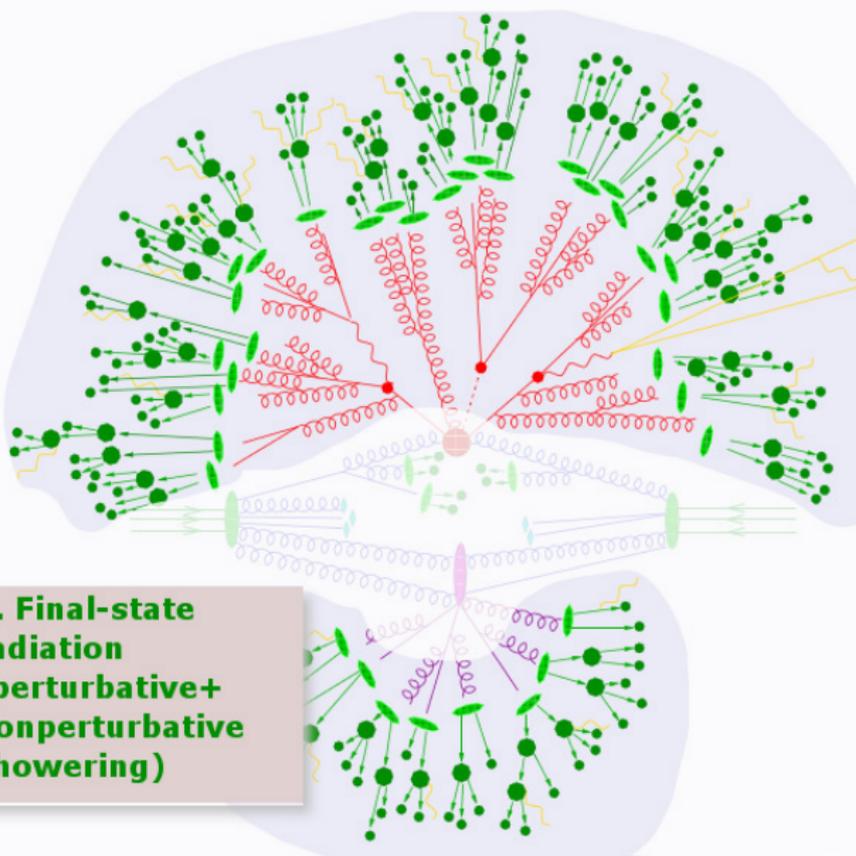
[Boughezal, Daleo, Gehrmann-De Ridder, Gehrmann,
Glover, Luisoni, Maitre, Monni, Pires, Ritzman]

Preliminary NNLO jet cross sections



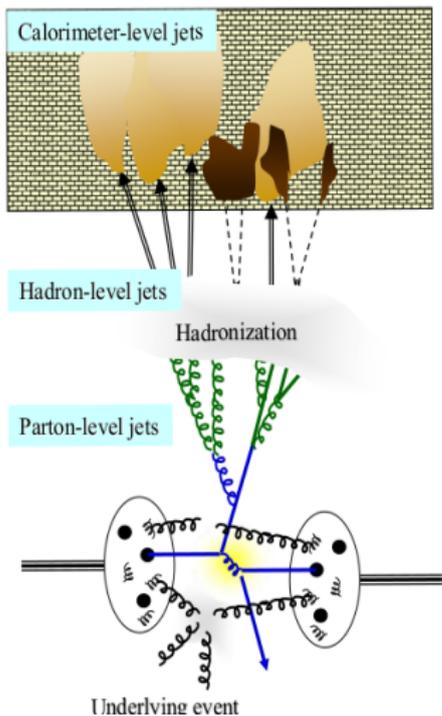
Note that this comparison uses a non-optimal QCD scale equal to p_T of the hardest jet. This may enhance the NNLO correction, as compared to the conventional scale equal to p_T of the single-inclusive jet in the bin.

Structure of the collision event



Dramatic advances
in jet algorithms and
understanding of jet
structure.

Jet Production



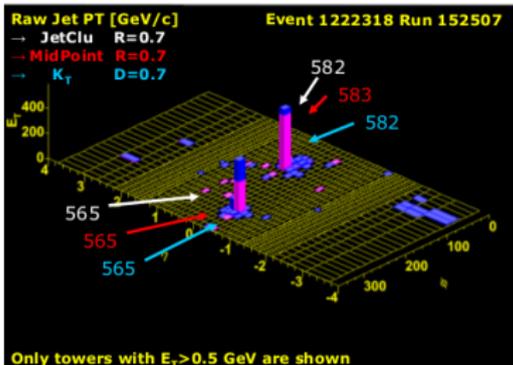
- Jets are collimated spray of hadrons originating from quarks/gluons coming from the hard scattering
(Jets are experimental signatures of quarks and gluons)
- Unlike photons, leptons etc, jets have to be defined by an algorithm for quantitative studies
- Need a well-defined algorithm that gives close relationship between calorimeter-level jets, hadron-level jets, and parton-level jets

Jet Clustering Algorithms

- Algorithms should be well-defined so that they map the experimental measurements with theoretical calculations as close as possible.
- Different algorithms with different parameters provide different sets of resulting jets.

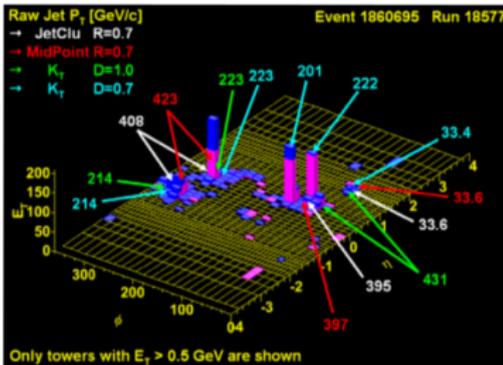
“Simple” event

(all algorithms give essentially the same results)



“Complicated” event

(Resulting jets depend on jet algorithms)



Jet “Definitions” - Algorithms at CDF

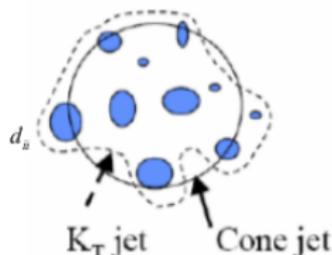
k_T algorithm

- Cluster objects in order of increasing their relative transverse momentum (k_T)

$$\square \quad d_{ii} = p_{T,i}^2, \quad d_{ij} = \min(p_{T,i}^2, p_{T,j}^2) \frac{\Delta R^2}{D^2}$$

until all objects become part of jets

- D parameter controls merging termination and characterizes size of resulting jets
- No issue of splitting/merging. Infrared and collinear safe to all orders of QCD.
- Every object assigned to a jet: concerns about vacuuming up too many particles.



Other clustering algorithm

- $p=1$
 - ◆ the regular k_T jet algorithm

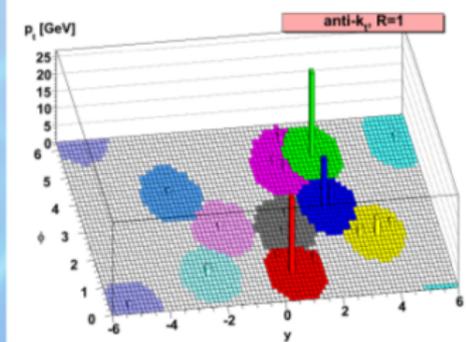
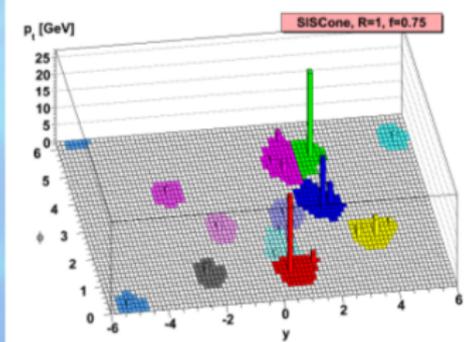
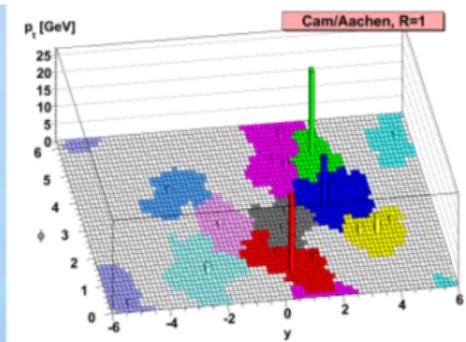
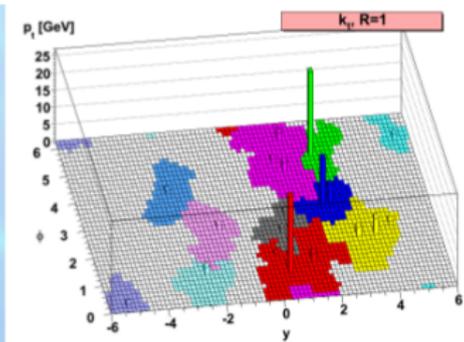
$$d_{ij} = \min(p_{T,i}^{2p}, p_{T,j}^{2p}) \frac{\Delta R_{ij}^2}{D^2}$$

- $p=0$
 - ◆ Cambridge-Aachen algorithm

$$d_{ii} = p_{T,i}^{2p}$$

- $p=-1$
 - ◆ anti- k_T jet algorithm
 - ◆ Cacciari, Salam, Soyez '08
 - ◆ also P-A Delsart '07
 - ◆ soft particles will first cluster with hard particles before clustering among themselves
 - ◆ no split/merge
 - ◆ leads mostly to constant area hard jets

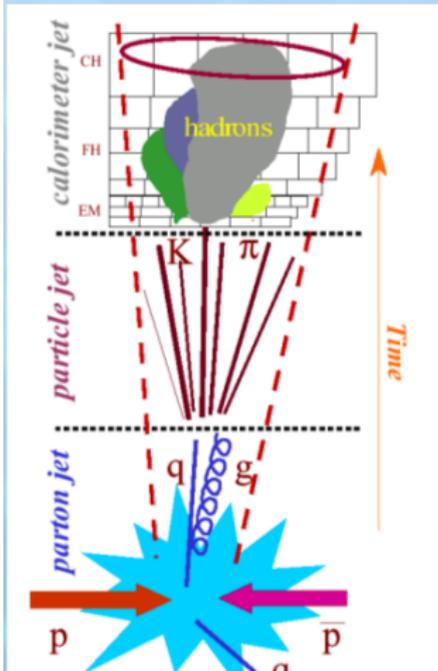
→ • #1 algorithm for ATLAS, CMS



Anti-Kt jet clustering algorithm

arXiv: 0802.1189

Jet Finding



• Calorimeter jet (cone)

- ◆ jet is a collection of energy deposits with a given cone R : $R = \sqrt{\Delta\phi^2 + \Delta\eta^2}$
- ◆ cone direction maximizes the total E_T of the jet
- ◆ various clustering algorithms

- correct for finite energy resolution
- subtract underlying event
- add out of cone energy

• Particle jet

- ◆ a spread of particles running roughly in the same direction as the parton after hadronization

Next lecture: Parton distribution functions

Where PDFs come from? \Rightarrow Global QCD analysis

How to use them properly?

