Radiative contributions to vector boson production

Pavel Nadolsky

Southern Methodist University
Dallas, Texas

July 13, 2011

Born cross section with decay effects

From yesterday's lecture: the Born cross section for
$q(p_a)\bar{q}(p_b) \to \gamma^*(q) \to e^-(p_1)e^+(p_2)$ is

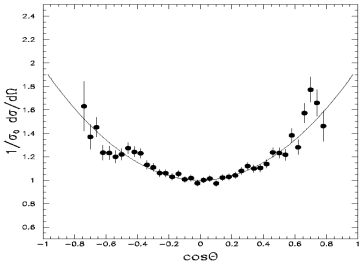
$$\frac{d\sigma}{dQ^2 dy d\Omega} = \frac{1}{16\pi N_c^2 s} \frac{Q^2}{(Q^2 - M_V^2)^2 + \Gamma_V^2 Q^4/M_V^2}$$

$$\times \sum_{j,\bar{k}=u,\bar{u},d,\bar{d},\ldots} \Big\{ (f_R^2 + f_L^2)(g_{L,j\bar{k}}^2 + g_{R,j\bar{k}}^2)(1+\cos^2\theta_*)\left[q_j(x_A)\bar{q}_{\bar{k}}(x_B) + \bar{q}_{\bar{k}}(x_A)q_j(x_B)\right]$$

$$+(f_R^2 - f_L^2)(g_{L,j\bar{k}}^2 - g_{R,j\bar{k}}^2)(2\cos\theta_*)\left[q_j(x_A)\bar{q}_{\bar{k}}(x_B) - \bar{q}_{\bar{k}}(x_A)q_j(x_B)\right] \Big\}$$

- The $(1+\cos\theta_*^2)$ dependence confirms the vector (spin-1) nature of low-Q Drell-Yan process

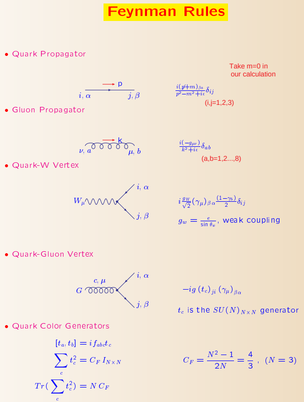
Let's derive it!



Derive the LO cross section for a spin-1 boson

Traditional path

Lagrangian⇒Feynman rules ⇒ $\sum_{spin}|\mathcal{M}|^2$⇒Tr $(\gamma^{\alpha_1}...\gamma^{\alpha_n})$⇒cross section



Helicity amplitudes

Lagrangian⇒"Feynman rules" for helicity amplitudes⇒$\mathcal{M}$⇒$\sum_{spin}|\mathcal{M}|^2$⇒ cross section

- Efficient computation of tree diagrams
- can be applied to 1-loop and 2-loop calculations (not discussed here)

- Many excellent reviews, e.g., *Mangano, Parke, Phys. Rep. 200, 301; Dixon, hep-ph/9601359*

Symmetries of the minimal Standard Model

Forces between particles emerge from the local $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$ symmetry of the quantum Lagrangian, broken as $SU(2)_L \otimes U(1)_Y \to U(1)_{EM}$ by interaction with Higgs scalar field doublet(s)

Spin-1 fields (force carriers)

- photons A^μ (electromagnetism)
- massive bosons $W^{\pm\mu}$, Z^μ (weak force)
- gluons $G^{a,\mu}$ (strong force)

Spin-1/2 fields ψ_f (matter fields)

	Charge		
	QCD	QED	Weak
quarks u, d, s, c, b, t	yes	yes	yes
charged leptons e, μ, τ	no	yes	yes
neutrinos ν_e, ν_μ, ν_τ	no	no	yes

The interaction Lagrangian

$$\mathcal{L}_{int}^{SM} = i\sum_{j,k}\sum_{V}\overline{\psi}_j\left[(1+\gamma_5)g_{R,jkV} + g_{L,jkV}(1-\gamma_5)\right]\gamma^{\mu}V_{\mu}\psi_k,$$

where

- ψ_j are fermion mass eigenstates
  - $(\gamma^{\mu}p_{\mu} - m_j)\psi_j = 0$; j, k run over all quark and lepton flavors
  - the weak and mass eigenstates for down-type quarks and neutrinos are related as

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = V^{CKM}\begin{pmatrix} d \\ s \\ b \end{pmatrix},\; V^{CKM}\left(V^{CKM}\right)^{\dagger} = 1$$

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = V^{MNS}\begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix},\; V^{MNS}\left(V^{MNS}\right)^{\dagger} = 1$$

V^{CKM}, V^{MNS}: mass mixing (Cabibbo-Kobayshi-Maskawa and Maki-Nakagawa-Sakata) matrices

The interaction Lagrangian

$$\mathcal{L}_{int}^{SM} = i \sum_{j,k} \sum_{V} \overline{\psi}_j \left[(1+\gamma_5) g_{R,jkV} + g_{L,jkV} (1-\gamma_5) \right] \gamma^\mu V_\mu \psi_k,$$

where

- $V = A^\mu, \mathcal{G}^\mu, W^{\pm\mu}, Z^\mu$
  - $\mathcal{G}_\mu \equiv G_\mu^a T^a$, T^a is the $SU(3)_C$ generator matrix ($\mathrm{Tr} T^a T^b = \delta^{ab}/2$)
- $g_{L,jkV}$, $g_{R,jkV}$: boson couplings to left- and right-handed fermions

The interaction Lagrangian

$$\mathcal{L}_{int}^{SM} = i\sum_{j,k}\sum_{V}\overline{\psi}_j\left[(1+\gamma_5)g_{R,jkV} + g_{L,jkV}(1-\gamma_5)\right]\gamma^\mu V_\mu \psi_k,$$

where

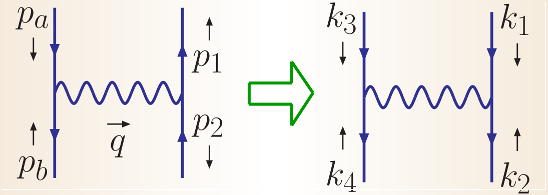
Fermions	Quarks	Leptons
Isospin $I_3 = 1/2$:	u, c, t	ν_1, ν_2, ν_3
$I_3 = -1/2$:	d, s, b	e^-, μ^-, τ^-

$g_{L,jkG} = g_{R,jkG}$	$g\frac{\delta_{jk}}{2}$	0
$g_{L,jkA} = g_{R,jkA}$	$ee_j\frac{\delta_{jk}}{2}$, $e_j \equiv I_3 + 1/6$	$ee_j\frac{\delta_{jk}}{2}$, $e_j \equiv I_3 - \frac{1}{2}$
$g_{L,jkW^+} = g^*_{L,kjW^-}$	$\frac{V_{jk}^{CKM} g_W}{2\sqrt{2}}$	$\frac{V_{jk}^{MNS} g_W}{2\sqrt{2}}$
$g_{R,jkW^+} = g^*_{R,kjW^-}$	0	0
$g_{L,jkZ}$	$\frac{g_W}{2c_W}(I_3 - e_j s_w^2)\delta_{jk}$	$\frac{g_W}{2c_W}(I_3 - e_j s_w^2)\delta_{jk}$
$g_{R,jkZ}$	$-\frac{g_W}{2c_W}e_j s_W^2$	$-\frac{g_W}{2c_W}e_j s_W^2$

$g = \sqrt{4\pi\alpha_S}$,
$e \equiv \sqrt{4\pi\alpha_{EM}}$,
$e = g_W \sin\theta_W$,
$c_W \equiv \cos\theta_W$,
$s_W \equiv \sin\theta_W$

Calculation of $\mathcal{M}\left(q(p_a)\bar{q}(p_b) \to \ell(p_1)\bar{\ell}(p_2)\right)$

1. Crossing



Compute $\mathcal{M}$ in an auxiliary process $\ell(k_1)\bar{\ell}(k_2)q(k_3)\bar{q}(k_4) \to 0$ to simplify the algebra; cross to the physical channel $q(p_a)\bar{q}(p_b) \to \ell(p_1)\bar{\ell}(p_2)$ at the very end

- Denote $k^\mu_{1,2} = -p^\mu_{1,2}$, $k^\mu_{3,4} = p^\mu_{a,b}$;
- Assume $m_i^2 = 0$, $i = 1, .., 4$
- Particle spins are $s_i \equiv \lambda_i/2$, $\lambda_i = \pm 1$
- Convenient notation: $\{k_i, \lambda_i\} \equiv k_i^{\lambda_i}$

Calculation of $\mathcal{M}\left(q(p_a)\bar{q}(p_b) \to \ell(p_1)\bar{\ell}(p_2)\right)$

2. Color decomposition

- Decompose $\mathcal{M}$ into a sum of products of color $SU(N_c)$ factors $\left(T^{a_1}...T^{a_n}\right)_{c_1 c_{n+1}}$ and kinematical partial amplitudes $A_4(k_1^{\lambda_1}, k_2^{\lambda_2}, k_3^{\lambda_3}, k_4^{\lambda_4})$
  - trivial in our case:

$$\mathcal{M}\left(\ell(k_1^{\lambda_1}), \bar{\ell}(k_2^{\lambda_2}), q^{c_3}(k_3^{\lambda_3}), \bar{q}^{c_4}(k_4^{\lambda_4})\right) = \mathcal{I}_{c_3 c_4} A_4(k_1^{\lambda_1}, k_2^{\lambda_2}, k_3^{\lambda_3}, k_4^{\lambda_4})$$

$$\mathrm{Tr}\mathcal{I} = N_c$$

  - general formulas are given in the above references
- $A_n(1...n)$ satisfy several helpful symmetries, which drastically reduce the number of independent amplitudes

$A_n(1, 2, ..., n)$ are gauge-invariant

$$A_n(1, ..., n) = (-1)^n A_n(n, n-1, ...1) \text{ (reflection identity)}$$

$$A_n(1^{\pm}, 2^{+}, ..., n^{+}) = 0 \text{ (effective supersymmetry)}$$

Massless spinor formalism in 4 dimensions

In the massless case, only 2 out of 4 components of the Dirac spinor field $\psi(k,\lambda)$ are independent

Introduce two 4-spinors $|k_i\pm\rangle \equiv |i\pm\rangle$:

$$|i\pm\rangle = u(k_i,\pm 1) = v(-k_i,\mp 1), \quad \langle i\pm| = \overline{u}(k_i,\mp 1) = \overline{v}(-k_i,\pm 1);$$

$$\frac{1}{2}(1\pm\gamma_5)|i\pm\rangle = |i\pm\rangle; \quad \langle i\pm|\frac{1}{2}(1\mp\gamma_5) = \langle i\pm|$$

On-shell conditions

$$\not{k}_i|i\pm\rangle = \langle i\pm|\not{k}_i = 0; \qquad \not{k}_i = |i+\rangle\langle i+| \; + \; |i-\rangle\langle i-|$$

Spinor products

$$\langle i-|j+\rangle \equiv \langle ij\rangle; \; \langle i+|j-\rangle \equiv [ij]$$

$$\langle ij\rangle^* = [ji]$$

$$\langle ij\rangle[ji] = 2k_i\cdot k_j \equiv s_{ij}$$

Tree amplitudes are rational functions of $\langle ij\rangle$ and $[ij]$

Some identities for spinor products

Gordon identity and projection operator:

$$\langle i^{\pm}|\gamma^{\mu}|i^{\pm}\rangle = 2k_i^{\mu}, \qquad |i^{\pm}\rangle\langle i^{\pm}| = \frac{1}{2}(1\pm\gamma_5)\,\not{k}_i \tag{19}$$

antisymmetry:

$$\langle j\,i\rangle = -\langle i\,j\rangle\,, \qquad [j\,i] = -[i\,j], \qquad \langle i\,i\rangle = [i\,i] = 0 \tag{20}$$

Fierz rearrangement:

$$\langle i^{+}|\gamma^{\mu}|j^{+}\rangle\langle k^{+}|\gamma_{\mu}|l^{+}\rangle = 2\,[i\,k]\,\langle l\,j\rangle \tag{21}$$

charge conjugation of current:

$$\langle i^{+}|\gamma^{\mu}|j^{+}\rangle = \langle j^{-}|\gamma^{\mu}|i^{-}\rangle \tag{22}$$

Schouten identity:

$$\langle i\,j\rangle\,\langle k\,l\rangle = \langle i\,k\rangle\,\langle j\,l\rangle + \langle i\,l\rangle\,\langle k\,j\rangle\,. \tag{23}$$

In an n-point amplitude, momentum conservation, $\sum_{i=1}^{n} k_i^{\mu} = 0$, provides one more identity,

$$\sum_{\substack{i=1\\ i\neq j,k}}^{n} [j\,i]\,\langle i\,k\rangle = 0. \tag{24}$$

Exercises

1. In Weyl representation,

$$\gamma^0 = \begin{pmatrix} 0 & \mathbf{I} \\ \mathbf{I} & 0 \end{pmatrix}, \vec{\gamma} = \begin{pmatrix} 0 & -\vec{\sigma} \\ \vec{\sigma} & 0 \end{pmatrix}, \gamma^5 = \begin{pmatrix} \mathbf{I} & 0 \\ 0 & -\mathbf{I} \end{pmatrix},$$

σ_i $(i = 1, 2, 3)$ are 2×2 Pauli matrices. The massless spinors satisfy

$$|p+\rangle = \begin{pmatrix} \xi_+(p) \\ 0 \end{pmatrix}, |p-\rangle = \begin{pmatrix} 0 \\ \xi_-(p) \end{pmatrix};$$

$$\langle p+| = \begin{pmatrix} 0 & \xi_+^\dagger(p) \end{pmatrix}, \langle p-| = \begin{pmatrix} \xi_-^\dagger(p) & 0 \end{pmatrix},$$

where $\xi_\lambda(p)$ is a 2-component spinor for a massless fermion with momentum p and helicity λ, normalized by $\xi^\dagger_{\lambda_1}(p)\xi_{\lambda_2}(p) = 2p^0\delta_{\lambda_1\lambda_2}$. Show that some spinor products vanish:

$$\langle p\pm|q\pm\rangle = \langle p\pm|\gamma^\mu|q\mp\rangle = 0.$$

Exercises

2. One possible representation for $\xi_\pm(p)$ is

$$\xi_\pm(p) = 2^{1/4} \begin{pmatrix} \pm\sqrt{p^\pm} e^{-i\varphi_P/2} \\ \sqrt{p^\mp} e^{i\varphi_p/2} \end{pmatrix},$$

where I introduced light-cone coordinates for p,

$$p^\pm \equiv \frac{p^0 \pm p^3}{\sqrt{2}},\ \vec{p}_T = \left\{ \sqrt{2p^+p^-} \cos\varphi_p,\ \sqrt{2p^+p^-} \sin\varphi_p \right\}.$$

We have $p^2 = 2p^+p^- - p_T^2 = 0$, $p \cdot q = p^+q^- + q^+p^- - \vec{p}_T \cdot \vec{q}_T$, etc.

(a) Check that $\xi^\dagger_{\lambda_1}(p)\xi_{\lambda_2}(p) = 2p^0\delta_{\lambda_1\lambda_2}$.

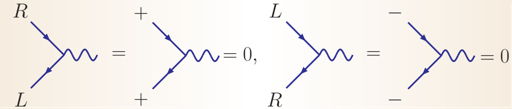
(b) Prove antisymmetry, Gordon identity, Fierz rearrangement on slide 9

Partial amplitudes

The rule

$$\langle p \pm | \gamma^{\mu} | q \mp \rangle = 0$$

reflects chirality conservation in the $\bar{\psi} \not{V} \psi$ vertex:

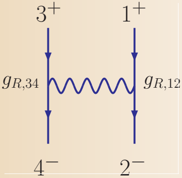


This condition and effective supersymmetry of massless QCD,

$$A_n(1^{\pm}, 2^{+}, ..., n^{+}) = 0,$$

imply that the only non-vanishing LO amplitudes are
$A_4(+-+-)$, $A_4(+--+)$, $A_4(-++-)$, $A_4(-+-+)$.

Partial amplitudes



Denote the couplings as $g_{P,12} \equiv f_P$ and $g_{P,34} \equiv g_P$ for $P = L, R$

$$A_4(+-+-) = -\frac{i}{q^2 - M_V^2} f_R g_R \langle 4+|\gamma^\mu|3+\rangle\langle 2+|\gamma_\mu|1+\rangle$$

$$= -\frac{i}{q^2 - M_V^2} g_{R,12} g_{R,34} [42]\langle 13\rangle$$

$$A_4(+--+) = -\frac{i}{q^2 - M_V^2} f_L g_R [41]\langle 23\rangle$$

$$A_4(-++-) = -\frac{i}{q^2 - M_V^2} f_R g_L [32]\langle 14\rangle$$

$$A_4(-+-+) = -\frac{i}{q^2 - M_V^2} f_L g_L [31]\langle 24\rangle$$

Spin sum

$$\sum_{spin} |A_4|^2 = \frac{1}{\left(q^2 - M_V^2\right)^2} \Big((f_R^2 g_R^2 + f_L^2 g_L^2) s_{42} s_{13} + (f_R^2 g_L^2 + f_L^2 g_R^2) s_{41} s_{23} \Big)$$

$$= \frac{1}{\left(q^2 - M_V^2\right)^2} \Big((f_R^2 g_R^2 + f_L^2 g_L^2) s_{13}^2 + (f_R^2 g_L^2 + f_L^2 g_R^2) s_{14}^2 \Big),$$

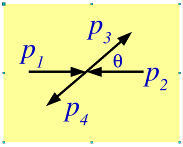
where I used

$$\langle ij \rangle [ji] = 2 p_i \cdot p_j = s_{ij},$$

$$s_{12} = s_{34},\ s_{13} = s_{24},\ s_{14} = s_{23}$$

A rest frame of the vector boson

Return to the physical channel and consider the rest frame of V:



$$p_a = \frac{Q}{2}(1,0,0,1)\,;\; p_b = \frac{Q}{2}(1,0,0,-1)\,;$$
$$p_1 = \frac{Q}{2}(1,0,0,\cos\theta_*)\,;\; p_2 = \frac{Q}{2}(1,0,0,-\cos\theta_*)\,;$$

For $q(p_a)\bar{q}(p_b) : p_a = -k_3,\, p_b = -k_4$

For $\bar{q}(p_a)q(p_b) : p_a = -k_4,\, p_b = -k_3$

$$|\mathcal{M}|^2 = \frac{1}{(q^2 - M_V^2)^2}\frac{Q^4}{4N_c}\Big[(f_R^2 + f_L^2)(g_L^2 + g_R^2)(1+\cos^2\theta_*)$$
$$+\,\epsilon_{q\bar{q}}(f_R^2 - f_L^2)(g_L^2 - g_R^2)(2\cos\theta_*)\Big],$$

$\epsilon_{q\bar{q}} = 1\ (-1)$ for $q\bar{q}$ $(\bar{q}q)$

Inclusive kinematics of the lepton pair

The momenta $p_1^\mu,\, p_2^\mu$ are fully specified by

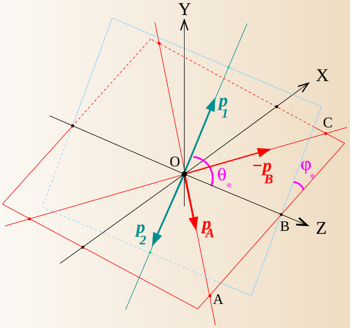
- the mass Q, transverse momentum Q_T, rapidity $y = \frac{1}{2}\ln\left(\frac{q^0+q^3}{q^0-q^3}\right)$ of the intermediate boson V **in the lab frame**
- angles θ_* and φ_* of lepton momenta **in the special rest frame of V** (Collins-Soper frame)

$$\frac{d^3\vec{p}_1}{2p_1^0}\frac{d^3\vec{p}_2}{2p_2^0} = \frac{1}{8}d^4q\,\underbrace{d\cos\theta_* d\varphi_*}_{d\Omega}$$

$$= \frac{\pi}{16}dQ^2 dy dQ_T d\Omega$$

At Born level, $Q_T = 0$

Collins−Soper frame



$\measuredangle AOB = \measuredangle BOC$

$p_A^x, p_B^x \propto -Q_T \qquad p_A^y = p_B^y = 0$

$p_1 = (Q/2)(1, \sin\theta_*\cos\varphi_*, \sin\theta_*\sin\varphi_*, \cos\theta_*)$

$p_2 = (Q/2)(1, -\sin\theta_*\cos\varphi_*, -\sin\theta_*\sin\varphi_*, -\cos\theta_*)$

Covariant definitions for Q_T and y

Exercise. Convince yourself that y and Q_T can be introduced in a covariant form as

$$y = \frac{1}{2}\ln\left(\frac{p_B \cdot q}{p_A \cdot q}\right),$$

$$Q_T^2 = -q_{t\mu} q_t^\mu, \text{ with}$$

$$q_t^\mu \equiv q^\mu - \frac{(p_A \cdot q)}{(p_A \cdot p_B)} p_B^\mu - \frac{(p_B \cdot q)}{(p_A \cdot p_B)} p_A^\mu$$

As a result, they can be a part of the Lorentz-invariant phase space

Born cross section with decay effects

Combining $|\mathcal{M}|^2$ with the phase space element and appropriate PDFs, obtain the full result:

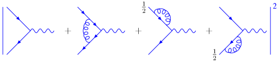
$$\frac{d\sigma}{dQ^2 dy d\Omega} = \frac{1}{16\pi N_c^2 s} \frac{Q^2}{(Q^2 - M_V^2)^2 + \Gamma_V^2 Q^4/M_V^2}$$

$$\times \sum_{j,\bar{k}=u,\bar{u},d,\bar{d},\ldots} \Big\{ (f_R^2 + f_L^2)(g_{L,j\bar{k}}^2 + g_{R,j\bar{k}}^2)(1+\cos^2\theta_*)\left[q_j(x_A)\bar{q}_{\bar{k}}(x_B) + \bar{q}_{\bar{k}}(x_A) q_j(x_B)\right]$$

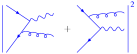
$$+ (f_R^2 - f_L^2)(g_{L,j\bar{k}}^2 - g_{R,j\bar{k}}^2)(2\cos\theta_*)\left[q_j(x_A)\bar{q}_{\bar{k}}(x_B) - \bar{q}_{\bar{k}}(x_A) q_j(x_B)\right] \Big\}$$

NLO cross section

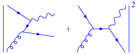
- NLO: $(\alpha_s^{(1)})$ virtual corrections $(q\overline{q'})_{virt}$



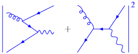
- NLO: $(\alpha_s^{(1)})$ real emission diagrams $(q\overline{q'})_{real}$



- NLO: $(\alpha_s^{(1)})$ real emission diagrams $(qG)_{real}$



- NLO: $(\alpha_s^{(1)})$ real emission diagrams $(G\overline{q'})_{real}$



Virtual contributions

In the first approximation, rescale the LO cross section; do not affect LO kinematics

$$\sigma_{tot}^{NLO} \sim \left[1 + \frac{\alpha_s}{2\pi} C_F \left(1 + \frac{4\pi^2}{3}\right)\right] \sigma_{tot}^{LO}$$
$$\sim \left[1 + 3.005\alpha_s\right] \sigma_{tot}^{LO}$$

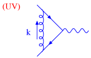
Keep photon's $Q_T = 0$

$2 \to 3$ contributions

Generate $Q_T \neq 0$, non-trivial θ_*, φ_* dependence

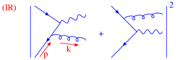
Immediate problems (Singularities)

- Ultraviolet singularity



$$\sim \int^{\infty} d^4k \frac{\not{k}\ \not{k}}{(k^2)\,(k^2)\,(k^2)} \to \infty$$

- Infrared singularities



$$\to \infty$$

as $k^\mu \to 0$ (soft divergence)

or $k^\mu \parallel p^\mu$ (collinear divergence)

- Solutions

Compute H_{ij} in pQCD in $n = 4 - 2\varepsilon$ dimensions (dimensional regularization)

(1) $n \neq 4 \Rightarrow$ UV & IR divergences appear as $\frac{1}{\varepsilon}$ poles in $\sigma_{ij}^{(1)}$ (Feynman diagram calculation)

(2) H_{ij} is IR safe $\Rightarrow$ no $\frac{1}{\varepsilon}$ in H_{ij}
(H_{ij} is UV safe after "renormalization".)

(Similar singularities also exist in virtual diagrams.)

- Treatment of collinear logarithms introduces dependence on the factorization scheme
- Residual soft logarithms in differential distributions may require resummation to all orders in α_s

A quiz on W boson kinematics

Consider $AB \to (W^+ \to e^+\nu_e)X$ decay in the lab frame.
The most probable transverse momentum Q_T of the W boson is

a) $Q_T = \sqrt{s}/2$

b) $Q_T = |\vec{p}_T^{\,e}| + \not{E}_T$

c) $Q_T = 0$

d) $Q_T = 2 - 5$ GeV,
depending on $\sqrt{s}$

A quiz on W boson kinematics

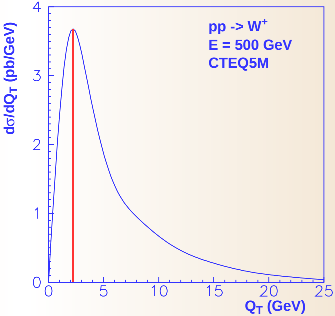
Consider $AB \to (W^+ \to e^+\nu_e)X$ decay in the lab frame.
The most probable transverse momentum Q_T of the W boson is

a) $Q_T = \sqrt{s}/2$

b) $Q_T = |\vec{p}_T^{\,e}| + \not{E}_T$

c) $Q_T = 0$

d) $Q_T = 2 - 5$ GeV, depending on $\sqrt{s}$



The LO condition $Q_T = 0$ (corresponding to no QCD radiation) is never realized because of self-suppression of very soft QCD contributions (Sudakov suppression). To predict $d\sigma/dQ_T$ at $Q_T \ll Q \sim M_W$, one needs to resum such soft contributions to all orders in α_S.

Factorization for one-scale cross sections

Scale dependence of the renormalized QCD charge $g(\mu)$ and fermion masses $m_f(\mu)$:

$$\mu \frac{dg(\mu)}{d\mu} = \beta(g(\mu)), \qquad \mu \frac{dm_f(\mu)}{d\mu} = -\gamma_m(g(\mu))m_f(\mu)$$

The RG equations predict that $\alpha_s(\mu) \to 0$ and $m_f(\mu) \to 0$ as $\mu \to \infty$

These features are employed to prove factorization for inclusive Drell-Yan cross sections *(Bodwin, PRD 31, 2616 (1985); Collins, Soper, Sterman, NPB 261, 104 (1985); B308, 833 (1988))*:

$$\frac{d\sigma(Q,\{m_f\})}{d\tau} = \sum_{a,b} \int_{x_A}^{1} d\xi_A \int_{x_B}^{1} d\xi_B \frac{d\widehat{\sigma}\left(\frac{Q}{\mu}, \frac{\tau}{\xi_A \xi_B}, \{m_f = 0\}\right)}{d\tau} f_{a/A}(\xi_A,\mu) f_{b/B}(\xi_B,\mu)$$
$$+ \mathcal{O}\left(\left\{m_f^2/Q^2\right\}\right)$$

assuming $\mu \sim Q \sim \sqrt{s} \gg \{m_f\}, \Lambda_{QCD}$

Factorization for one-scale cross sections

$$\frac{d\sigma(Q,\{m_f\})}{d\tau} = \sum_{a,b}\int_{x_A}^{1} d\xi_A \int_{x_B}^{1} d\xi_B \,\frac{d\widehat{\sigma}\left(\frac{Q}{\mu}, \frac{\tau}{\xi_A\xi_B}, \{m_f=0\}\right)}{d\tau} f_{a/A}(\xi_A,\mu) f_{b/B}(\xi_B,\mu)$$
$$+\,\mathcal{O}\left(\left\{m_f^2/Q^2\right\}\right)$$

- The hard cross section $\widehat{\sigma}$ is infrared-safe: $\lim_{\{m_f\to 0\}}\widehat{\sigma}(\{m_f\})$ is finite and can be computed as a series in $\alpha_s(\mu)$
- Collinear logarithms are subtracted from $\widehat{\sigma}$ and resummed in $f(\xi,\mu)$ using DGLAP equations
- Soft-gluon singularities in $\widehat{\sigma}$ vanish when the sum of all Feynman diagrams is integrated over all phase space (Kinoshita-Lee-Nauenberg theorem)

Factorization for Q_T distributions (two scales)

- Differential distributions may still contain integrable soft singularities of the type $\alpha_s^k \ln^m(Q^2/p_i\cdot p_j)$, e.g., $L\equiv \ln(Q^2/Q_T^2)\gg 1$:

$$\left.\frac{d\sigma}{dQ^2\,dy\,dQ_T^2}\right|_{Q_T\to 0} \approx \frac{1}{Q_T^2}\Big\{ \alpha_S\,(L+1) + \alpha_S^2\left(L^3+L^2+L+1\right) + \alpha_S^3\left(L^5+L^4+L^3+L^2+L+1\right) + \ldots\Big\}.$$

The purpose of Q_T resummation is to reorganize this series as

$$\left.\frac{d\sigma}{dQ^2dy\,dQ_T^2}\right|_{Q_T\to 0} \approx \frac{1}{Q_T^2}\left\{\alpha_S Z_1 + \alpha_S^2 Z_2 + \ldots\right\},$$

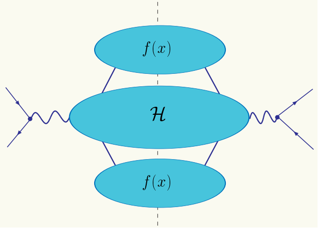
where $\alpha_S^{n+1}Z_{n+1} \ll \alpha_S^n Z_n$:

$$\begin{array}{lcll}
\alpha_S Z_1 & \sim & \alpha_S(L+1) + \alpha_S^2(L^3+L^2) + \alpha_S^3(L^5+L^4) + \ldots & |\,A_1, B_1, \mathcal{C}_0\ ; \\
\alpha_S^2 Z_2 & \sim & \alpha_S^2(L+1) + \alpha_S^3(L^3+L^2) + \ldots & |\,A_2, B_2, \mathcal{C}_1\ ; \\
\alpha_S^3 Z_3 & \sim & \alpha_S^3(L+1) + \ldots & |\,A_3, B_3, \mathcal{C}_2\ . \\
& \ldots & &
\end{array}$$

QCD factorization at large and small Q_T

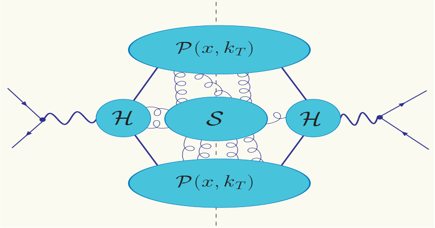
Finite-order (FO) factorization

$$\Lambda^2_{QCD} \ll q_T^2 \sim Q^2$$

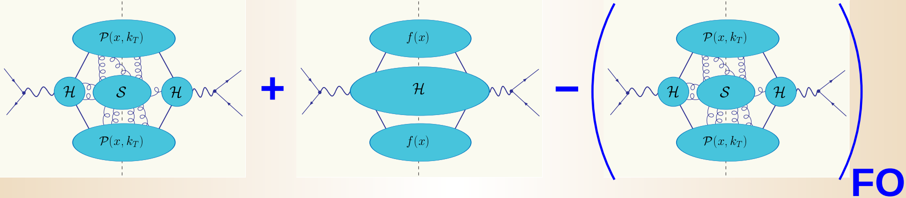


Small-q_T factorization

$$\Lambda^2_{QCD} \ll q_T^2 \ll Q^2$$



Solution for all q_T:



Factorization at $Q_T \ll Q$

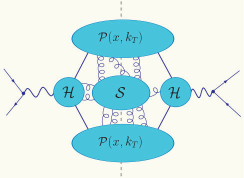
(Collins, Soper, Sterman, 1985)

Realized in space of the impact parameter b

$$\left.\frac{d\sigma_{AB\to VX}}{dQ^2 dy dq_T^2}\right|_{q_T^2 \ll Q^2} = \sum_{flavors} \int \frac{d^2 b}{(2\pi)^2} e^{-i\vec{q}_T \cdot \vec{b}} \widetilde{W}_{ab}(b, Q, x_A, x_B)$$

$$\widetilde{W}_{ab}(b, Q, x_A, x_B) = |\mathcal{H}_{ab}|^2 \, e^{-\mathcal{S}(b,Q)} \overline{\mathcal{P}}_a(x_A, b) \overline{\mathcal{P}}_b(x_B, b)$$

$\mathcal{H}_{ab}$ is the hard vertex, $\mathcal{S}$ is the soft (Sudakov) factor, $\overline{\mathcal{P}}_a(x, b)$ is the unintegrated PDF



For $b \ll 1$ GeV^{-1}, $\widetilde{W}_{ab}(b, Q, x_A, x_B)$ is calculable in perturbative QCD; at $Q \sim M_Z$, this region dominates the resummed cross section

Nonperturbative contributions at large b

At $b \gtrsim 1\ \mathrm{GeV}^{-1}$, the leading nonperturbative contribution is approximated as $\exp\left(-a(Q)b^2\right)$, where $a(Q)$ is an effective "nonperturbative parton $\langle k_T^2\rangle/4$" inside the proton

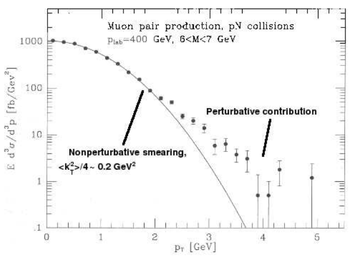
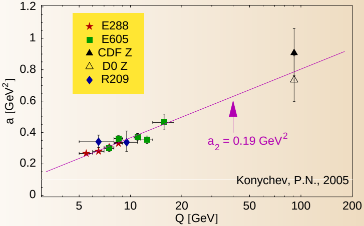


Fig. 9.2. The lepton pair transverse momentum from the CFS collaboration [4]. The curve corresponds to a Gaussian intrinsic k_T distribution for the annihilating

The RG invariance suggests that

$$a(Q) \approx a_1 + a_2 \ln Q,$$

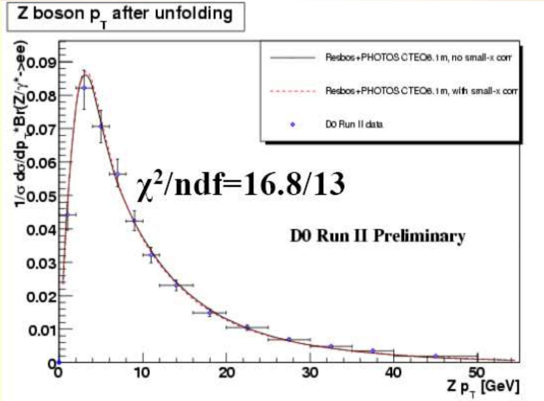
where $a_{1,2} \sim \Lambda^2_{QCD}$, and a_2 is process-independent



The $\ln Q$ growth of $a(Q)$ is indeed observed in the Drell-Yan and Z p_T data

An example of the resummed cross section

Z production at the Tevatron vs. resummed NLO (*Balazs, Ladinsky, PN, Yuan*)



These predictions for $d\sigma/dQ_T$ are employed in M_W measurements