Problem Set

1. Consider a spin 1/2 massive Majorana fermion whose annihilation rate factor is mass-dependent: $< \sigma v >= 5.0 \times 10^{-21}/m^2 GeV^2\, cm^3/s$. For $m = 300$ GeV [Use $g_{*eff} = 345/4$], find: (a) (Γ/H) at T_f, (b)Ωh^2.

2. For relativistic neutrinos and charged leptons, a typical process that maintains thermal equilibrium, like $\nu_e + e^+ \to \nu_\mu + \mu^+$ has cross-section $\alpha_w^2 E^2/M_W^4$, where E can be written as T since the average energy is proportional to T. Find the interaction rate Γ which is $\sigma|v|n$. (a) Compare the interaction rate with Hubble expansion rate $H = 1.66\sqrt{g_*}\, T^2/M_{\rm Pl}$ (where $M_{\rm Pl} = 2.44 \times 10^{18}$ GeV) and determine the decoupling temperature. (b) Find the ratio at present of neutrino temperature to photon temperature. (c) Find the relic density (Ωh^2) for neutrino in terms of neutrino mass m. Assuming that the neutrinos do not overclose the universe (i.e., $\Omega h^2 < 1$), find the bound on neutrino mass.